Indicators of many-body quantum chaos and time scales for equilibration

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DYNAMICS

Quench Dynamics

Quench Dynamics:

$$H = H_0 + \lambda V$$

Total Hamiltonian
Describes the system
Determines the evolution

H |α> = E_α |α>

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle$$

Perturbation

Dynamics: Survival Probability

Survival Probability
Return Probability
Fidelity

$$\left|\langle \Psi(0) \mid \Psi(t) \rangle\right|^2 = \left|\langle \Psi(0)| e^{-iHt} |\Psi(0)\rangle\right|^2$$

~~Loschmidt Echo~~

$$LE(t) = \left|\langle \Psi(0)| e^{i(H+\delta)t} e^{-iHt} |\Psi(0)\rangle\right|^2$$

Survival Probability: Numerics

Survival Probability
Return Probability
Fidelity

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\langle \Psi(0) | e^{-iHt} | \Psi(0) \rangle\right|^2$$

$$SP(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\sum_\alpha \left|C_\alpha^{ini}\right|^2 e^{-iE_\alpha t}\right|^2$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle$$

$$H = H_0 + \lambda V$$

For H written in the basis of eigenstates of H_0

	α_1	α_2	α_3	α_4
	C_1^1	C_2^1	C_3^1	C_4^1
$\Psi(0)=$	C_1^2	C_2^2	C_3^2	C_4^2
	C_1^3	C_2^3	C_3^3	C_4^3
	C_1^4	C_2^4	C_3^4	C_4^4

Survival Probability: Numerics

Survival Probability
Return Probability
Fidelity

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\langle \Psi(0)| e^{-iHt} |\Psi(0)\rangle\right|^2$$

$$SP(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\sum_\alpha \left|C_\alpha^{ini}\right|^2 e^{-iE_\alpha t}\right|^2$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle$$

$$SP(t) = \left(\sum_\alpha |C_\alpha^{ini}|^2 \cos(E_\alpha t)\right)^2 + \left(\sum_\alpha |C_\alpha^{ini}|^2 \sin(E_\alpha t)\right)^2$$

Survival Probability: Universal Behavior at Short Times

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\langle \Psi(0)| e^{-iHt} |\Psi(0)\rangle\right|^2$$

$$|\langle \Psi(0)|e^{-iHt}|\Psi(0)\rangle|^2 = \langle \Psi(0)|(1 - iHt - \frac{H^2}{2}t^2|\Psi(0)\rangle\langle \Psi(0)|(1 + iHt - \frac{H^2}{2}t^2|\Psi(0)\rangle$$

$$SP(t) = 1 - \Gamma^2 t^2$$

$$\sum_{\alpha} |\alpha\rangle\langle\alpha|$$

$$\langle \Psi(0)|H^2|\Psi(0)\rangle - \langle \Psi(0)|H|\Psi(0)\rangle^2$$

$$\sum_{\alpha} |C_{ini}^{\alpha}|^2 E_{\alpha}^2 - \left(\sum_{\alpha} |C_{ini}^{\alpha}|^2 E_{\alpha}\right)^2 = \sum_{\alpha} |C_{ini}^{\alpha}|^2 (E_{\alpha} - E_{ini})^2$$

$$\sum_{k} \langle \Psi(0)|H|\phi_k\rangle\langle \phi_k|H|\Psi(0)\rangle - \langle \Psi(0)|H|\Psi(0)\rangle^2 = \sum_{k \neq ini} |\langle \phi_k|H|\Psi(0)\rangle|^2$$

Hamiltonian matrix: Spin-1/2 model NN couplings

$$H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \sigma_n^z \sigma_{n+1}^z)$$

	110000	101000	100100	100010	100001	011000	010100	010010	010001	001100	001010	001001	000110	000101	000011
110000	H_{11}	J/2	0	0	0	0	0	0	0	0	0	0	0	0	0
101000	J/2	H_{22}	J/2	0	0	J/2	0	0	0	0	0	0	0	0	0
100100	0	J/2	H_{33}	J/2	0	0	J/2	0	0	0	0	0	0	0	0
100010	0	0	J/2	H_{44}	J/2	0	0	J/2	0	0	0	0	0	0	0
100001	0	0	0	J/2	H_{55}	0	0	0	J/2	0	0	0	0	0	0
011000	0	J/2	0	0	0	H_{66}	J/2	0	0	0	0	0	0	0	0
010100	0	0	J/2	0	0	J/2	H_{77}	J/2	0	J/2	0	0	0	0	0
010010	0	0	0	J/2	0	0	J/2	H_{88}	J/2	0	J/2	0	0	0	0
010001	0	0	0	0	J/2	0	0	J/2	H_{99}	0	0	J/2	0	0	0
001100	0	0	0	0	0	0	J/2	0	0	H_{1010}	J/2	0	0	0	0
001010	0	0	0	0	0	0	0	J/2	0	J/2	H_{1111}	J/2	J/2	0	0
001001	0	0	0	0	0	0	0	0	J/2	0	J/2	H_{1212}	0	J/2	0
000110	0	0	0	0	0	0	0	0	0	0	J/2	0	H_{1313}	J/2	0
000101	0	0	0	0	0	0	0	0	0	0	0	J/2	J/2	H_{1414}	J/2
000011	0	0	0	0	0	0	0	0	0	0	0	0	0	J/2	H_{1515}

Survival Probability and LDOS

Survival Probability
Return Probability
Fidelity

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\langle \Psi(0) | e^{-iHt} | \Psi(0) \rangle\right|^2$$

$$SP(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 e^{-iE_{\alpha} t}\right|^2 \cong \left|\int \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$

Survival probability is the Fourier transform of the LDOS

$$|\Psi(0)\rangle = \sum_{\alpha} C_{\alpha}^{ini} |\alpha\rangle$$

$$\rho_{ini}(E) = \sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 \delta(E - E_{\alpha})$$

Energy distribution of the initial state
LDOS (Local density of states)
Strength function

LFS, Borgonovi, Izrailev
PRL **108**, 094102 (2012)
PRE **85**, 036209 (2012)
Torres, Vyas, LFS
NJP**16**, 063010 (2014)

Torres & LFS
PRA **89**, 043620 (2014)
PRA **90**, 033623 (2014)
AIP**1619**, 171 (2014)

Survival Probability for GOE matrices

Quench Dynamics: $$H = H_0 + \lambda V$$

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

H_0 Diagonal matrix of the GOE full random matrix

V Off-diagonal matrix of the GOE full random matrix

Survival Probability for GOE matrices

Quench Dynamics: $H = H_0 + \lambda V$

$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$

	α_1	α_2	α_3	α_4
	C_1^1	C_2^1	C_3^1	C_4^1
$\Psi(0)=$	C_1^2	C_2^2	C_3^2	C_4^2
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H_0 Diagonal matrix of the GOE full random matrix

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$$SP(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\sum_\alpha \left|C_\alpha^{ini}\right|^2 e^{-iE_\alpha t}\right|^2 \cong \left|\int \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$

$$\rho_{ini}(E) = \sum_\alpha \left|C_\alpha^{ini}\right|^2 \delta(E - E_\alpha)$$

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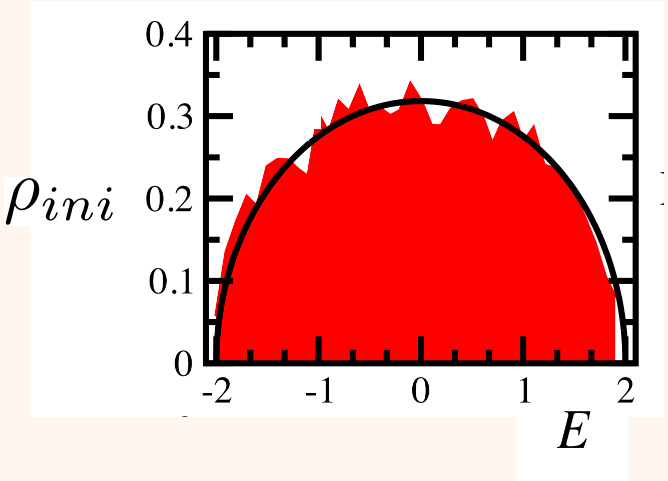
Full GOE Random Matrix

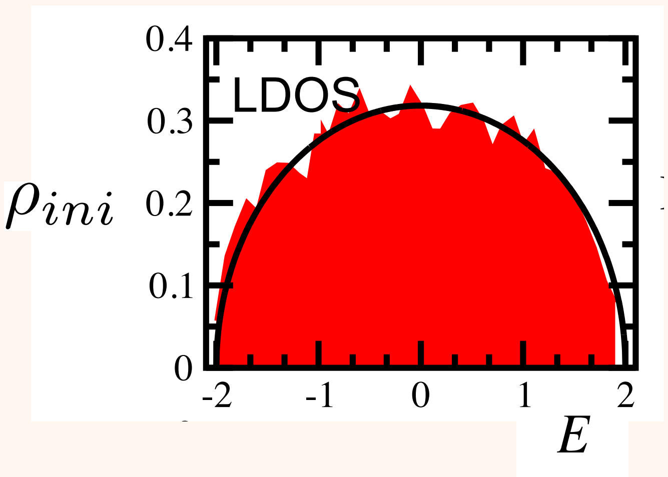
Full Random Matrices



$$DOS = \sum_{\alpha} \delta(E - E_{\alpha})$$

Full GOE Random Matrix

Full Random Matrices

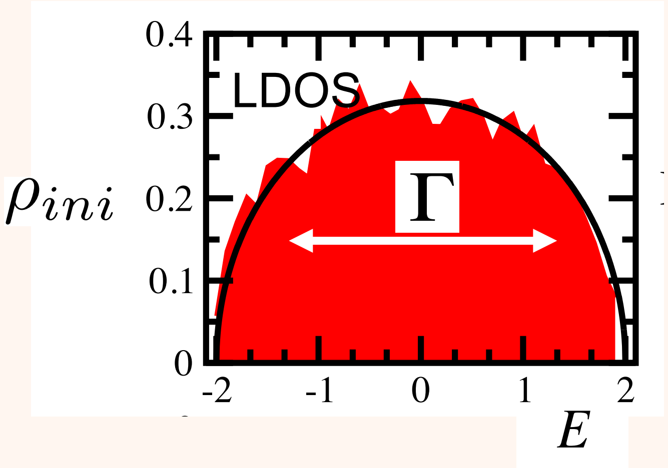


$$DOS = \sum_{\alpha} \delta(E - E_{\alpha})$$

$$\rho_{ini}(E) = \sum_{\alpha} \left| C_{\alpha}^{ini} \right|^{2} \delta(E - E_{\alpha})$$

Full GOE Random Matrix

Full Random Matrices



$$DOS = \sum_{\alpha} \delta(E - E_{\alpha})$$

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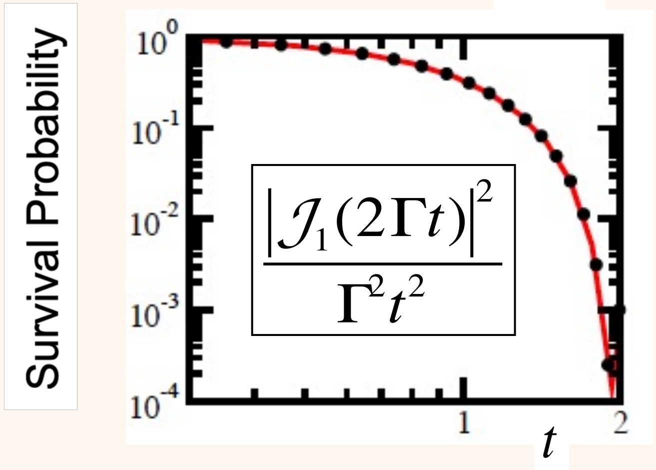
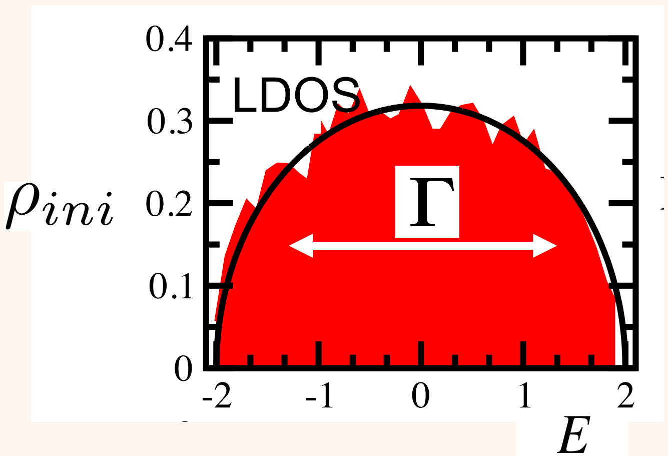


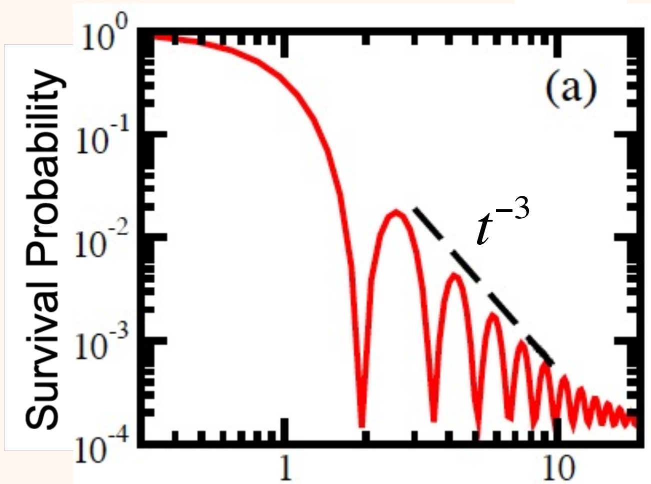
Γ Width of the distribution

Torres-Herrera & LFS
PRA, NJP (2014)

Full GOE Random Matrix

Full Random Matrices





$$DOS = \sum_{\alpha} \delta(E - E_\alpha)$$

$$\rho_{ini}(E) = \sum_{\alpha} \left| C_\alpha^{ini} \right|^2 \delta(E - E_\alpha)$$

Γ Width of the distribution

$$\frac{\left|\mathcal{J}_1(2\Gamma t)\right|^2}{\Gamma^2 t^2} \rightarrow t^{-3}$$

Exercise LDOS-GOE

*) Obtain LDOS for one initial state and one realization of random matrix.
*) Compute SP(t) averaged over 100 initial states with energy in the middle of the spectrum.
*) Compare with the analytical expression.
*) Indicate the **infinite-time average**.

PRA **94**, 041603R (2016)
PRA **95**, 013604 (2017)

Survival Probability: Infinite-time average

$$SP(t) = \left| \sum_{\alpha} |C_{\alpha}^{ini}|^2 e^{-iE_{\alpha}t} \right|^2$$

$$\langle SP(t) \rangle = \langle \sum_{\alpha,\beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_{\alpha}-E_{\beta})t} \rangle$$ **(averages)**

$$\langle SP(t) \rangle = \langle \sum_{\alpha \neq \beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_{\alpha}-E_{\beta})t} \rangle + \langle \sum_{\alpha} |C_{\alpha}^{ini}|^4 \rangle$$

↓

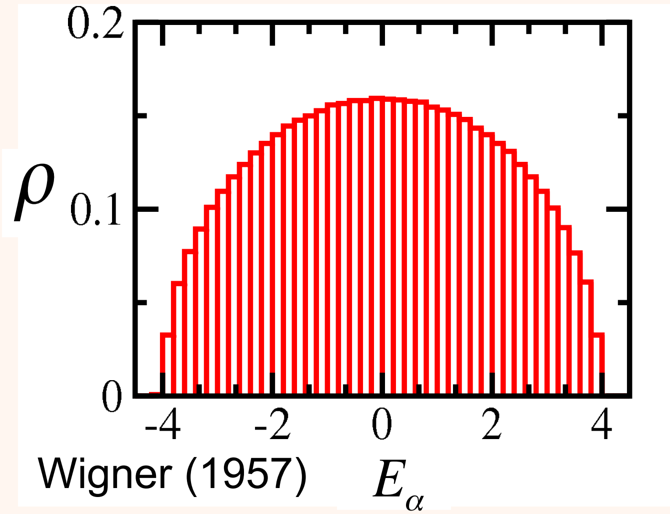
$$IPR^{ini} = \overline{SP} = \frac{3}{D}$$

Full Random Matrices vs Two-Body Interaction

Density of States (Energy Distribution)

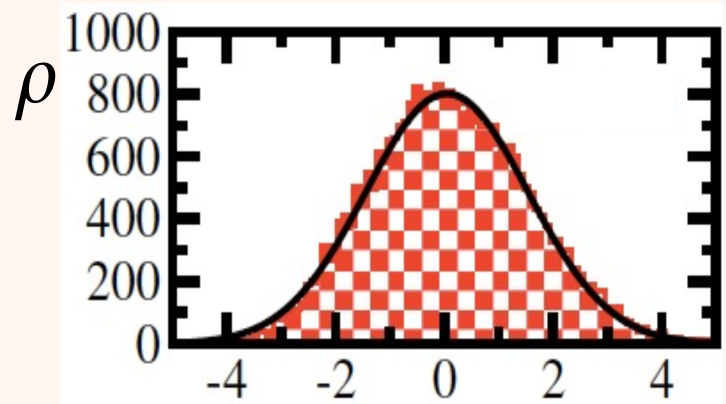
$$DOS = \sum_{\alpha} \delta(E - E_{\alpha})$$

Full random matrices: semicircular



Wigner (1957)

Two-body interactions: Gaussian



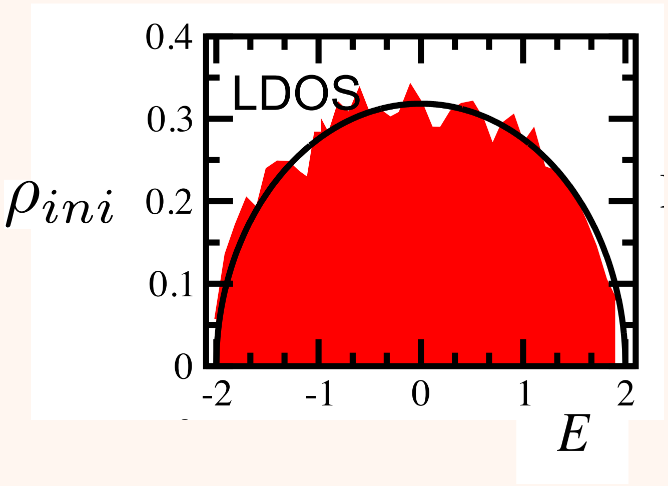
French & Wong, PLB (1970) E_α

Full Random Matrices vs Two-Body Interaction

Local Density of States

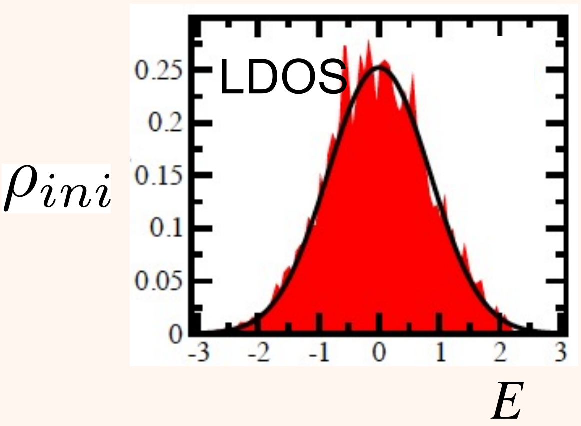
$$\rho_{ini}(E) = \sum_{\alpha} \left| C_{\alpha}^{ini} \right|^2 \delta(E - E_{\alpha})$$

