Computational Methods in Research

Assignment 1

Exercise 1

Another ball dropped from a tower

A ball is dropped from a tower of height h with initial velocity zero. Write a program that asks the user to enter the height in meters of the tower and then calculates and prints the time the ball takes until it hits the ground, ignoring air resistance. Use your program to calculate the time for a ball dropped from a 100 m high tower.

Exercise 2

Altitude of a satellite

A satellite is to be launched into a circular orbit around the Earth so that it orbits the planet once every T seconds. T is the period of the rotational motion of the satellite around Earth.

The altitude h above the Earth's surface that the satellite must have is

$$h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R,$$

where $G = 6.67 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$ is Newton's gravitational constant, $M = 5.97 \times 10^{24}\,\mathrm{kg}$ is the mass of the Earth, and $R = 6371\,\mathrm{km}$ is its radius.

The equation above comes from equating the gravitational force F between the satellite of mass m_{sat} and Earth of mass M

$$F = \frac{GMm_{sat}}{(h+R)^2}$$

with the equation for the centripetal force F felt by the satellite, which depends on its velocity $v = \dfrac{2\pi(R+h)}{T}$ as

$$F = \frac{m_{sat}v^2}{(h+R)}$$

1. Write a program that asks the user to enter the desired value of T and then calculates and prints out the correct altitude in meters. Use for this test $T = 7200s$, which means 2hrs, since $2 \times 60 \times 60 = 7200$.
2. Use your program to calculate the altitudes of satellites that orbit the Earth

   (i) once a day (so-called "geosynchronous" orbit),

   (ii) once every 90 minutes,

   (iii) once every 45 minutes. What do you conclude from the last of these calculations?

   BE CAREFUL WITH THE UNITS!! CONSTANTS and ANSWERS HAVE TO HAVE CONSISTENT UNITS.

Exercise 3

Catalan numbers

The Catalan numbers C_n are a sequence of integers $1, 1, 2, 5, 14, 42, 132\ldots$ that play an important role in quantum mechanics and the theory of disordered systems. (They were central to Eugene Wigner's proof of the so-called semicircle law.) They are given by

$$C_0 = 1, \qquad C_{n+1} = \frac{4n+2}{n+2}\, C_n.$$

Write a program that prints in increasing order all Catalan numbers less than 60000.
NOTE: print the numbers as INTEGERS.

Exercise 4

Dot Product

Write a code to calculate

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} + 2 \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

NOTE: all elements are integers

Exercise 5

Import Data and Manipulate

Import the file "Lec05_RandomVector.txt" as an array. Let us call it "VecRand".

1) Find the maximum, the minimum, and how many elements VecRand has.
2) Sort the elements of VecRand in increasing order.
3) What is the sum of the elements of VecRand?
4) What is the mean value of VecRand?
5) What is the mean of the square of VecRand? [Hint: you can multiply the vector by itself.]
6) What is the variance of VecRand? [Look online what "variance" means]
7) Multiply the third element of VecRand by the last one.
8) What is the dot product of VecRand with itself?
9) Create a new vector with only 5 elements starting from the second element of VecRand. Let us call it "VecSlice".
10) Create a 5x5 matrix called "MatVec", where the first three rows are equal to VecSlice and the last two have only elements "1.".
11) What is the value of
ln[(MatVec$_{2,3}$ + MatVec$_{5,5}$] - cos(20 Degrees)?
Careful! Remember that in Python the first element is the ZERO element.

Exercise 6

Sums and Products: use FOR-LOOPS

1) Compute the sum of the reciprocals of the odd numbers from 1 to 31.
2) Compute the sum of the first 20 numbers that are multiples of 10 (starting from 10).
3) Compute the product of the square root of the first 10 even numbers.
4) Compute the factorial of 8.
5) Compute the binomial coefficient

$$C(n,k) = \frac{n!}{k!(n-k)!}$$

where $n = 10$ and $k = 4$.

6) The binomial coefficient can also be expressed as

$$\left(\frac{n}{k}\right)\left(\frac{n-1}{k-1}\right)\left(\frac{n-2}{k-2}\right)\cdots\left(\frac{n-k+1}{1}\right)$$

Use this expression in the for-loop and confirm that your result agrees with the result in the exercise above for $n = 10$ and $k = 4$.

Exercise 7

Construction of vectors and matrices

NOTE: For this Exercise 7, you cannot just type the elements! The idea is to use FOR-LOOPs. You can also use arrays available at numpy, such as "zero" and "one".

1) Construct a vector with 30 elements, such that the first and last elements are equal to 10 and the remaining elements alternate between 2 and -1,
$\{10, 2, -1, 2, -1, 2, -1, \ldots 10\}$

2) Construct a 5x5 matrix where each element is the sum of the PYTHON indexes that indicate its position in the row and in the column.

3) Construct a 5x5 matrix that looks like this

$$\begin{pmatrix} 7 & 7 & 7 & 7 & 7 \\ 7 & 0 & 0 & 0 & 7 \\ 7 & 0 & 0 & 0 & 7 \\ 7 & 0 & 0 & 0 & 7 \\ 7 & 7 & 7 & 7 & 7 \end{pmatrix}$$

4) Construct a 5x5 matrix that looks like this

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Exercise 8

Dot product between a matrix and a vector done manually

Given the matrix

$$mat = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

and the vector

$$vec = \begin{pmatrix} 6 \\ 5 \\ 4 \end{pmatrix}$$

Use FOR-LOOPs to compute

$$mat.vec$$

Compare your result with the dot product from numpy.

Exercise 9

Dot product between two matrices done manually

Given the matrix

$$mA = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

and the matrix

$$mB = \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix}$$

Use FOR-LOOPs to compute

$$mA.mB$$

Compare your result with the dot product from numpy.

Exercise 10 (2.7 from the book)

The semi-empirical mass formula

In nuclear physics, the semi-empirical mass formula is a formula for calculating the approximate nuclear binding energy B of an atomic nucleus with atomic number Z and mass number A:

$$B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \frac{a_5}{A^{1/2}},$$

where, in units of millions of electron volts, the constants are $a_1 = 15.8$, $a_2 = 18.3$, $a_3 = 0.714$, $a_4 = 23.2$, and

$$a_5 = \begin{cases} 0 & \text{if } A \text{ is odd,} \\ 12.0 & \text{if } A \text{ and } Z \text{ are both even,} \\ -12.0 & \text{if } A \text{ is even and } Z \text{ is odd.} \end{cases}$$

1. Write a program that takes as its input the values of A and Z, and prints out the binding energy for the corresponding atom. Use your program to find the binding energy of an atom with $A = 58$ and $Z = 28$. (Hint: The correct answer is around 490 MeV.)
2. Modify your program to print out not the total binding energy B, but the binding energy per nucleon, which is B/A.
3. Now modify your program so that it takes as input just a single value of the atomic number Z and then goes through all values of A from $A = Z$ to $A = 3Z$, to find the one that has the largest binding energy per nucleon. This is the most stable nucleus with the given atomic number. Have your program print out the value of A for this most stable nucleus and the value of the binding energy per nucleon.
4. Modify your program again so that, instead of taking Z as input, it runs through all values of Z from 1 to 100 and prints out the most stable value of A for each one. At what value of Z does the maximum binding energy per nucleon occur? (The true answer, in real life, is $Z = 28$, which is nickel. You should find that the semi-empirical mass formula gets the answer roughly right, but not exactly.)