Assignment 4: Introduction to Solid State Physics

1) Use Eqs.(4.3), (4.5), and (4.6) to derive the electrical conductivity, Eq.(4.9). Note that \( N \) in your book corresponds to electron concentration, that is, the number of electrons per unit volume. The notation we have been using for this quantity is \( n \).

2) Why is the collision time \( \tau \) in Eq.(4.6) also called relaxation time? Give a quantitative explanation by showing how to obtain Eq. (4.10)

3) Explain why \( 1/\tau \) in the expression for resistivity, Eq.(4.15), can be written as \( 1/\tau = 1/\tau_{ph} + 1/\tau_{i} \). Which term depends on the temperature and when is it dominant? Estimate the temperature dependence of the resistivity at high temperatures.

4) At first we might expect the total heat capacity in metals to be equal to \( C = 9R/2 \). How do we arrive at this result and why is it wrong? Estimate how to correct it by obtaining Eq. (4.31).

5) Why the plot for the Fermi-Dirac distribution vs. \( E \) at \( T \neq 0 \) is very similar to the distribution function at \( T=0 \)? Sketch the behavior of the distribution.

6) What is the Fermi energy? Find its expression for the 1D and 3D case. What is the Fermi surface?

7) Use Table 4.1 and calculate the Fermi temperature for Cu, Na, and Ag. Also calculate the ratio \( T/T_{F} \) for \( T = 300^\circ K \). Estimate the fraction of electrons excited above the Fermi level at \( T = 20^\circ C \).

8) Derive the thermal conductivity of a metal as caused by the electrons. What is the Lorenz number?

9) Use Lorentz force to derive the relation between the Hall constant and the electron concentration

10) Find the real and imaginary components of the conductivity in the case of an AC field.

11) Derive Eq. (4.19)
12) The atom He\textsuperscript{3} has spin 1/2 and is a fermion. It consists of two protons and one neutron. The density of liquid He\textsuperscript{3} is 0.081 g cm\textsuperscript{-3} near absolute zero. Calculate the Fermi energy and the Fermi temperature.

More about He\textsuperscript{3}: http://en.wikipedia.org/wiki/Helium-3

13) Explain how the energy bands appear in crystals. What are the gaps?

14) What does it mean to say that solid wave functions are delocalized?

15) Use the spin-1/2 Heisenberg model

$$H = J[\sigma^z_n \sigma^z_{n+1} + \sigma^x_n \sigma^x_{n+1} + \sigma^y_n \sigma^y_{n+1}]$$

and find the eigenvectors and eigenvalues for the sector with a single spin pointing up and all the others pointing down.