Full random matrices: semicircular



NOT universal

Two-body interactions: Gaussian



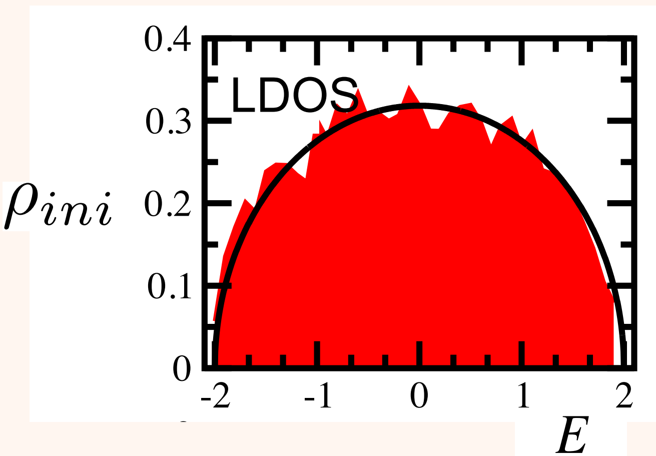
General:

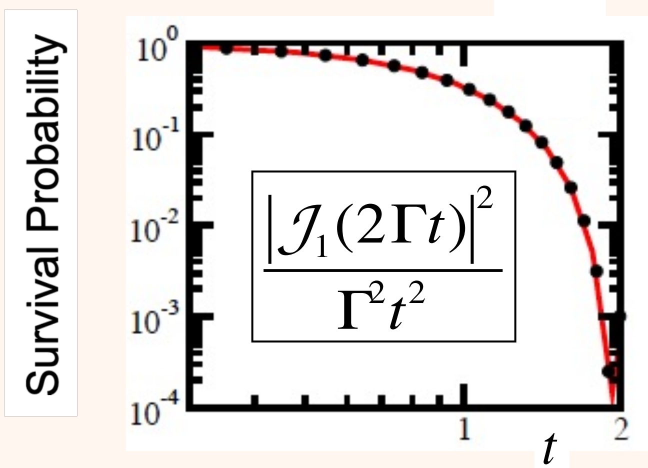
- Chaotic many-body systems with 2-body interactions
- Perturbed far from equilibrium
- Initial state close to middle

Initial Fast Decay

$$\left| \int \rho_{ini}(E) e^{-iEt} \, dE \right|^2$$

Full Random Matrices



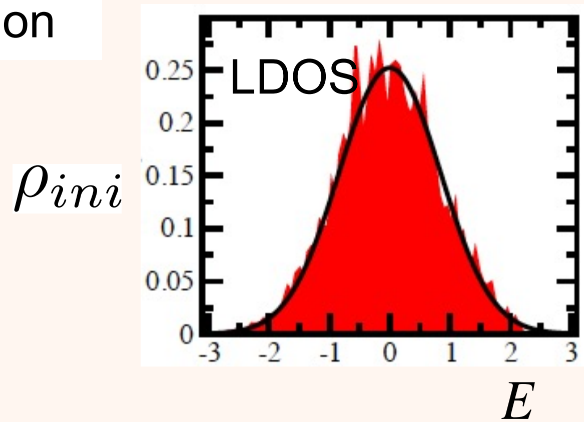


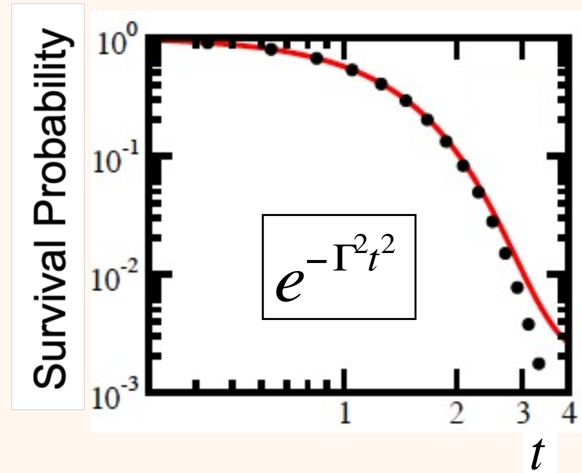
Γ

Width of the distribution

$1/\Gamma$

Spin Model





Torres-Herrera & LFS
PRA, NJP (2014)

Initial State: Néel State

Strong perturbation

$$H_{initial} = \sum_{n=1}^{L-1} S_n^z S_{n+1}^z$$

$$\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$$

Néel state

$$H_{final} = H_{NN} + \lambda H_{NNN}$$

$$H_{NN} = J\sum_{n=1}^{L-1}(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) + J\Delta\sum_{n=1}^{L-1} S_n^z S_{n+1}^z$$

$$H_{NNN} = \left(J\sum_{n=1}^{L-2}(S_n^x S_{n+2}^x + S_n^y S_{n+2}^y) + J\Delta\sum_{n=1}^{L-2} S_n^z S_{n+2}^z \right)$$

$\Delta = 1, \lambda = 0$ ▲ (red)

$\Delta = 0.5, \lambda = 0$ ▼ (blue)

$\Delta = 1, \lambda = 0.4$ X

$\Delta = 1, \lambda = 1$ ● (purple)

$\Delta = 0.5, \lambda = 1$ ■

Far from equilibrium

Initial State: Néel State

Strong perturbation

$$H_{initial} = \sum_{n=1}^{L-1} S_n^z S_{n+1}^z$$

$\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

Néel state

$$H_{final} = H_{NN} + \lambda H_{NNN}$$

$$H_{NN} = J \sum_{n=1}^{L-1} (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) + J\Delta \sum_{n=1}^{L-1} S_n^z S_{n+1}^z$$

$$H_{NNN} = \left(J \sum_{n=1}^{L-2} (S_n^x S_{n+2}^x + S_n^y S_{n+2}^y) + J\Delta \sum_{n=1}^{L-2} S_n^z S_{n+2}^z \right)$$

10101010
directly coupled with L-1 states

01101010
11001010
…
10101100
10101001

$$\Gamma^2 = \frac{J^2}{4}(L-1)$$

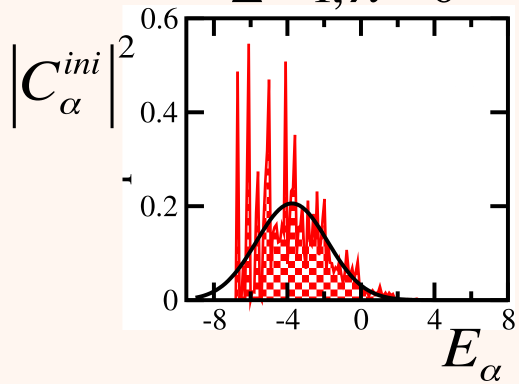
$\Delta = 1, \lambda = 0$ ▲ (red)
$\Delta = 0.5, \lambda = 0$ ▼ (blue)
$\Delta = 1, \lambda = 0.4$ X
$\Delta = 1, \lambda = 1$ ● (purple)
$\Delta = 0.5, \lambda = 1$ ■ (black)

Energy of Initial State and LDOS

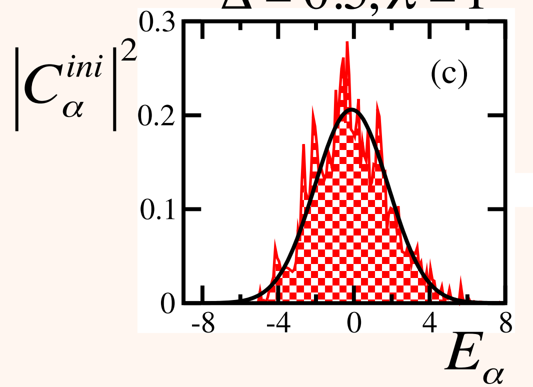
Néel state $\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$

Energy of the initial state: $\frac{J\Delta}{4}\left[-(L-1)+(L-2)\lambda\right]$

$\Delta = 1, \lambda = 0$



$\Delta = 0.5, \lambda = 1$



$\Delta = 1, \lambda = 0$ ▲

$\Delta = 0.5, \lambda = 0$ ▼

$\Delta = 1, \lambda = 0.4$ X

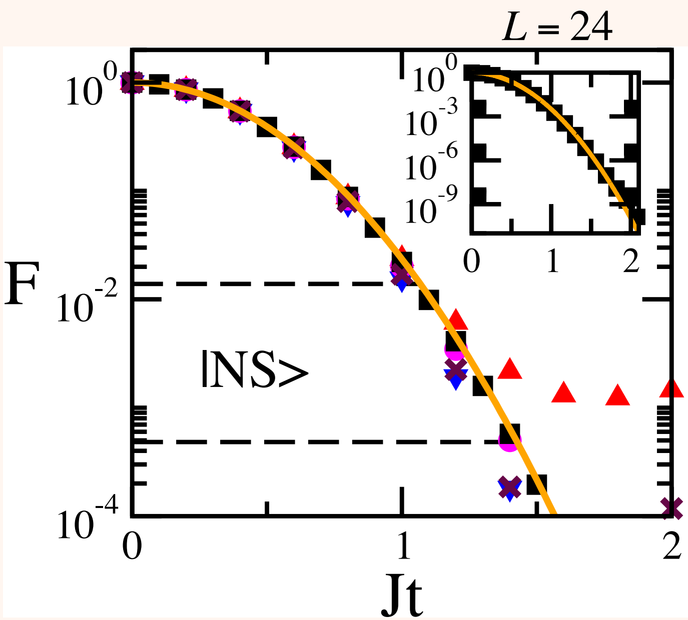
$\Delta = 1, \lambda = 1$ ●

$\Delta = 0.5, \lambda = 1$ ■

Gaussian Decay and Energy of Initial State

Energy of the initial state: $\frac{J\Delta}{4}\left[-(L-1)+(L-2)\lambda\right]$

Néel state ↑↓↑↓↑↓↑↓



Strong perturbation

$L=16, S^z=0$

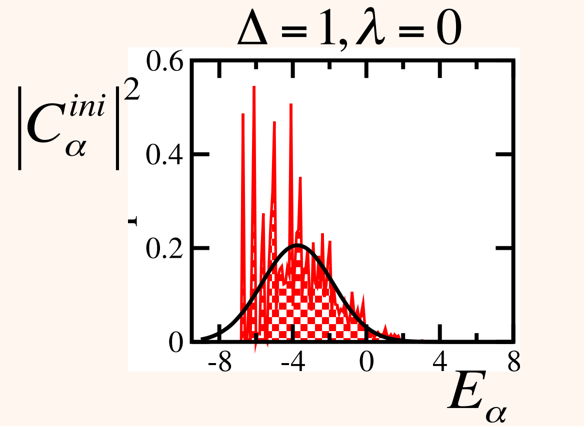
$\Delta = 1, \lambda = 0$ ▲

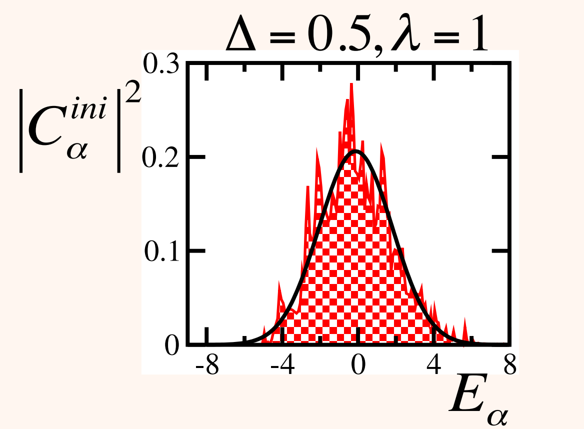
$\Delta = 0.5, \lambda = 0$ ▼

$\Delta = 1, \lambda = 0.4$ X

$\Delta = 1, \lambda = 1$ ●

$\Delta = 0.5, \lambda = 1$ ■





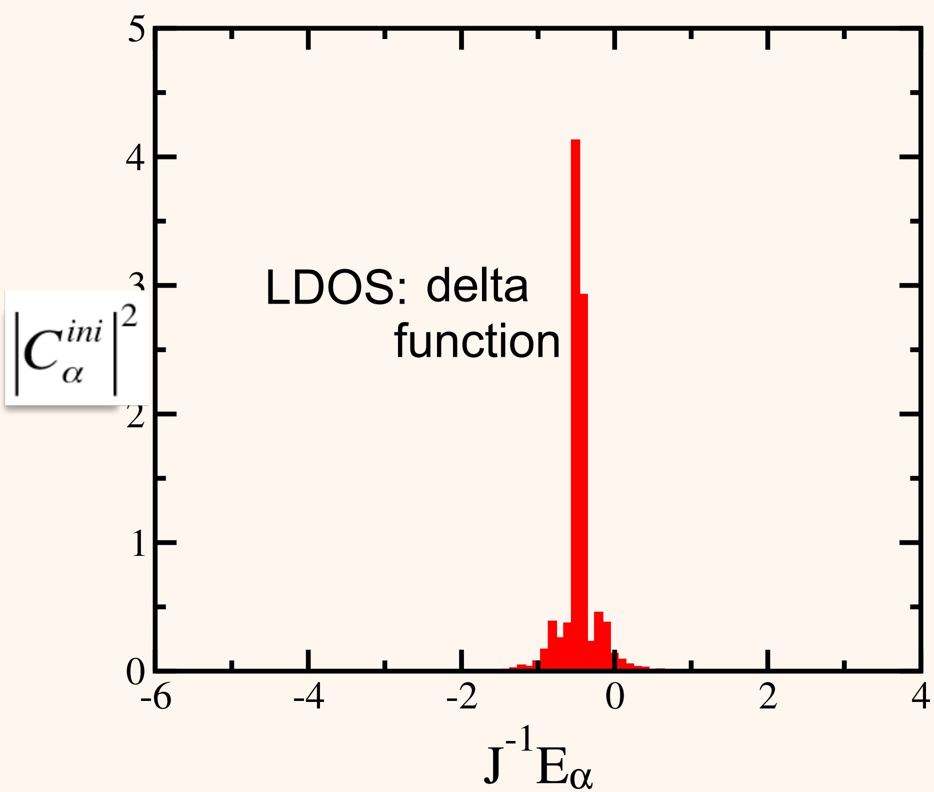
Exercise LDOS-SP

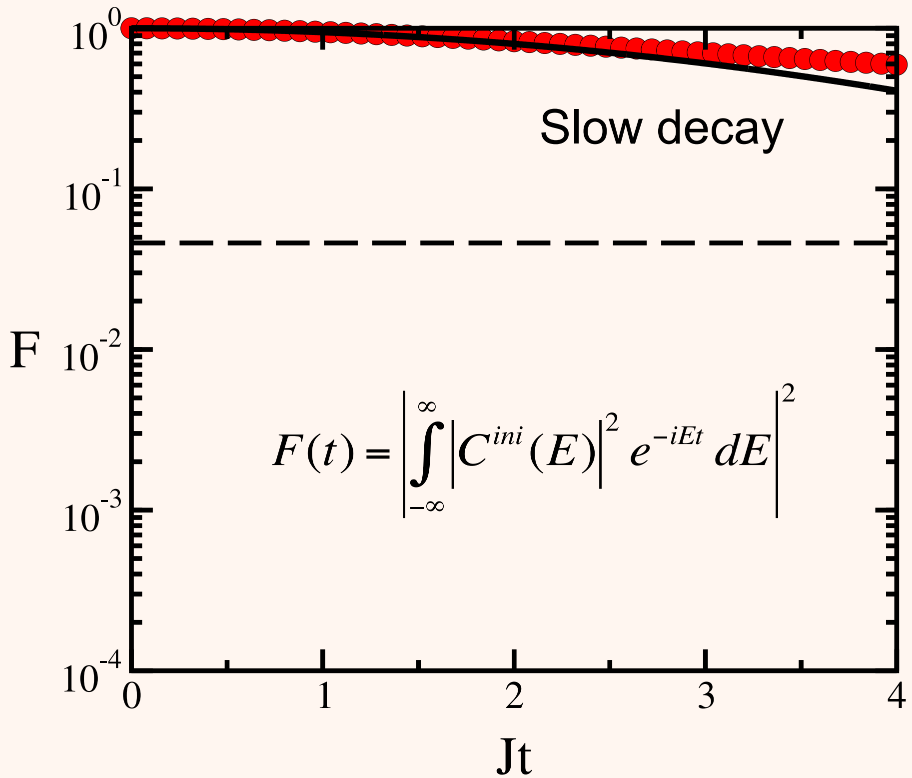
*) Reproduce these figures for the LDOS
*) Reproduce these figures for the SP

Perturbation increases Survival Probability decays faster

$$H_{initial} = H_{XXZ} \xrightarrow{quench} H_{final} = H_{XXZ} + \lambda H_{NNN}$$

$$\lambda = 0.2$$





L=16, 8 up spins, T=7.1 $\Delta = 0.5$

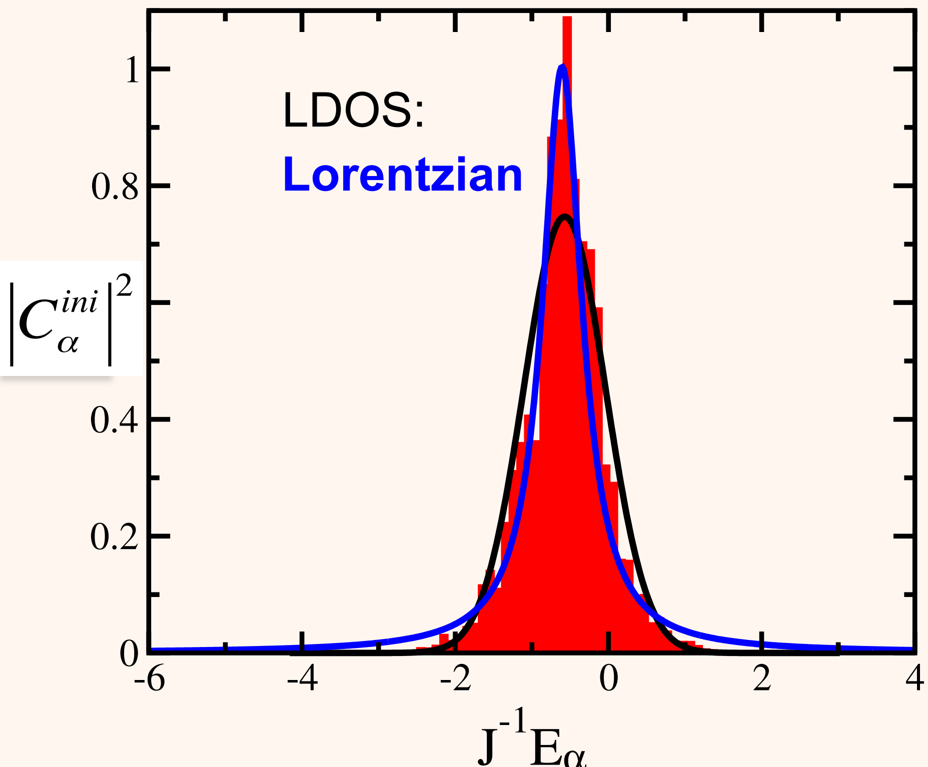
Perturbation increases Survival Probability decays faster

