Sunday, October 28, 2018 2:41 AM

→ In a linear equation

$$y = mx + b$$

$b = y(0)$ and

m is the slope, which is given by the ratio

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

because

$$\left.\begin{aligned} y_1 &= m x_1 + b \\ y_2 &= m x_2 + b \end{aligned}\right\} \quad m(x_2 - x_1) = y_2 - y_1$$

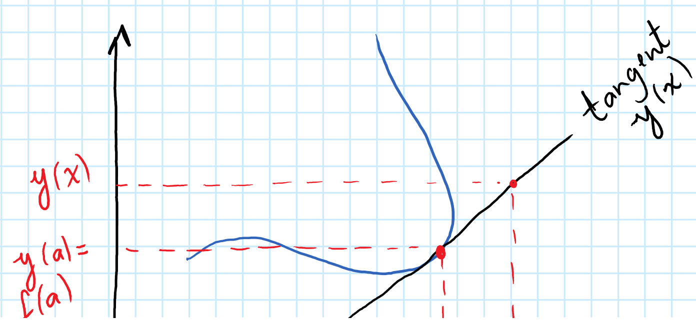
$$\hookrightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$$

→ If x_2 is very close to x_1

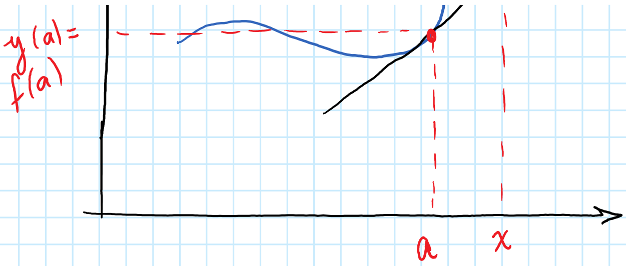
$$\left.\begin{aligned} x_1 &= a \\ x_2 &= a + h \quad \text{with } h \to 0 \end{aligned}\right\} \quad m = y'(x)\Big|_{x=a}$$

that is, m is the derivative of $y'(x)$ at the point $x = a$

→ How to write the tangent to a curve $f(x)$ at the point $x = a$?



The tangent is the linear equation $y(x)$ that passes through the point $(a, f(a))$



The slope for the tangent line is

$$m = f'(x)\Big|_{x=a}$$

Or equivalently,

$$m = \frac{y(x) - y(a)}{x - a}$$

Therefore, the equation for the tangent is

$$\boxed{y(x) = \underbrace{f(a)}_{y(a)} + f'(x)\Big|_{x=a}(x-a)}$$

⇒ The Taylor expansion is a polynomial approximation to the function $f(x)$ at $x = a$

$$\underbrace{f(a) + \frac{f'(x)}{1!}\Big|_{x=a}(x-a)}_{\text{at first order, this polynomial is the tangent line}} + \frac{f''(x)}{2!}\Big|_{x=a}(x-a)^2 + \frac{f'''(x)}{3!}\Big|_{x=a}(x-a)^3 + \dots$$

The higher the order considered, the closer we get to the function.