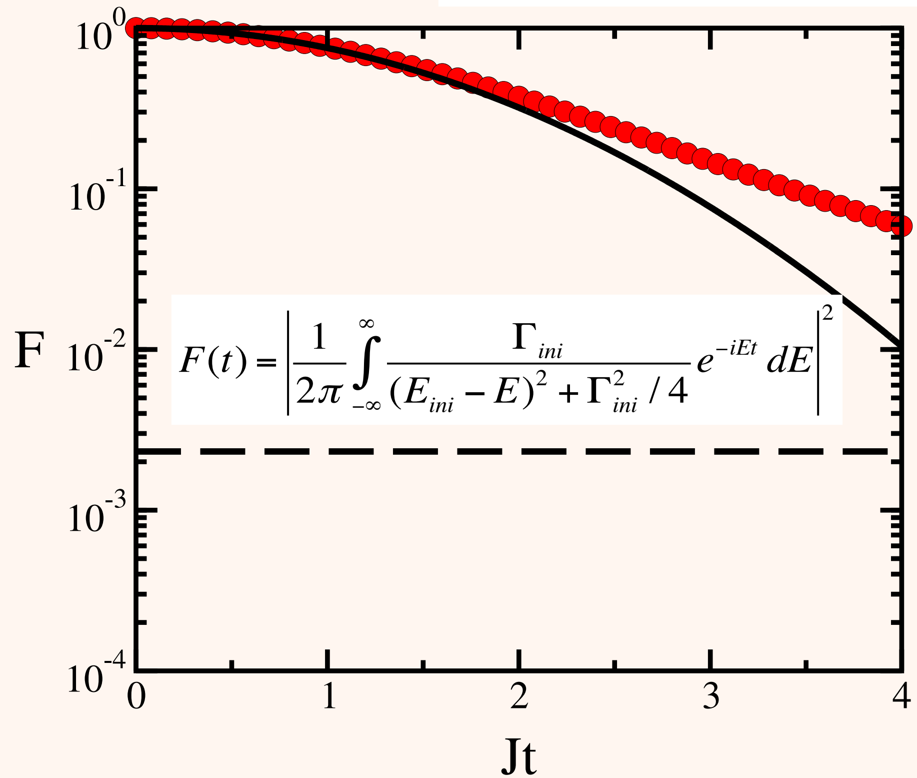
$$H_{initial} = H_{XXZ} \xrightarrow{quench} H_{final} = H_{XXZ} + \lambda H_{NNN}$$

$$\lambda = 0.45$$

$$\frac{1}{2\pi} \frac{\Gamma_{ini}}{(E_{ini} - E)^2 + \Gamma_{ini}^2 / 4}$$

$$F(t) = \exp(-\Gamma_{ini} t)$$





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L=16, 8 up spins, T=7.1

$\Delta = 0.5$

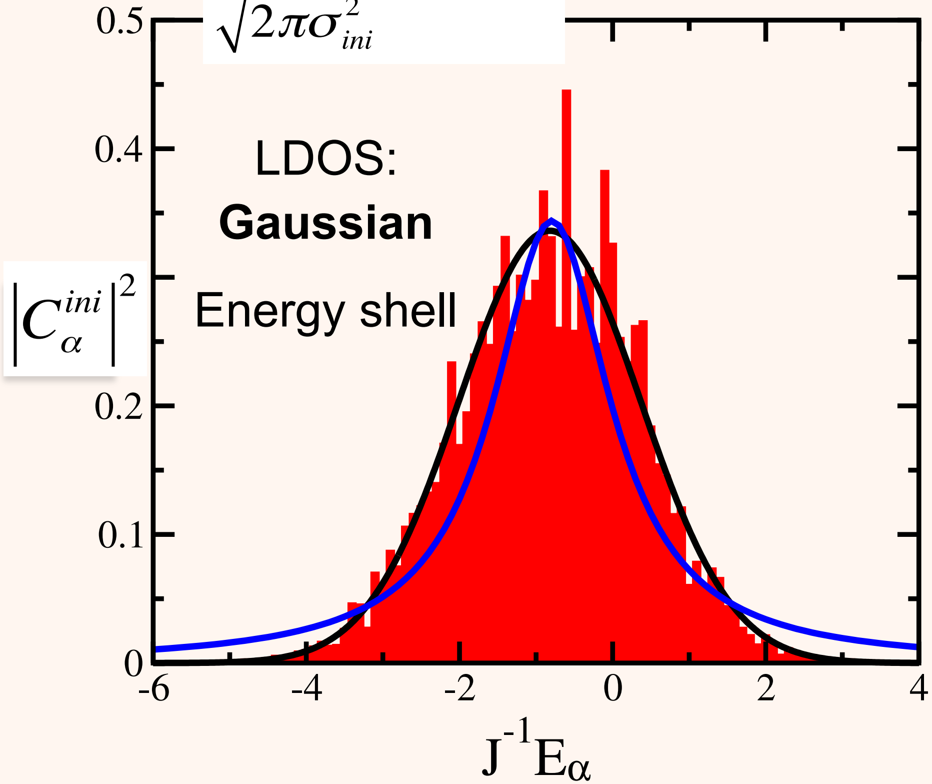
Perturbation increases Survival Probability decays faster

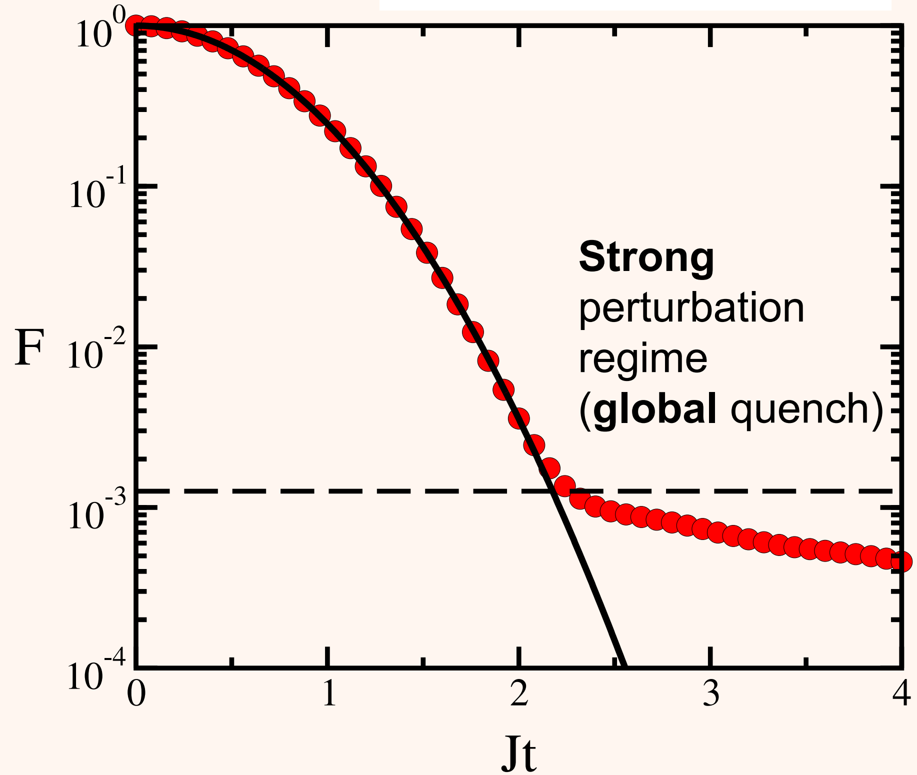
$$H_{initial} = H_{XXZ} \xrightarrow{quench} H_{final} = H_{XXZ} + \lambda H_{NNN}$$

$$\lambda = 1$$

$$\frac{1}{\sqrt{2\pi\sigma_{ini}^2}} e^{-\frac{(E-E_{ini})^2}{2\sigma_{ini}^2}}$$

$$F(t) = \exp(-\sigma_{ini}^2 t^2)$$





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L=16, 8 up spins, T=7.1

$\Delta = 0.5$

Survival Probability & LDOS

LDOS	Fidelity
Breit-Wigner $P_{\mathrm{BW}}^{\mathrm{ini}}(E) = \frac{1}{2\pi}\frac{\Gamma_{\mathrm{ini}}}{(E_{\mathrm{ini}}-E)^2+\Gamma_{\mathrm{ini}}^2/4}$	$F_{\mathrm{BW}}(t) = \exp(-\Gamma_{\mathrm{ini}} t)$
Gaussian $P_{\mathrm{G}}^{\mathrm{ini}}(E) = \frac{1}{\sqrt{2\pi\sigma_{\mathrm{ini}}^2}} \exp\left[-\frac{(E-E_{\mathrm{ini}})^2}{2\sigma_{\mathrm{ini}}^2}\right]$	$F_{\mathrm{G}}(t) = \exp(-\sigma_{\mathrm{ini}}^2 t^2)$
Semicircle $P_{\mathrm{SC}}^{\mathrm{ini}}(E) = \frac{2}{\pi\mathscr{E}}\sqrt{1-\left(\frac{E}{\mathscr{E}}\right)^2}$	$F_{\mathrm{SC}}(t) = \frac{[\mathscr{J}_1(2\sigma_{\mathrm{ini}} t)]^2}{\sigma_{\mathrm{ini}}^2 t^2}$
Two Gaussians $P_{\mathrm{TG}}^{\mathrm{ini}}(E) = \frac{1}{2}\frac{\exp\left[-\frac{(E-E_1)^2}{2\sigma^2}\right]}{\sqrt{2\pi\sigma^2}} + \frac{1}{2}\frac{\exp\left[-\frac{(E-E_2)^2}{2\sigma^2}\right]}{\sqrt{2\pi\sigma^2}}$	$F_{\mathrm{TG}}(t) = \exp(-\sigma^2 t^2)\cos^2\left[\frac{E_2-E_1}{2}t\right]$

Torres & LFS
PRA **90** (2014)

BW and Gaussian LDOS
H^{final}: Chaotic or Integrable

$$H_{XX} \xrightarrow{\Delta} H_{XXZ} \qquad H_{XXZ} \xrightarrow{\lambda} H_{XXZ} + \lambda H_{NNN}$$

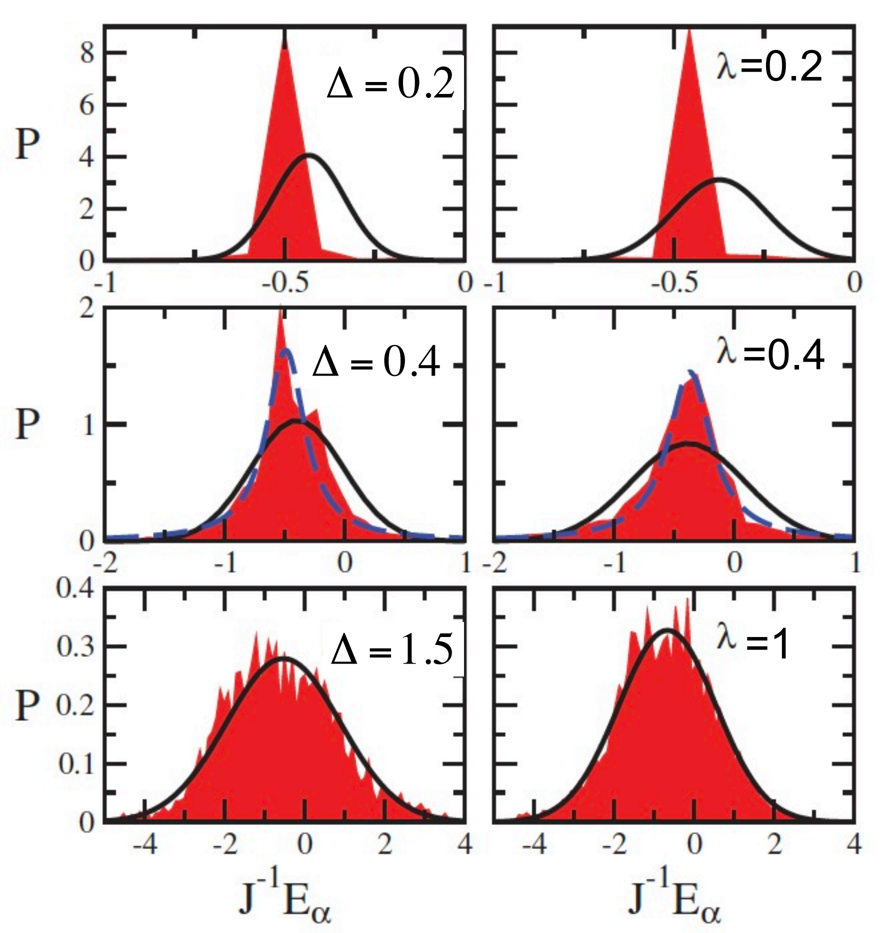
Integrable
to
integrable

$$\sum_{n=1}^{L-1} J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y)$$

$$\downarrow \Delta$$

$$\sum_{n=1}^{L-1} J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta S_n^z S_{n+1}^z)$$

L=18, 6 up spins



Integrable
to
chaotic

Exponential and Gaussian SP(t)
H^{final}: Chaotic or Integrable

$$H_{XX} \xrightarrow{\Delta} H_{XXZ} \qquad H_{XXZ} \xrightarrow{\lambda} H_{XXZ} + \lambda H_{NNN}$$

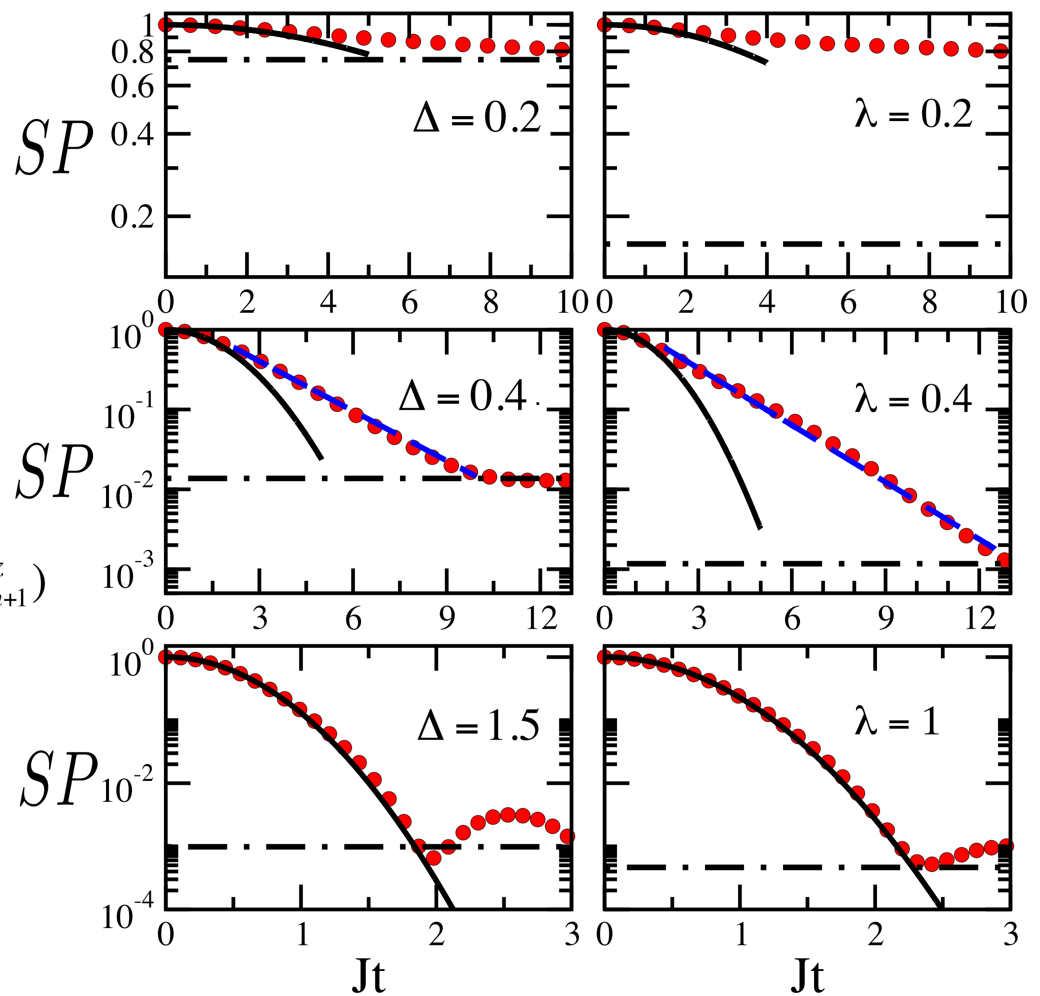
Integrable
to
integrable

$$\sum_{n=1}^{L-1} J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y)$$

$$\downarrow \Delta$$

$$\sum_{n=1}^{L-1} J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta S_n^z S_{n+1}^z)$$

L=18, 6 up spins



Integrable
to
chaotic

ENTIRE EVOLUTION OF THE SURVIVAL PROBABILITY

GOE FULL RANDOM MATRICES

Dynamics: Survival Probability

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

$$|\Psi(t)\rangle = \sum_{\alpha} C_{\alpha}^{ini} e^{-iE_{\alpha}t} |\alpha\rangle$$

$$SP(t) = \left| \sum_{\alpha} |C_{\alpha}^{ini}|^2 e^{-iE_{\alpha}t} \right|^2$$

$$\langle SP(t) \rangle = \langle \sum_{\alpha,\beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_{\alpha}-E_{\beta})t} \rangle$$ **(averages)**

Survival Probability
vs
Spectral Form Factor

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

$$|\Psi(t)\rangle = \sum_{\alpha} C_{\alpha}^{ini} e^{-iE_{\alpha}t} |\alpha\rangle$$

$$SP(t) = \left| \sum_{\alpha} |C_{\alpha}^{ini}|^2 e^{-iE_{\alpha}t} \right|^2$$

$$\langle SP(t) \rangle = \langle \sum_{\alpha,\beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_{\alpha}-E_{\beta})t} \rangle$$

True DYNAMICAL quantity!!

Spectral form factor: $SFT(t) = \langle \sum_{\alpha,\beta} e^{-i(E_{\alpha}-E_{\beta})t} \rangle$

Analysis of spectrum in the time domain

Survival Probability

$$\langle SP(t)\rangle = \langle \sum_{\alpha\neq\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t}\rangle + \langle \sum_{\alpha} |C_\alpha^{ini}|^4 \rangle$$

$$IPR^{ini} = \overline{SP} = \frac{3}{D}$$

Survival Probability

$$\langle SP(t) \rangle = \langle \sum_{\alpha \neq \beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t} \rangle + \langle \sum_{\alpha} |C_\alpha^{ini}|^4 \rangle$$

$$\overline{SP}$$

$$\left\langle \int \sum_{\alpha \neq \beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 \delta\big(E - (E_\alpha - E_\beta)\big) e^{-iEt} dE \right\rangle$$

Components are uncorrelated with the eigenvalues.
Eigenstates and eigenvalues are statistically independent.

$$\sum_{\alpha \neq \beta} \langle |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 \rangle \int \langle \delta\big(E - (E_\alpha - E_\beta)\big) \rangle e^{-iEt} dE$$

Survival Probability

$$\sum_{\alpha\neq\beta}\langle|C_\alpha^{ini}|^2|C_\beta^{ini}|^2\rangle\int\langle\delta\big(E-(E_\alpha-E_\beta)\big)\rangle e^{-iEt}dE$$

P: Probability density function

$$\langle\delta\big(E-(E_\alpha-E_\beta)\big)\rangle=\int\delta\big(E-(E_\alpha-E_\beta)\big)P(E_1,\ldots E_\alpha,\ldots E_\beta\ldots E_D)dE_1\ldots dE_D$$

R_2: two-point spectral correlation function

$$\langle\delta\big(E-(E_\alpha-E_\beta)\big)\rangle=\frac{(D-2)!}{D!}\int\delta\big(E-(E_\alpha-E_\beta)\big)R_2(E_\alpha,E_\beta)dE_\alpha dE_\beta$$

R_2: Probability density of finding a level (regardless of labeling) around each of the points E_α, E_β, while the positions of the remaining levels are unobserved.

$$R_2(E_\alpha,E_\beta)=R_1(E_\alpha)R_1(E_\beta)-T_2(E_\alpha,E_\beta)$$

Survival Probability

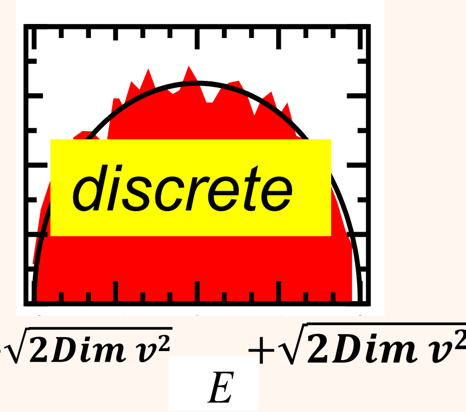
$$\sum_{\alpha\neq\beta}\langle|C_\alpha^{ini}|^2|C_\beta^{ini}|^2\rangle\int\langle\delta\big(E-(E_\alpha-E_\beta)\big)\rangle e^{-iEt}dE$$

$$\frac{(D-2)!}{D!}\int\int\delta\big(E-(E_\alpha-E_\beta)\big)R_1(E_\alpha)R_1(E_\beta)dE_\alpha dE_\beta$$

R_1: density of states (semicircle)

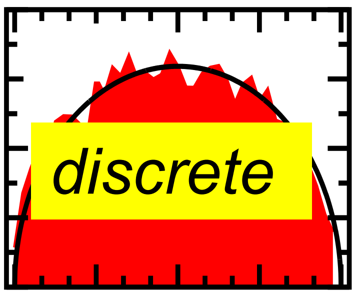
$$\frac{1}{D(D-1)}\left|\int e^{-iE_\alpha t}R_1(E_\alpha)dE_\alpha\right|^2$$

$$\frac{D}{D-1}\frac{\mathcal{J}_1^2(2\Gamma t)}{\Gamma^2t^2}$$



Survival Probability

$$\sum_{\alpha\neq\beta}\langle|C_\alpha^{ini}|^2|C_\beta^{ini}|^2\rangle\int\langle\delta\big(E-(E_\alpha-E_\beta)\big)\rangle e^{-iEt}dE$$



$$\frac{(D-2)!}{D!}\int\int\delta\big(E-(E_\alpha-E_\beta)\big)R_1(E_\alpha)R_1(E_\beta)dE_\alpha dE_\beta$$

R_1: density of states (semicircle)

$$\frac{1}{D(D-1)}\left|\int e^{-iE_\alpha t}R_1(E_\alpha)dE_\alpha\right|^2$$

$$-\frac{(D-2)!}{D!}\int\int\delta\big(E-(E_\alpha-E_\beta)\big)T_2(E_\alpha,E_\beta)dE_\alpha dE_\beta$$

T_2: two-level cluster function

related with the level number variance $\Sigma(l)^2$

$$\frac{D}{D-1}\,\frac{\mathcal{J}_1^2(2\Gamma t)}{\Gamma^2 t^2}$$

Survival Probability

$$\sum_{\alpha \neq \beta} \langle |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 \rangle \int \langle \delta\big(E - (E_\alpha - E_\beta)\big) \rangle e^{-iEt} dE$$

$$-\frac{(D-2)!}{D!} \int \int \delta\big(E - (E_\alpha - E_\beta)\big) T_2(E_\alpha, E_\beta) dE_\alpha dE_\beta$$

$$-\frac{(D-2)!}{D!} \int e^{-i(E_\alpha - E_\beta)t} T_2(E_\alpha, E_\beta)\, dE_\alpha\, dE_\beta$$

(see Mehta's book)

b_2: two-level form factor

$$-\frac{1}{D-1} b_2 \left(\frac{t}{2\pi R_1(0)} \right)$$

$$b_2(t) = \begin{cases} 1 - 2t + t\ln(1+2t), & t \le 1 \\ t\ln\left(\dfrac{2t+1}{2t-1}\right) - 1, & t > 1 \end{cases}$$

Survival Probability

$$\langle SP(t)\rangle = \langle \sum_{\alpha\neq\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t}\rangle + \langle \sum_{\alpha} |C_\alpha^{ini}|^4 \rangle$$

disconnected

connected

$$\langle SP(t)\rangle = \sum_{\alpha\neq\beta} \langle |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 \rangle \frac{1}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{\Gamma^2 t^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] - \overline{SP}$$

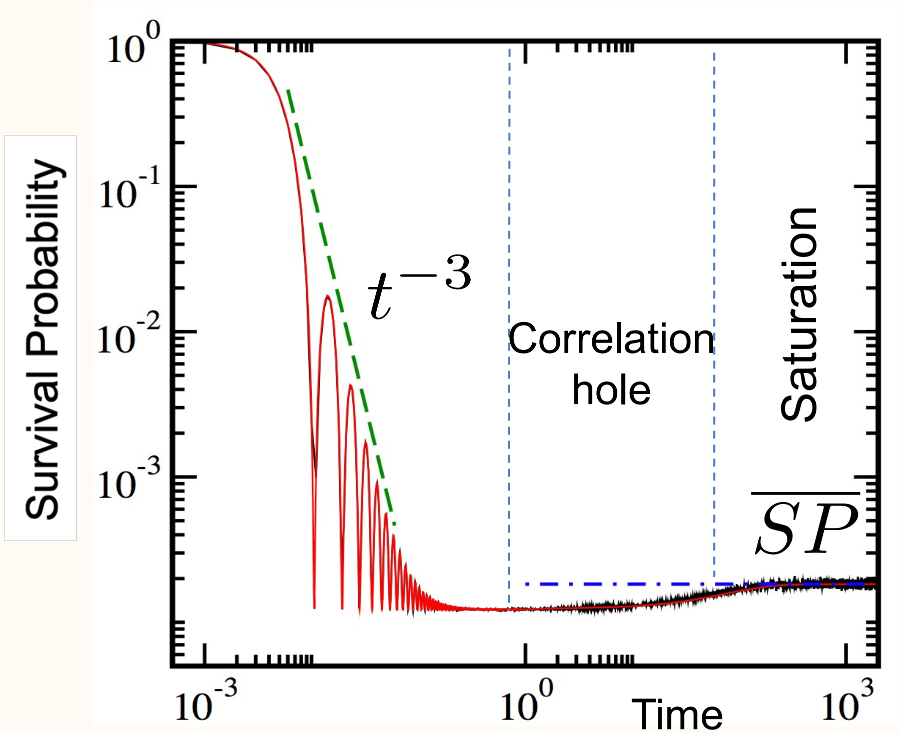
$$\langle \sum_{\alpha\neq\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 \rangle = \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 \rangle - \sum_{\alpha} |C_\alpha^{ini}|^4$$

$$\langle \sum_{\alpha\neq\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 \rangle = 1 - \overline{SP}$$

Survival Probability $\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$

GOE Full Random Matrices

$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$



Torres, Garcia, LFS
PRB **97**, 060303 (R) (2018)

Schiulaz, Torres & LFS,
PRB **99**, 174313 (2019)

Correlation Hole

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

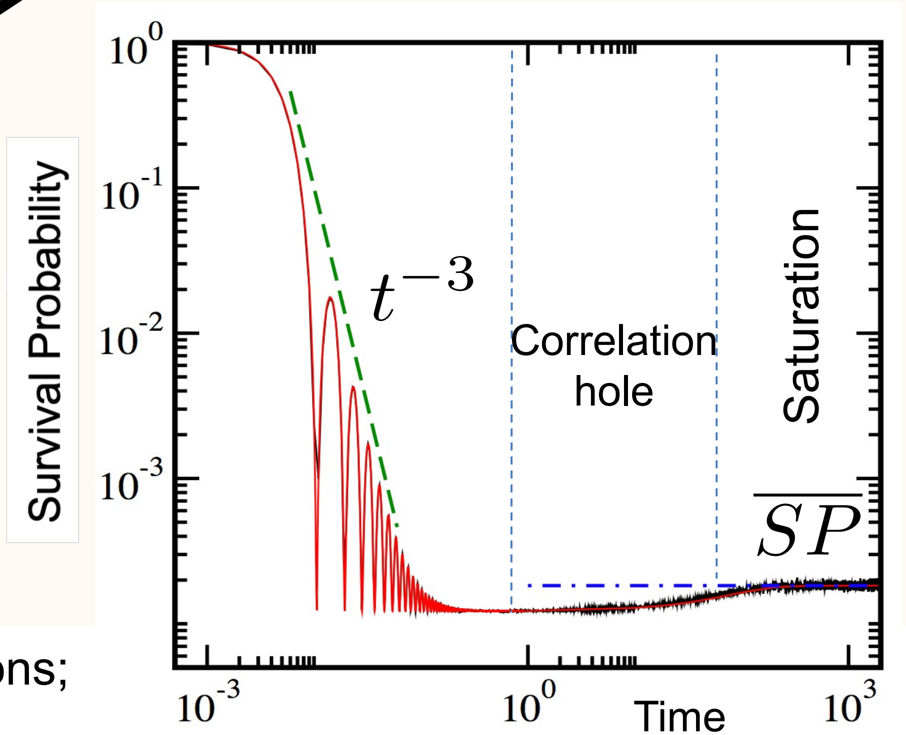
GOE Full Random Matrices

$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$

CORRELATION HOLE (ramp)

True dynamical manifestation of spectral correlations

experiments



(i) Detect short- and long-range correlations;
(ii) No unfolding;
(iii) Do not need to separate the eigenvalues by symmetry sectors

Manifestations of Chaos in the Dynamics

Level statistics is a good approach when we have access to the spectrum: Nuclear Physics

How about experiments with cold atoms and ion traps?

CORRELATION HOLE (ramp)

True dynamical manifestation of spectral correlations

BUT it takes time for the dynamics to resolve the discreteness of the spectrum!

Survival Probability $\left|\langle\Psi(0)|\Psi(t)\rangle\right|^2$

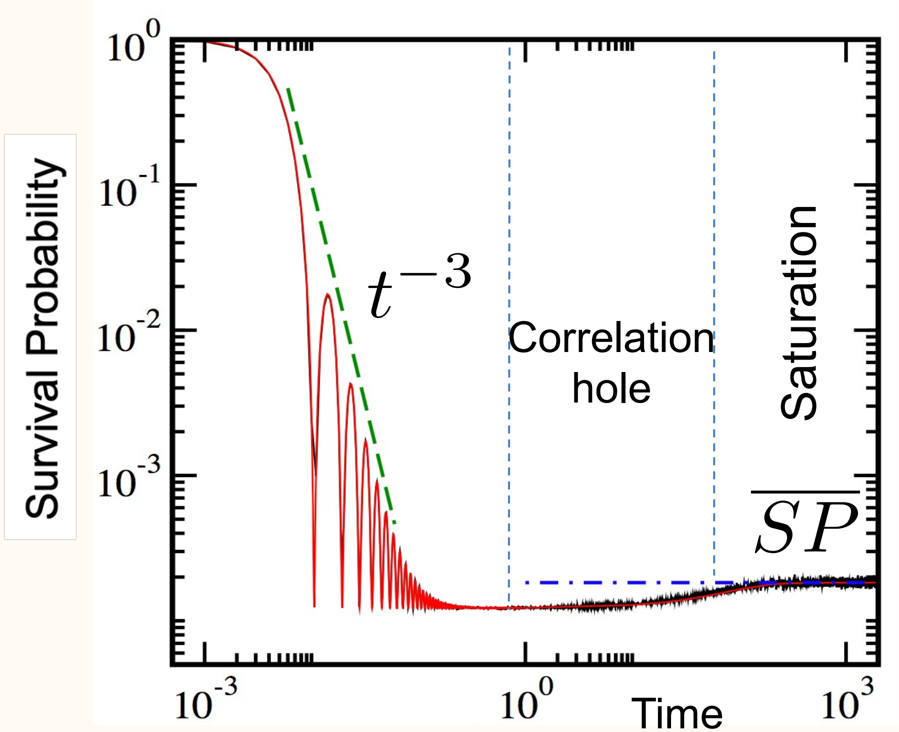
GOE Full Random Matrices

$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2}-b_2\left(\frac{\Gamma t}{2D}\right)\right]+\overline{SP}$$

Exercise SP-GOE

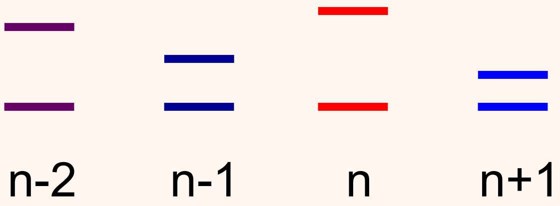
Evolve the survival probability under GOE full random matrices and show that it agrees with the analytical expression above.

ATTENTION!
Averages are needed for seeing the correlation hole.



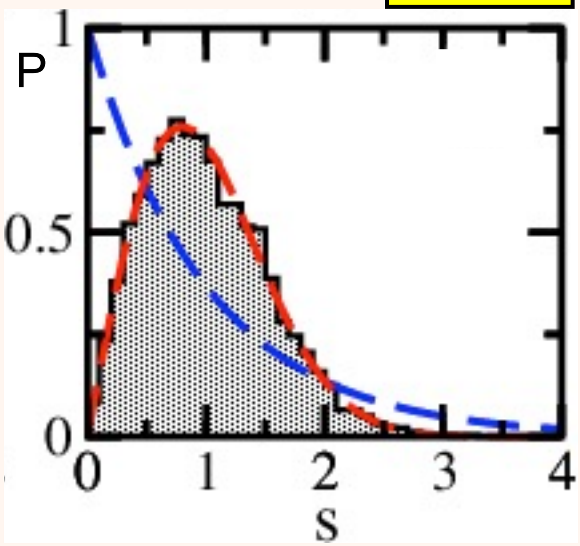
1D Disordered Spin-1/2 Model

$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n=1}^{L} \frac{J}{4}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \sigma_n^z\sigma_{n+1}^z)$$

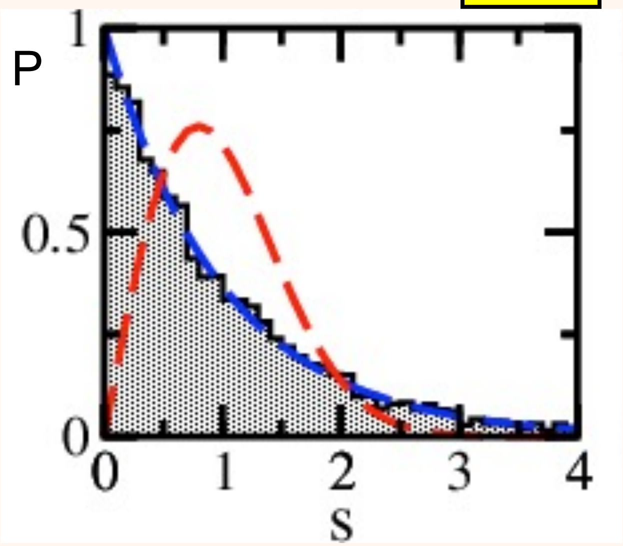


$h_n \in [-h, h]$

Chaotic h=0.5J



Localized $h>h_c$



LFS, Rigolin, Escobar
Entanglement versus chaos in disordered spin chains
PRA **69**, 042304 (2004)

Models

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

$$H = H_0 + V$$

$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$

GOE Full Random Matrices

H_0 Diagonal matrix,
Gaussian real random numbers

$\langle H_0^2 \rangle = 2$

V Off-diagonal matrix,
Gaussian real random numbers

$\langle V^2 \rangle = 1$

Strong level repulsion

Disordered Spin Model

$$H_0 = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z$$

Random numbers
$h_n \in [-h, h]$

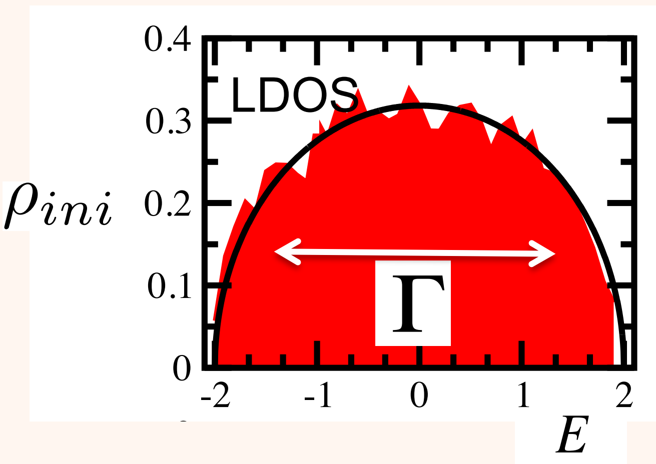
$$V = \sum_{n=1}^{L} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \sigma_n^z \sigma_{n+1}^z)$$

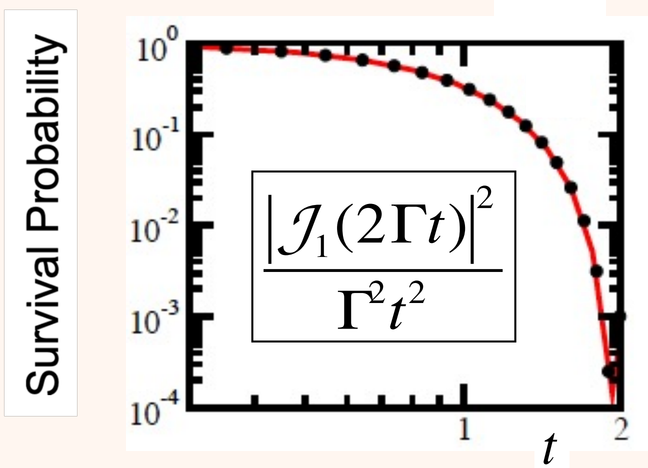
Strong level repulsion for *h~J*

AVERAGES
10^4-10^5 data

Initial Fast Decay

Full Random Matrices

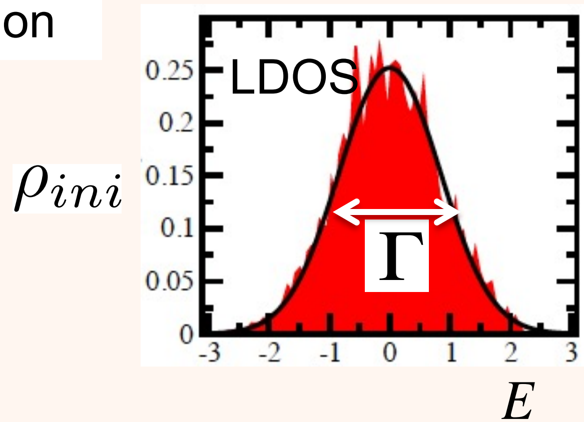


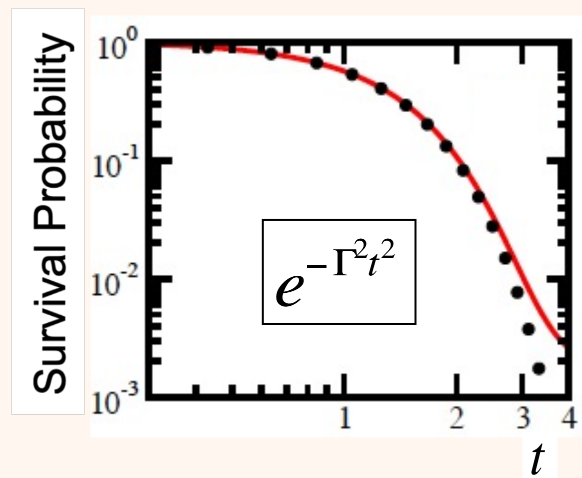


Γ Width of the distribution

$1/\Gamma$

Spin Model

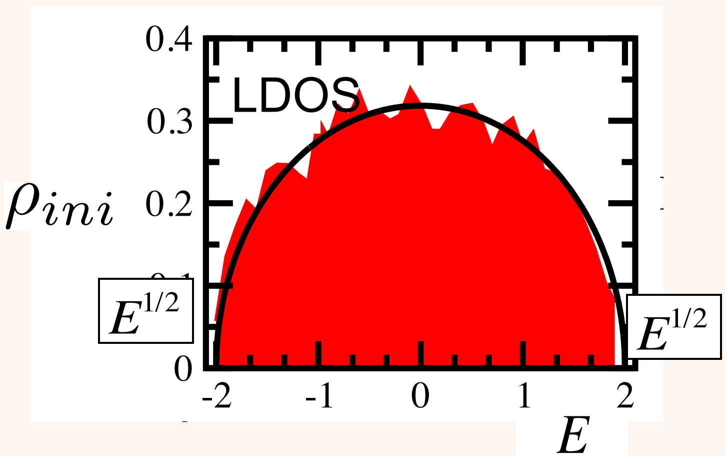


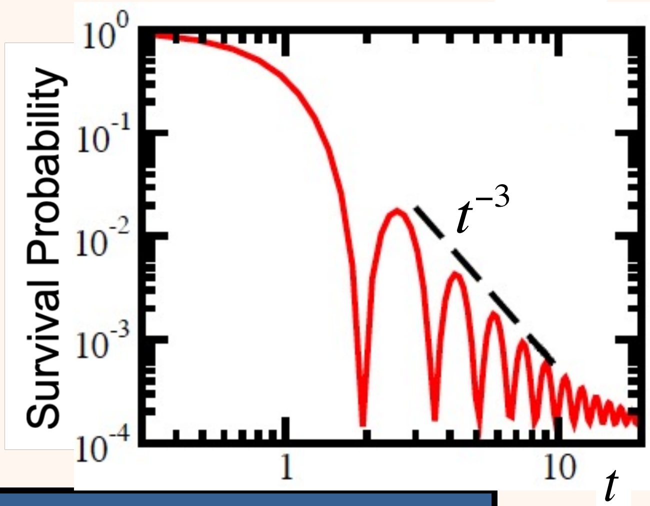


Torres-Herrera & LFS
PRA, NJP (2014)

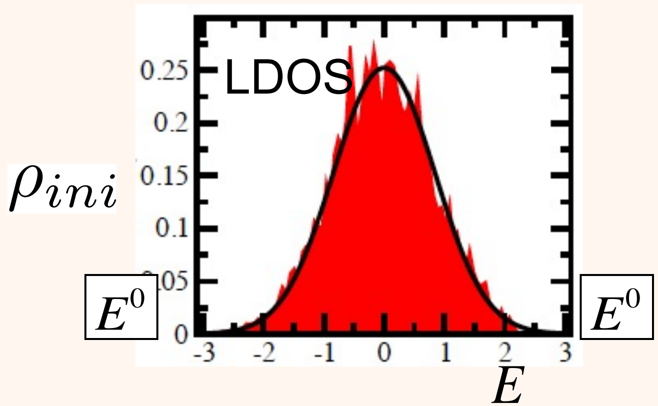
Power-law Decay

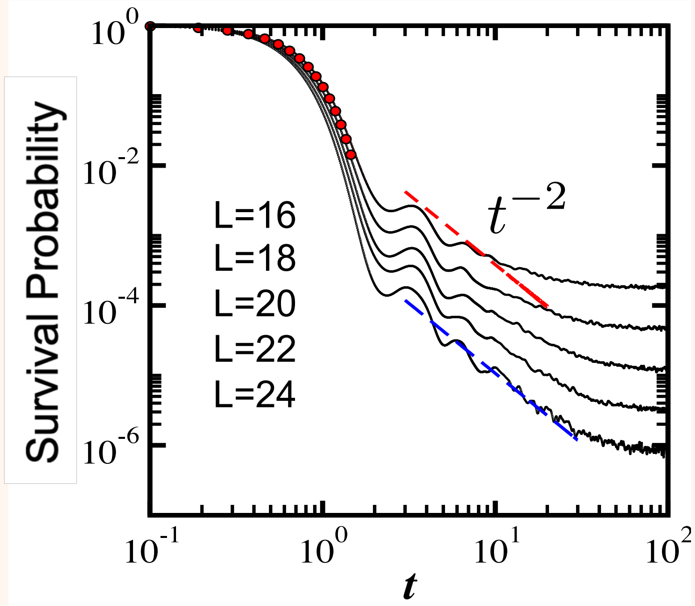
Full Random Matrices





Spin Model





Távora, Torres, LFS
PRA **94**, 041603R (2016)
PRA **95**, 013604 (2017)

Lea F. Santos, Yeshiva University

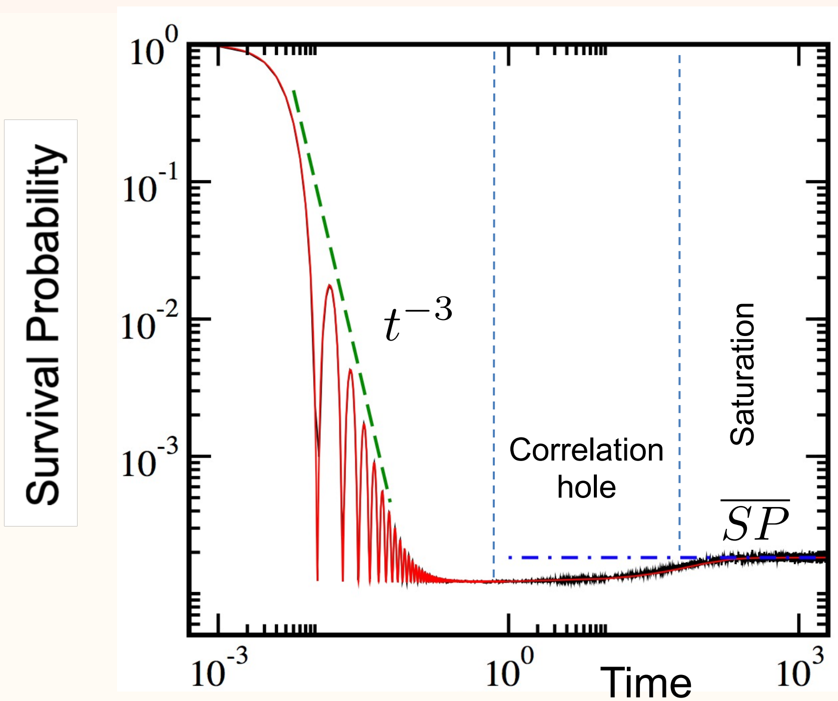
Survival Probability

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

GOE Full Random Matrices

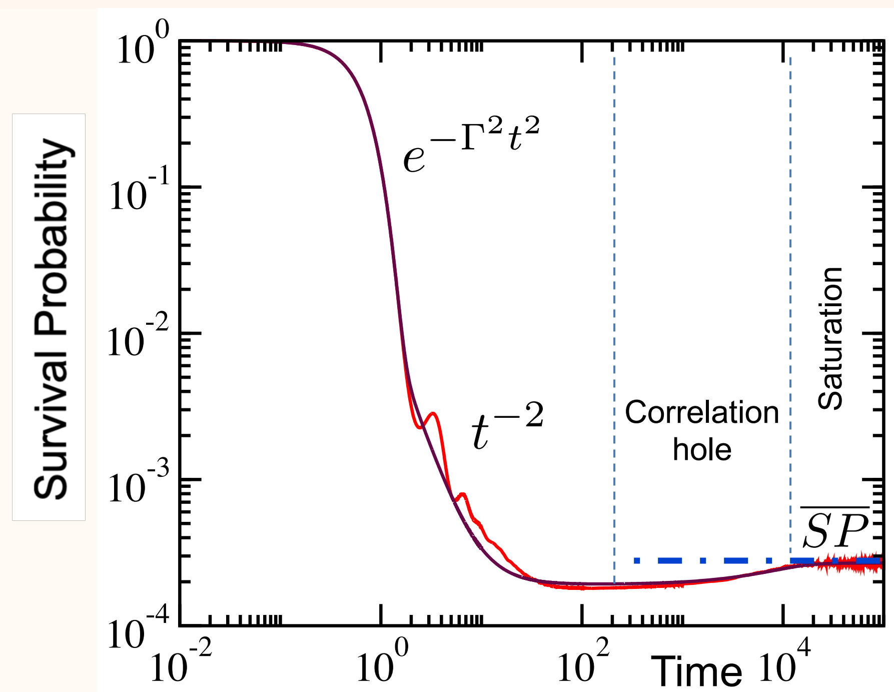
$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$



Disordered Spin Model

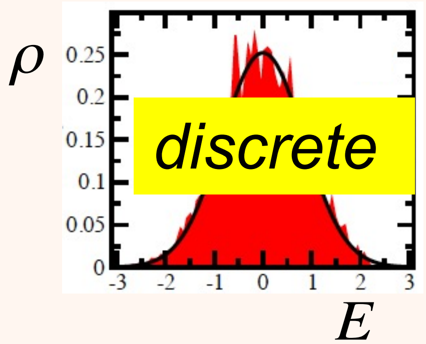
$$\frac{1-\overline{SP}}{(D-1)}\left[\frac{De^{-\Gamma^2 t^2}}{\mathcal{N}^2}\mathcal{F}(t) - b_2\left(\frac{\Gamma t}{\sqrt{2\pi}D}\right)\right] + \overline{SP}$$

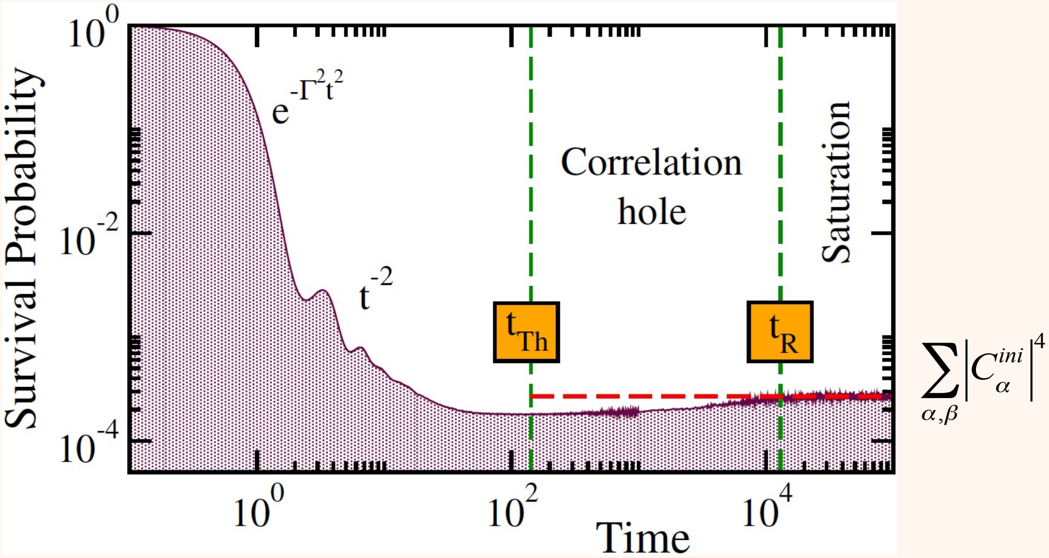
$$\mathcal{F}(t) = \left|\operatorname{erf}\left(\frac{E_{\max} + it\Gamma^2}{\sqrt{2}\Gamma}\right) - \operatorname{erf}\left(\frac{E_{\min} + it\Gamma^2}{\sqrt{2}\Gamma}\right)\right|^2$$



Correlation Hole

$$\langle SP(t)\rangle = \langle \sum_{\alpha\neq\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t}\rangle + \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^4 \rangle$$





Fourier transform of the 2-level cluster function:

2-level form factor b_2

$$b_2(t) = \begin{cases} 1 - 2t + t\ln(1+2t), & t \le 1 \\ t\ln\left(\frac{2t+1}{2t-1}\right) - 1, & t > 1 \end{cases}$$

Correlation hole = ramp

Schiulaz, Torres & LFS, PRB **99**, 174313 (2019)

Survival Probability

Realistic chaotic models

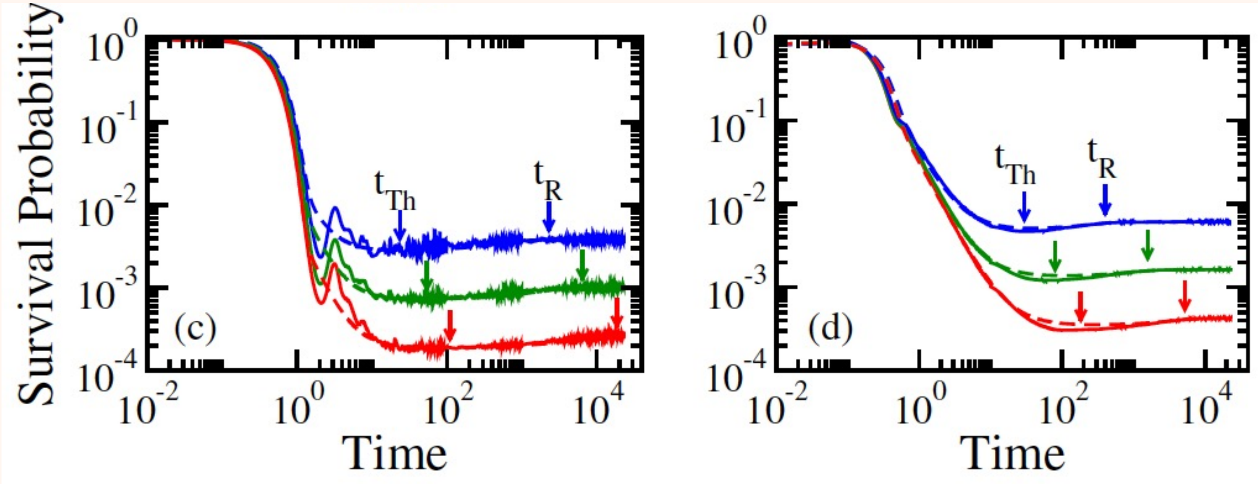
Exercise SP-spin

Check that the semi-analytical expression describes well the
*) XXZ+onsite disorder (h=0.5)
*) NN+NNN model
*) NN+NNN for sparse band random matrix
Do AVERAGES over disorder realizations and initial states!
Initial states close to the middle of the spectrum!

$$H^{\mathrm{cl}} = H_0^{\mathrm{cl}} + V^{\mathrm{cl}},$$

$$H_0^{\mathrm{cl}} = J\Delta \sum_{k=1}^{L} \left(S_k^z S_{k+1}^z + \lambda S_k^z S_{k+2}^z \right),$$

$$V^{\mathrm{cl}} = J \sum_{k=1}^{L} \left[S_k^x S_{k+1}^x + S_k^y S_{k+1}^y + \lambda \left(S_k^x S_{k+2}^x + S_k^y S_{k+2}^y \right) \right].$$



Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

TIME SCALES
EQUILIBRATION TIME

How long does it take for
isolated many-body quantum systems
perturbed far from equilibrium
to finally reach equilibrium?

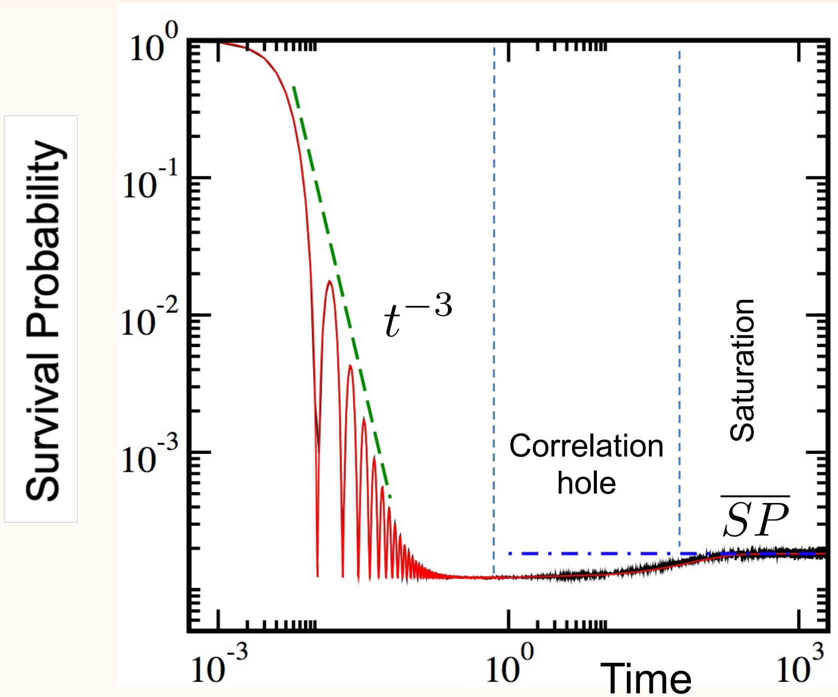
Different behaviors/ different **timescales**

Time to Reach the Minimum of the Correlation Hole

PRB **99**, 174313 (2019)

<u>GOE Full Random Matrices</u>

$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$



$$D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} \rightarrow \frac{D}{\pi(\Gamma t)^3} \quad \text{for} \quad \Gamma t \gg 1$$

$$b_2\left(\frac{\Gamma t}{2D}\right) \rightarrow 1 - \frac{\Gamma t}{D} \quad \text{for} \quad \frac{\Gamma t}{D} \ll 1$$

$$\left.\frac{d\langle P_S(t)\rangle}{dt}\right|_{t=t_{\text{Th}}^{\text{GOE}}} \simeq \frac{1-\overline{P_S}}{D-1}\left.\left[-3\frac{D}{\pi\Gamma^3 t^4} + \frac{\Gamma}{D}\right]\right|_{t=t_{\text{Th}}^{\text{GOE}}} = 0$$

$$t_{\text{Th}}^{\text{GOE}} = \left(\frac{3}{\pi}\right)^{1/4}\frac{\sqrt{D}}{\Gamma} = \left(\frac{3}{\pi}\right)^{1/4}$$

$$\langle P_S(t)\rangle|_{t=t_{\text{Th}}^{\text{GOE}}} \approx \frac{2}{D}$$

Time to Reach the Minimum of the Correlation Hole

PRB **99**, 174313 (2019)

Disordered Spin Model

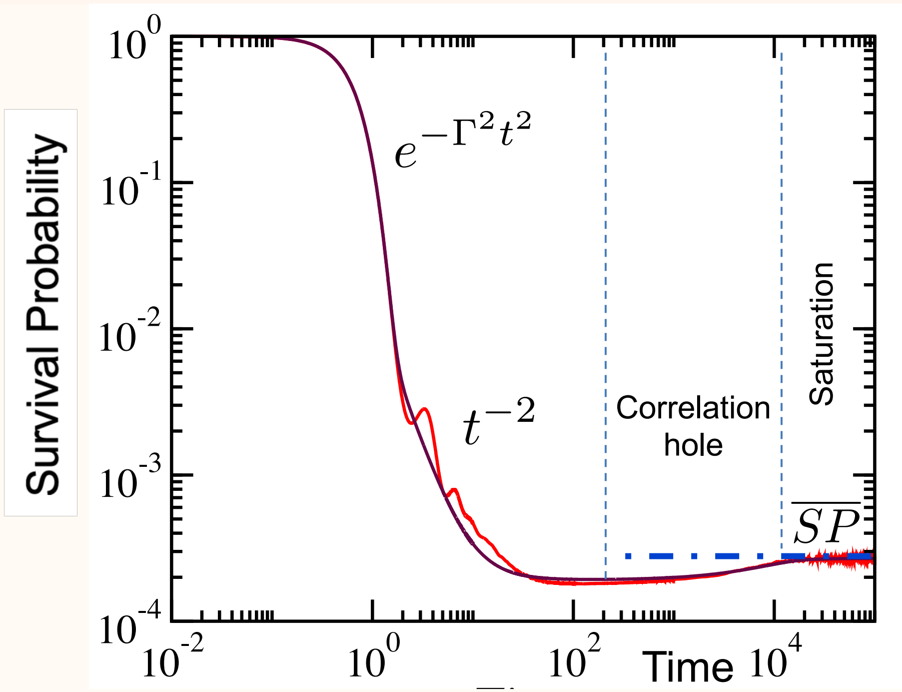
$$\frac{De^{-\Gamma^2 t^2}}{4\mathcal{N}^2}\mathcal{F}(t) \to \frac{D}{\Gamma^2 t^2} \quad \text{for} \quad \Gamma t \gg 1$$

$$b_2\left(\frac{\Gamma t}{\sqrt{2\pi}D}\right) \to 1 - 2\frac{\Gamma t}{\sqrt{2\pi}D} \quad \text{for} \quad \frac{\Gamma t}{D} \ll 1$$

$$t_{\mathrm{Th}} \propto \frac{D^{2/3}}{\Gamma}$$

$$\frac{1-\overline{SP}}{(D-1)}\left[\frac{De^{-\Gamma^2 t^2}}{\mathcal{N}^2}\mathcal{F}(t) - b_2\left(\frac{\Gamma t}{\sqrt{2\pi}D}\right)\right] + \overline{SP}$$

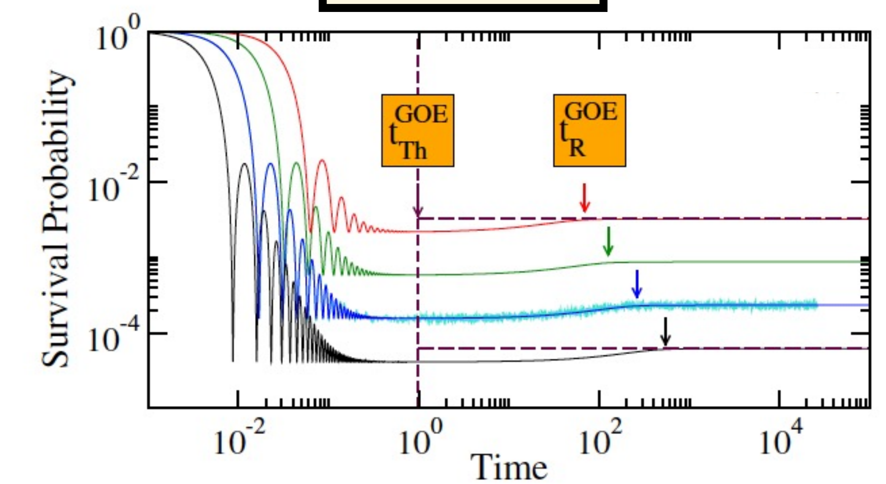
$$\mathcal{F}(t) = \left|\mathrm{erf}\left(\frac{E_{\max} + it\Gamma^2}{\sqrt{2}\Gamma}\right) - \mathrm{erf}\left(\frac{E_{\min} + it\Gamma^2}{\sqrt{2}\Gamma}\right)\right|^2$$



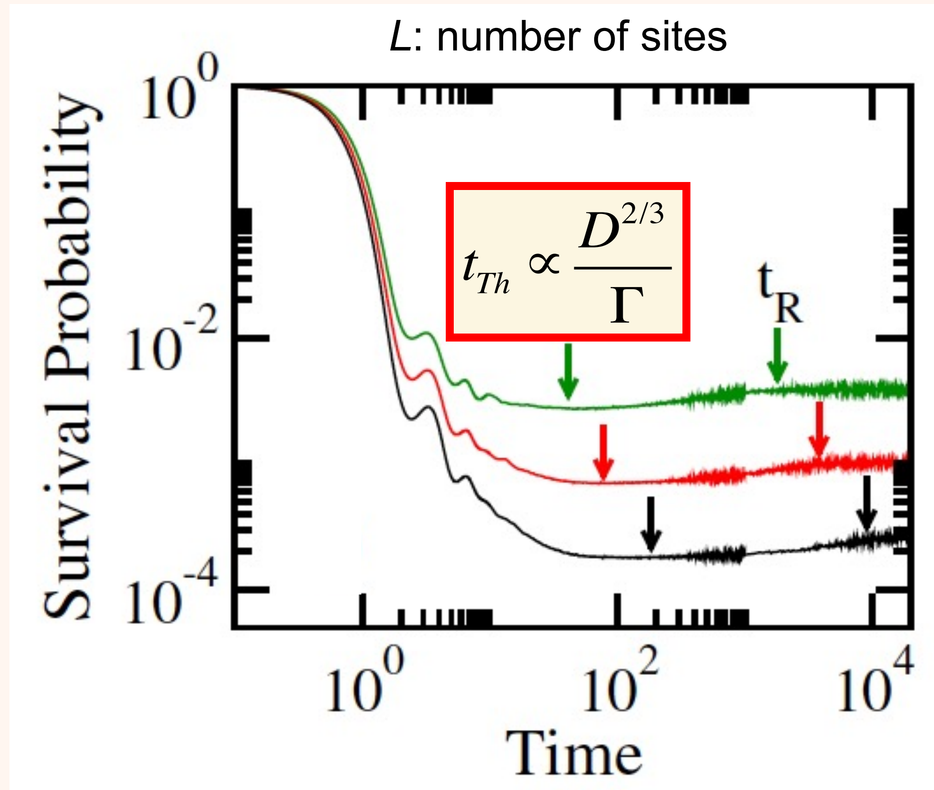
Time to Reach the Minimum of the Hole

GOE full random matrix

$$t_{Th} = \left(\frac{3}{\pi}\right)^{1/4}$$



Realistic CHAOTIC spin-1/2 model



Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Spread in the many-body Hilbert space

$$|\Psi(0)\rangle = |\uparrow\downarrow\downarrow\downarrow\uparrow\downarrow\uparrow\rangle$$

$$|\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\uparrow\rangle \qquad |\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\rangle$$

$$|\uparrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\rangle \qquad \ldots \qquad \ldots$$

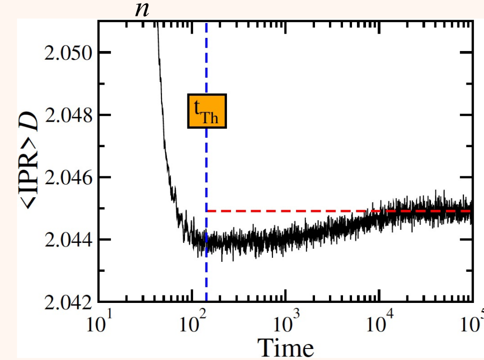
$$D = \frac{L!}{(L/2)!^2}$$

L: number of sites

$$V = \sum_{n=1}^{L} \frac{J}{4}\left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \sigma_n^z \sigma_{n+1}^z\right)$$

Spread in the many-body Hilbert space
THOULESS TIME

$$IPR(t) = \sum_n \left| \langle n | e^{-iHt} | \Psi(0) \rangle \right|^4$$



$$|\Psi(0)\rangle = |\uparrow\downarrow\downarrow\downarrow\uparrow\downarrow\uparrow\rangle$$

$$|\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\uparrow\rangle \qquad |\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\rangle$$

$$|\uparrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\rangle \qquad \dots \qquad \dots$$

dimension of Hilbert space

$$D = \frac{L!}{(L/2)!^2}$$

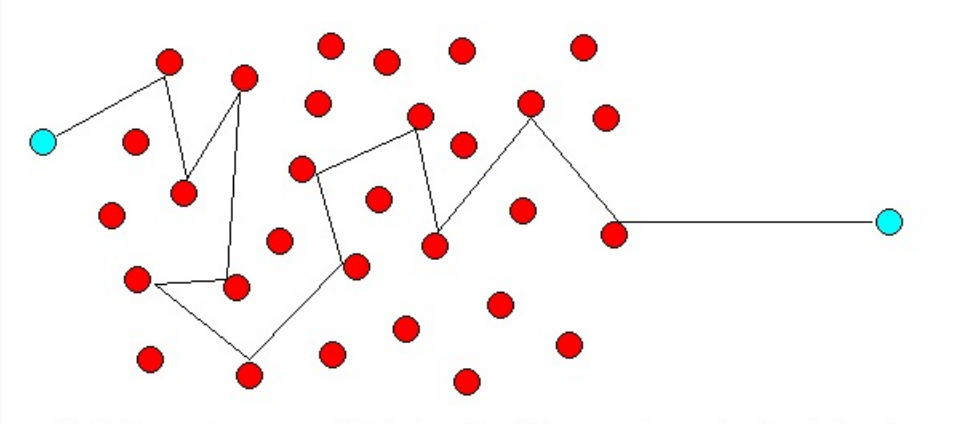
L: number of sites

$$V = \sum_{n=1}^{L} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \sigma_n^z \sigma_{n+1}^z)$$

Non-Interacting Systems
THOULESS TIME

*) Thouless time is the time for an excitation to diffusive through the system and reach its borders. If this time is larger than the maximal time scale of the system (that is, the Heisenberg time), then we have localization.

→ So, delocalization implies $(t_R)/(t_{Th})>1$.



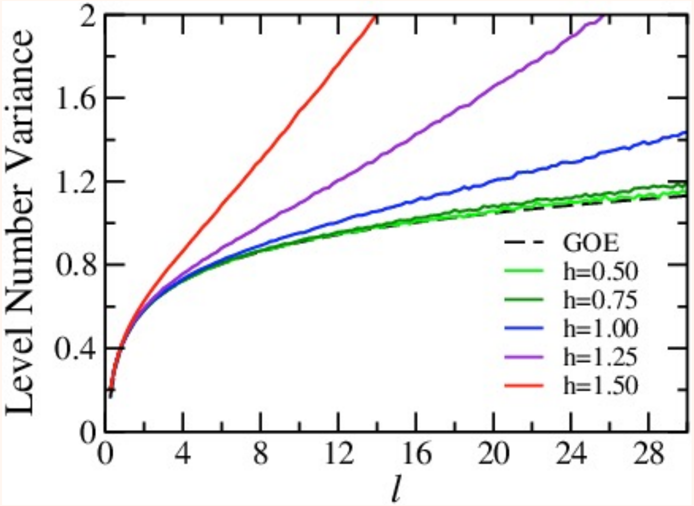
*) Thouless energy is the width of the resonances. To explain diffusion via microstates (through resonances between these microstates), their mean level spacing has to be smaller than E_{Th}.
If the mean level spacing is larger than this width, we cannot explain diffusion with microstates.

→ So, delocalization implies that (E_{Th})/(mean level spacing)>1.

→ The states that are within this energy scale satisfy the properties of full random matrices (level number variance)

Why Thouless?
Relation with the Thouless Energy

Level number variance



Bertrandt & García-García
Phys. Rev. B **94**, 144201 (2016).

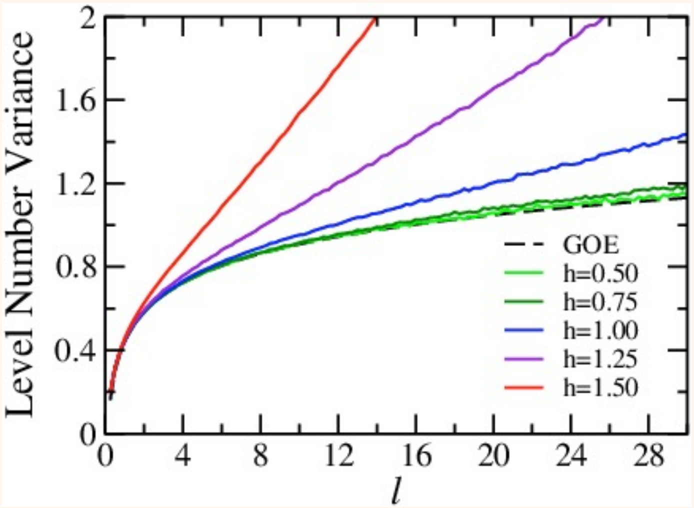
The level statistics of disordered systems follows that of full random matrices up to the Thouless energy

$$E_{Th} \propto \frac{1}{t_{Th}}$$

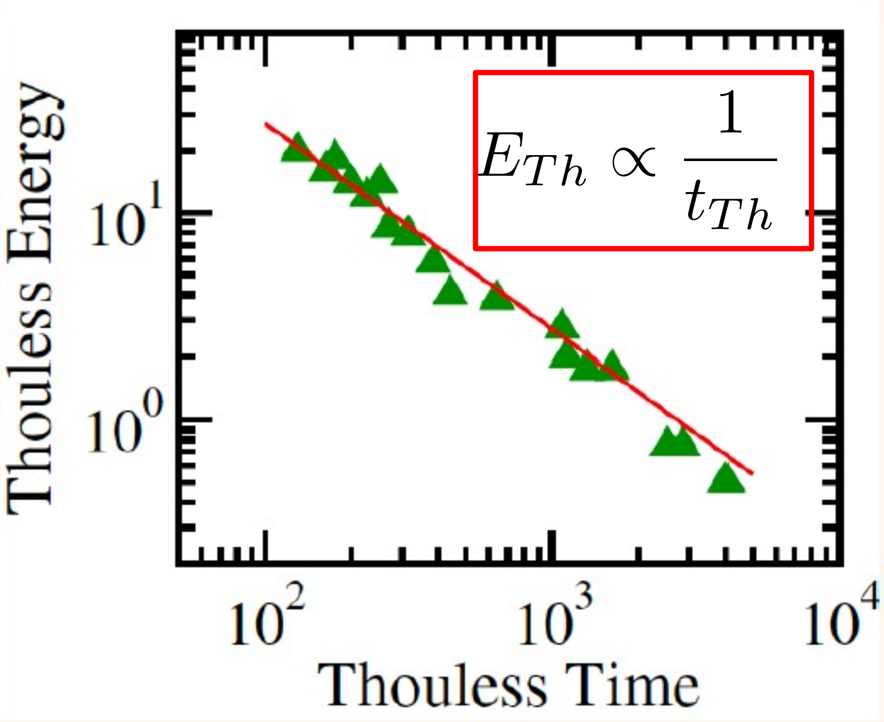
Schiulaz, Torres-Herrera & LFS,
PRB **99**, 174313 (2019)

Why Thouless Time?
Relation with the Thouless Energy

Level number variance



The level statistics of disordered systems follows that of full random matrices up to the Thouless energy



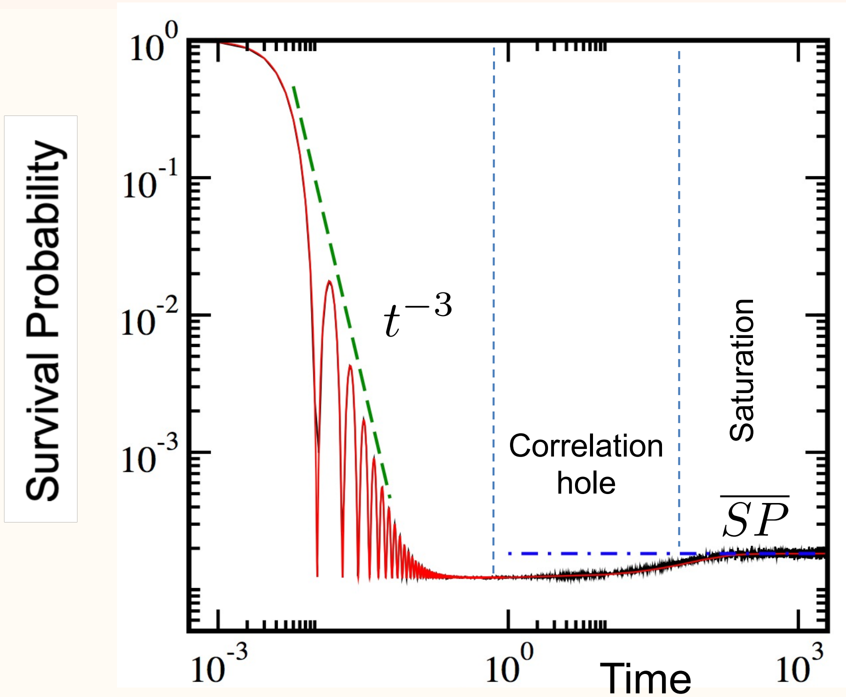
Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Time for Saturation of the Dynamics

GOE Full Random Matrices

PRB **99**, 174313 (2019)

$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2}-b_2\left(\frac{\Gamma t}{2D}\right)\right]+\overline{SP}$$



$$b_2\left(\frac{\Gamma t}{2D}\right)\to\frac{D^2}{3\Gamma^2 t^2}\quad \text{for}\quad \frac{\Gamma t}{D}\gg 1$$

$$\frac{|\langle P_S(t)\rangle-\overline{P_S}|}{\overline{P_S}}\approx\frac{1-\overline{P_S}}{\overline{P_S}(D-1)}\frac{D^2}{3\Gamma^2 t^2}\approx\left(\frac{D}{3\Gamma t}\right)^2$$

$$\frac{|\langle P_S(t_R)\rangle-\overline{P_S}|}{\overline{P_S}}\sim\delta$$

$$t_R\propto\frac{D}{\Gamma\sqrt{\delta}}$$

HEISENBERG time = inverse of the mean level spacing

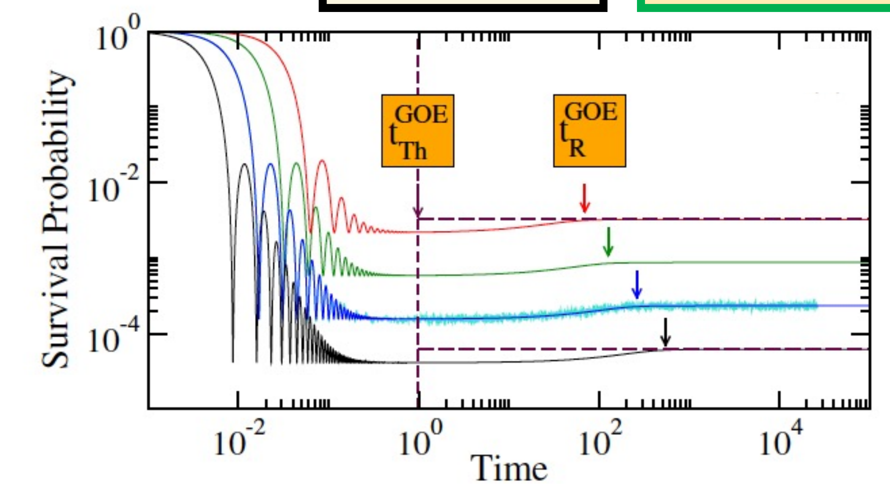
Any model

Thermalization time = Heisenberg time

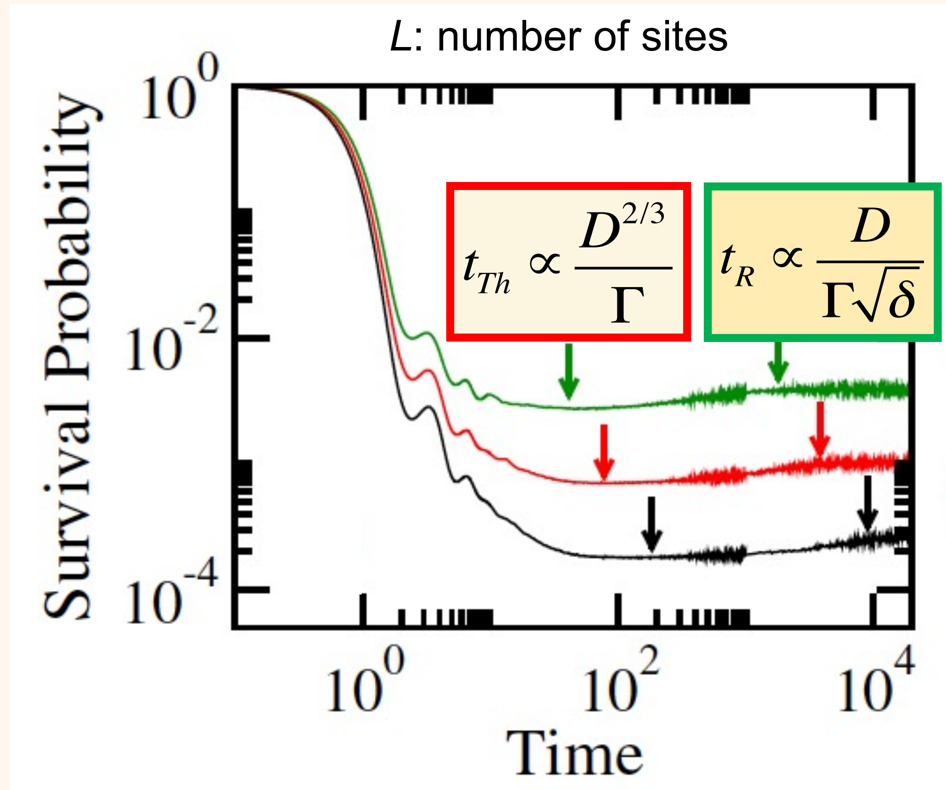
GOE full random matrix

$$t_{Th} = \left(\frac{3}{\pi}\right)^{1/4}$$

$$t_R \propto \frac{D}{\Gamma\sqrt{\delta}}$$



Realistic CHAOTIC spin-1/2 model



Schiulaz, Torres-Herrera & LFS,
PRB **99**, 174313 (2019)

Realistic spin-1/2 model

Chaotic h=0.5J

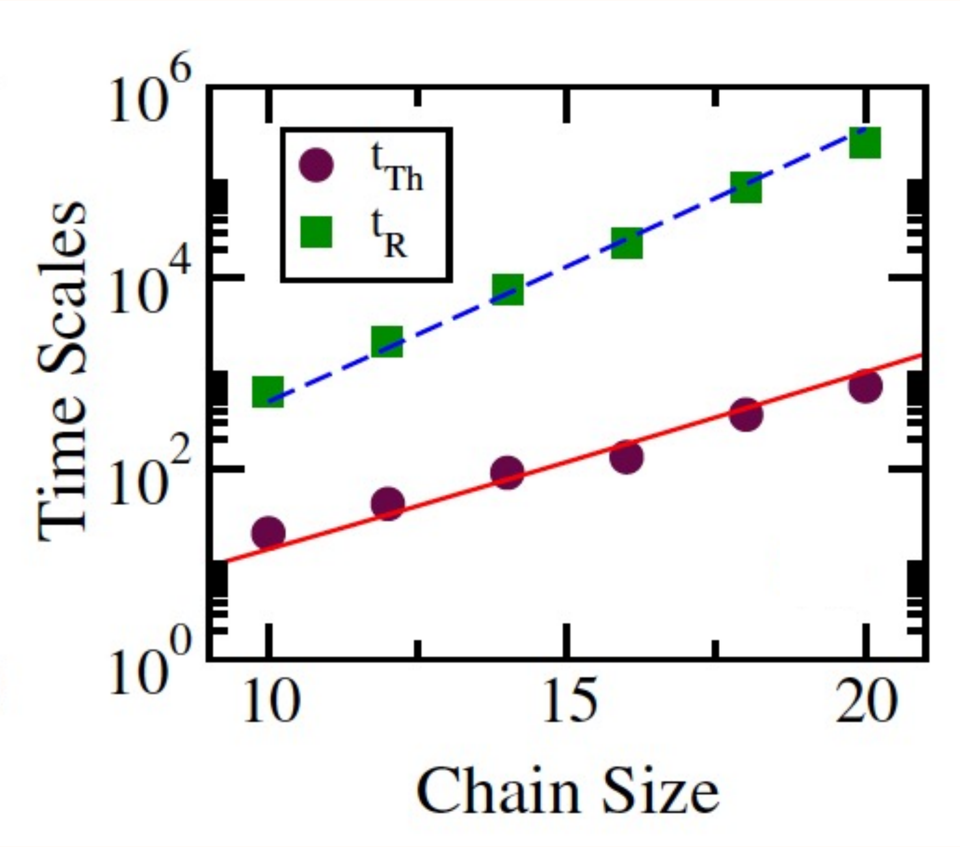
Thouless time

$$t_{Th} \propto \frac{D^{2/3}}{\Gamma}$$

$$\Gamma \propto \sqrt{L} \qquad D = \frac{L!}{(L/2)!^2}$$

Relaxation time

$$t_R \propto \frac{D}{\Gamma\sqrt{\delta}}$$



$$t_R \propto D^{1/3} t_{Th}$$

hole stretches with L

Exercise PR(t) and Shannon(t)

*) Compute the evolution of PR and Shannon by considering the spread of the initial state in other basis vectors.
*) Use GOE random matrices
*) Use spin models
*) Do averages.

Spin Autocorrelation Function

Spin Autocorrelation Function

$$I(t) = \frac{1}{L} \sum_{k=1}^{L} \langle \Psi(0) | \sigma_k^z e^{iHt} \sigma_k^z e^{-iHt} | \Psi(0) \rangle$$

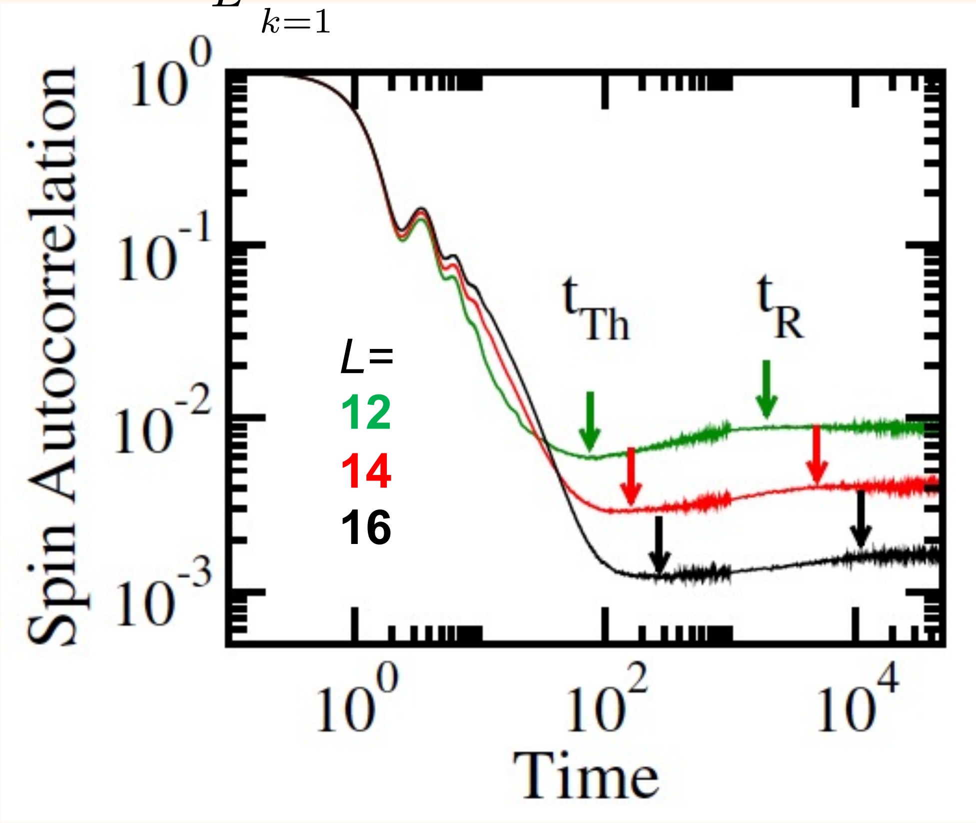
Local in space
Nonlocal in time
Similar to the density imbalance (experimental)

Schiulaz, Torres-Herrera & LFS,
PRB **99**, 174313 (2019)

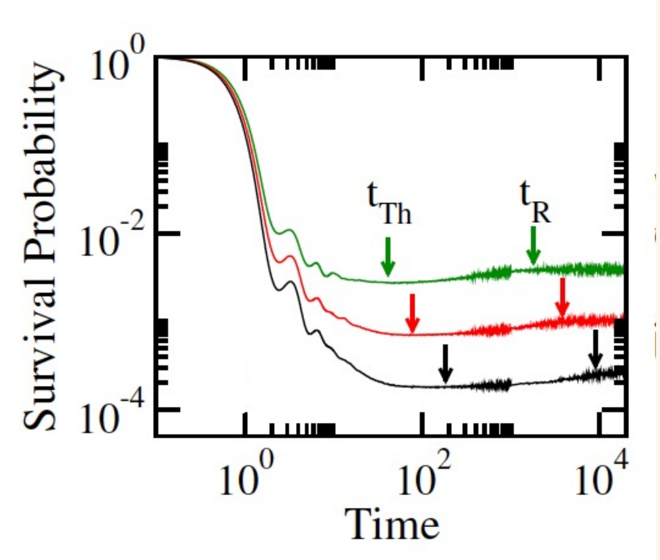
Spin Autocorrelation Function

Chaotic h=0.5J

$$I(t) = \frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\sigma_k^z e^{iHt}\sigma_k^z e^{-iHt}|\Psi(0)\rangle$$



Torres, García-García, LFS
PRB **97**, 060303 (R) (2018)

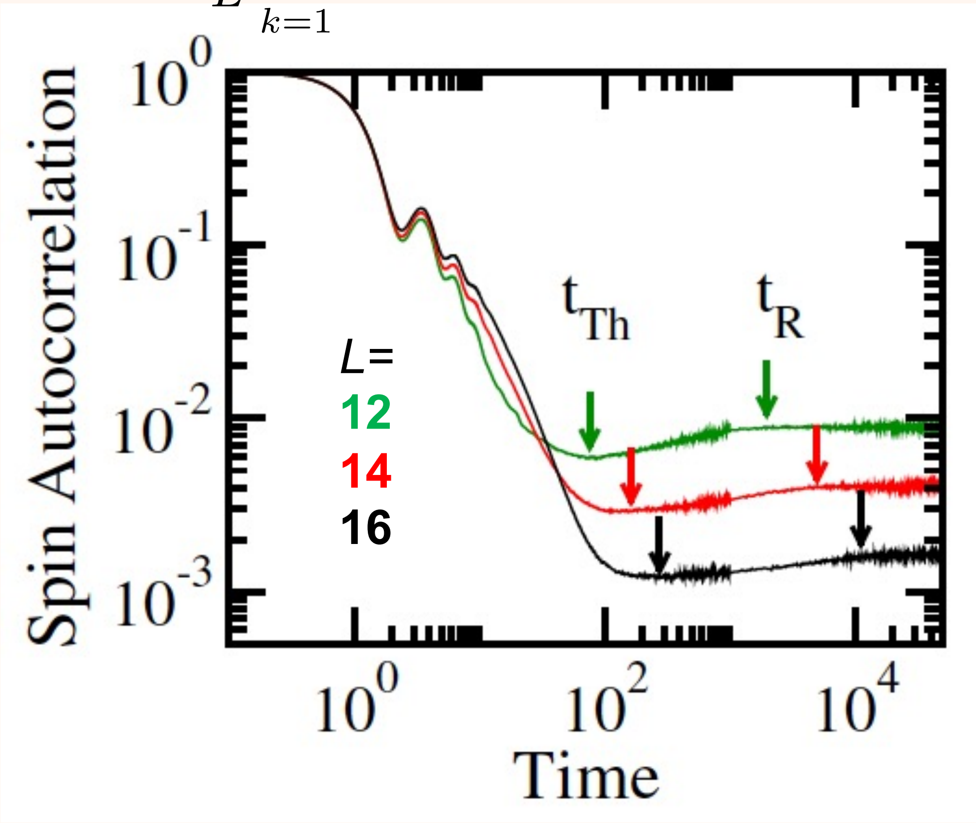


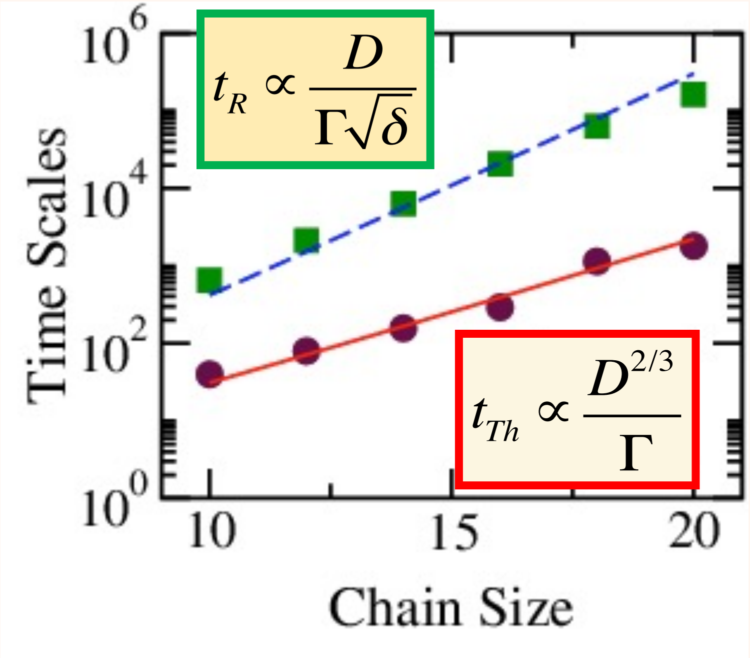
Schiulaz, Torres-Herrera, LFS,
PRB **99**, 174313 (2019)

Spin Autocorrelation Function

Chaotic h=0.5J

$$I(t) = \frac{1}{L}\sum_{k=1}^{L} \langle \Psi(0)|\sigma_k^z e^{iHt} \sigma_k^z e^{-iHt}|\Psi(0)\rangle$$





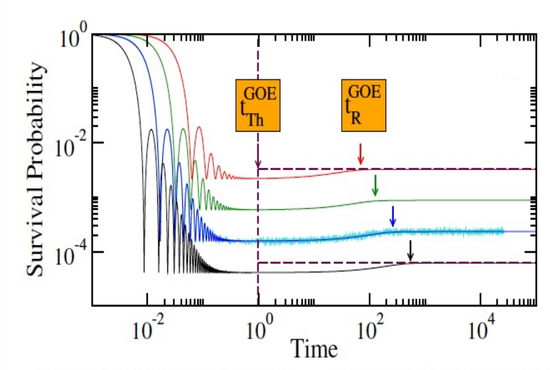
Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

EQUILIBRATION TIME

Correlation Hole and System Size

Relative depth of the correlation hole:

$$\kappa = \frac{\langle \overline{O} \rangle - \langle O \rangle_{\min}}{\langle \overline{O} \rangle}$$



Survival probability for GOE matrices:

$$\overline{SP} = 3/D$$

$$SP_{min} = 2/D$$

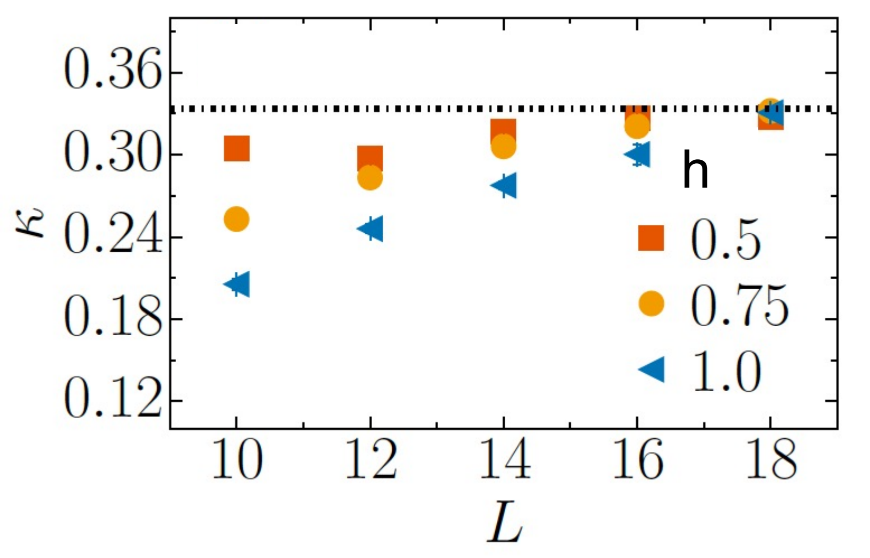
$$\kappa = 1/3$$

Thermalization time in many-body quantum systems
Lezama, Torres, Bernal, Bar Lev & LFS,
PRB **104**, 085117 (2021)

Correlation Hole for the Survival Probability

Relative depth of the correlation hole:

$$\kappa = \frac{\langle \overline{O} \rangle - \langle O \rangle_{\min}}{\langle \overline{O} \rangle}$$



Survival probability for realistic chaotic systems:

$$\kappa = 1/3$$

$$h \leq J = 1$$

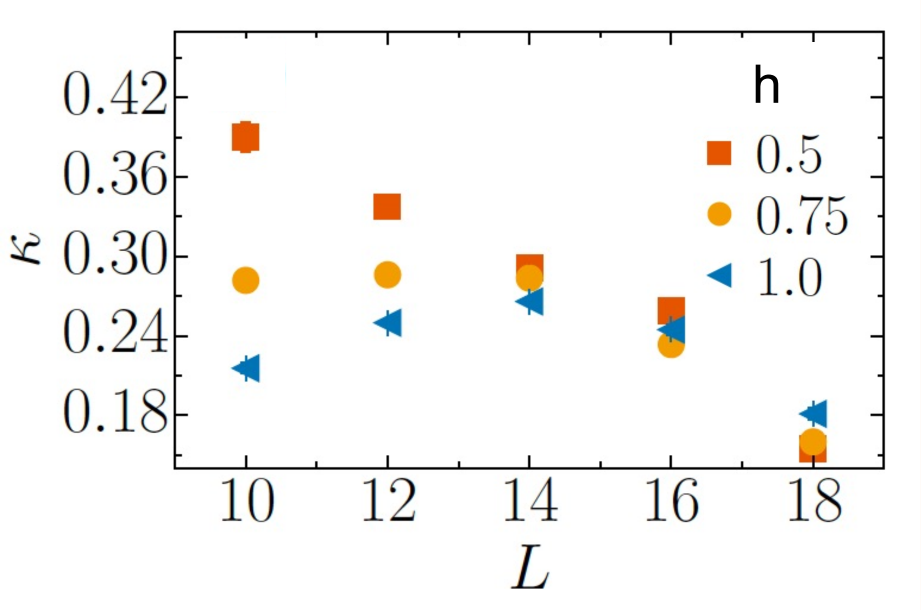
Thermalization time in many-body quantum systems
Lezama, Torres, Bernal, Bar Lev & LFS,
PRB **104**, 085117 (2021)

Correlation Hole for the Spin Autocorrelation Function

Relative depth of the correlation hole: $$\kappa = \frac{\langle \overline{O} \rangle - \langle O \rangle_{\min}}{\langle \overline{O} \rangle}$$

Spin autocorrelation function for realistic chaotic systems:

$h \leq J = 1$

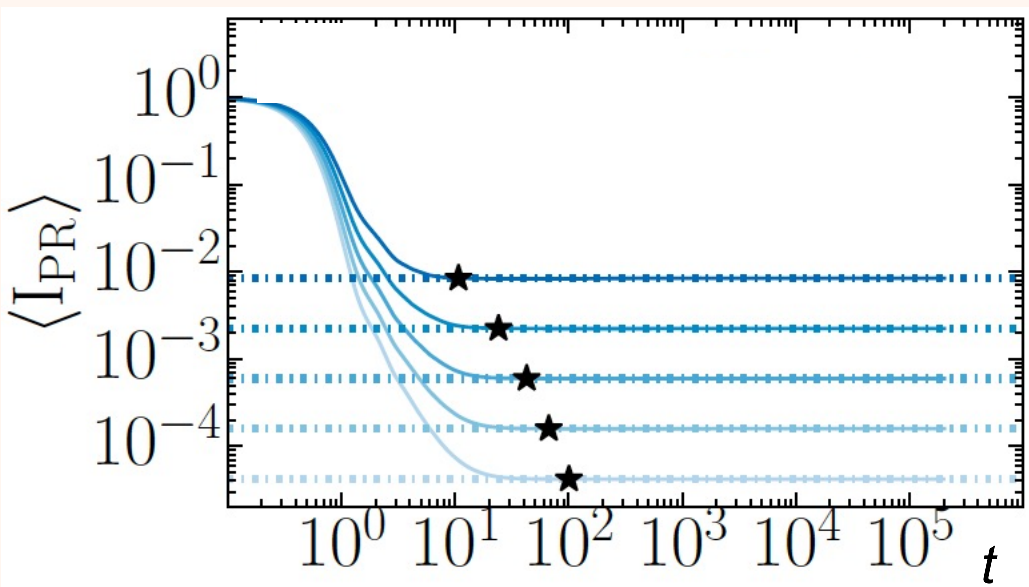


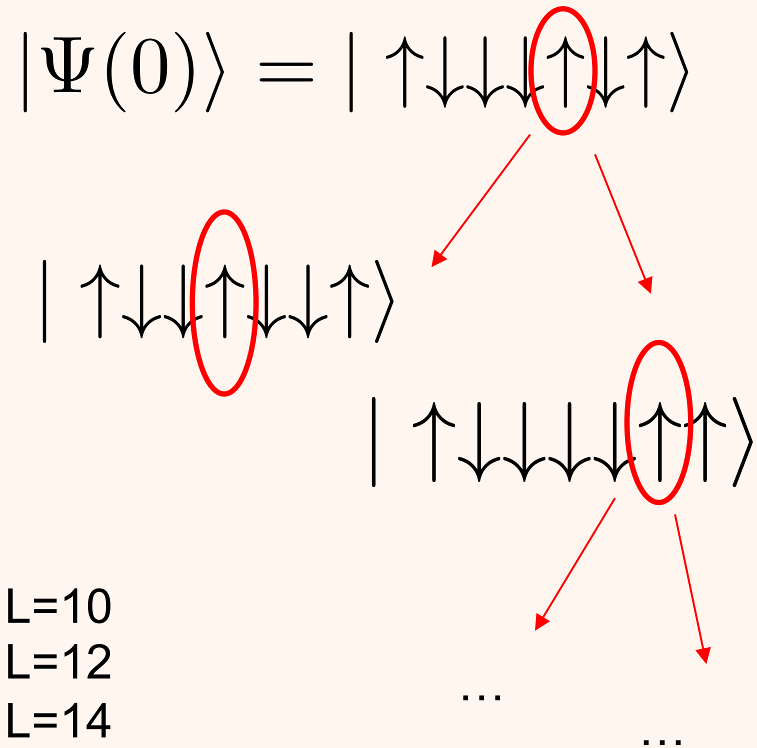
Thermalization time in many-body quantum systems
Lezama, Torres, Bernal, Bar Lev & LFS,
PRB **104**, 085117 (2021)

Inverse Participation Ratio

$$\mathrm{I_{PR}}(t) = \sum_n |\langle \phi_n | \Psi(t) \rangle|^4$$

-Log(IPR) =
2nd-order Rényi entropy





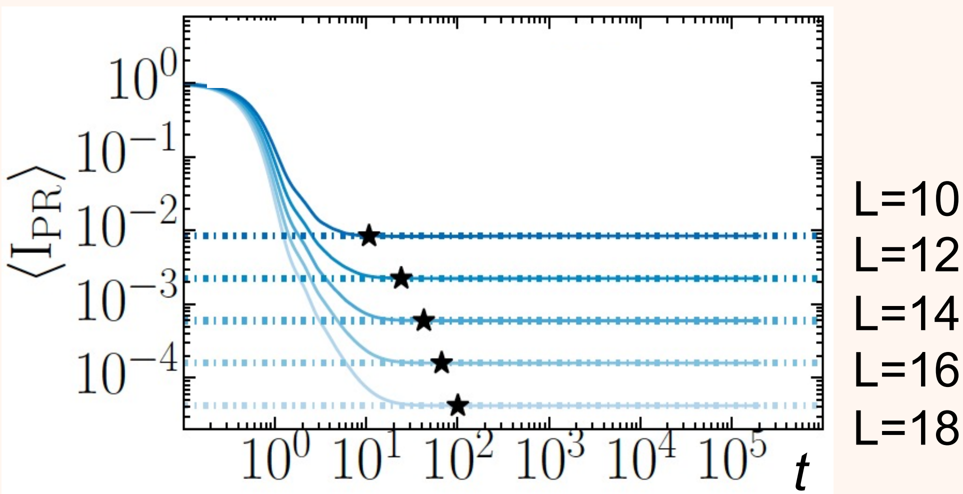
Nonlocal in space
Local in time

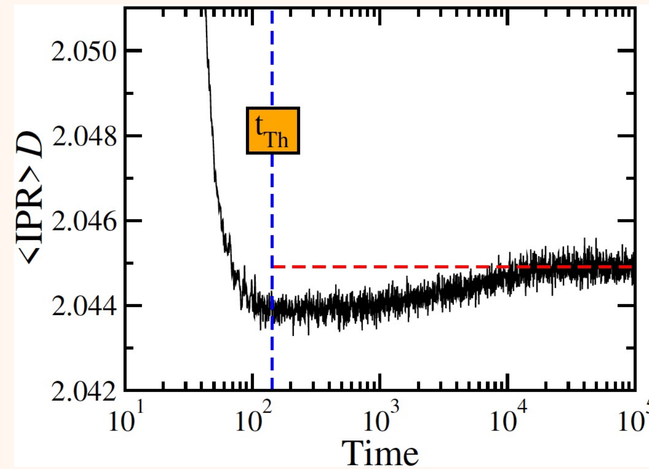
Lea F. Santos, Yeshiva University

PRB **104**, 085117 (2021)

Inverse Participation Ratio

$$I_{PR}(t) = \sum_{n} |\langle \phi_n | \Psi(t) \rangle|^4$$





Thermalization time in many-body quantum systems
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PRB **104**, 085117 (2021)

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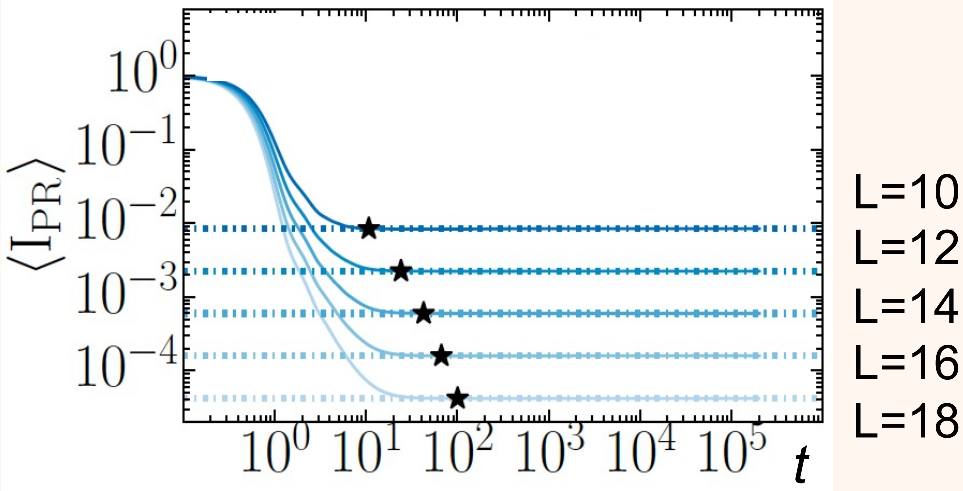
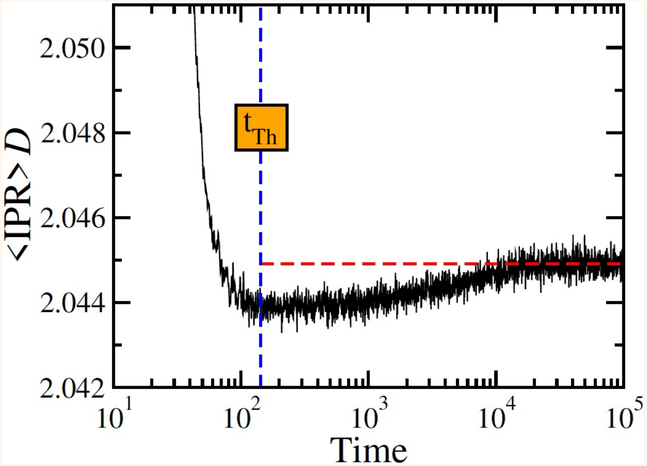
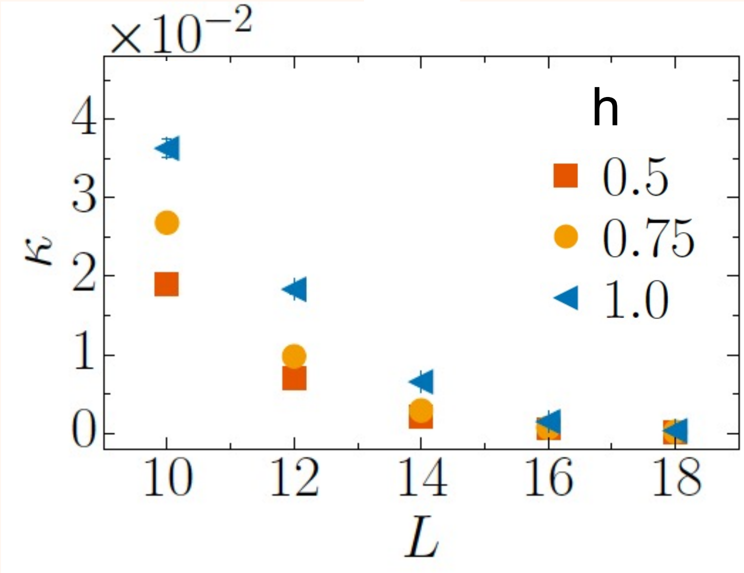
$$I_{PR}(t) = \sum_n |\langle \phi_n | \Psi(t) \rangle|^4$$





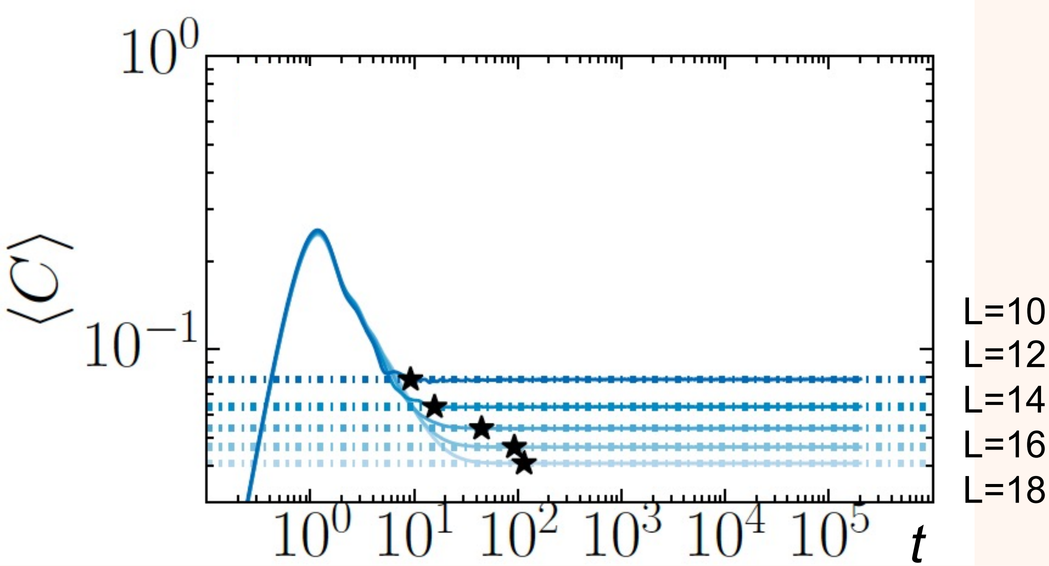


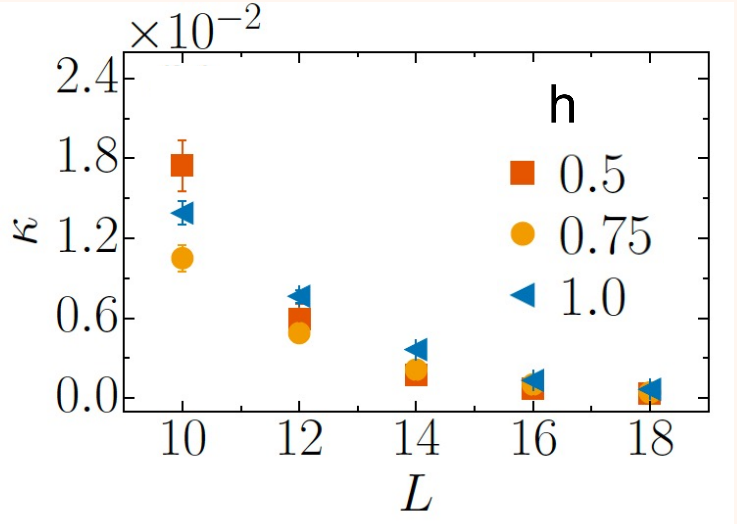
PRB **104**, 085117 (2021)

Lea F. Santos, Yeshiva University

Spin-Spin Correlation Function

$$C(t) = \frac{4}{L}\sum_{i=1}\left[\langle\Psi(t)|\,\hat{S}_i^z\hat{S}_{i+1}^z\,|\Psi(t)\rangle - \langle\Psi(t)|\,\hat{S}_i^z\,|\Psi(t)\rangle\,\langle\Psi(t)|\,\hat{S}_{i+1}^z\,|\Psi(t)\rangle\right]$$

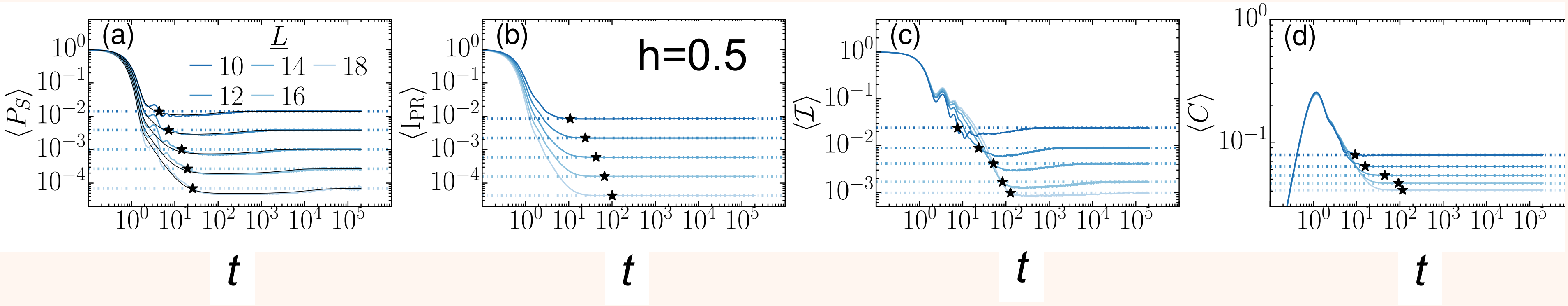


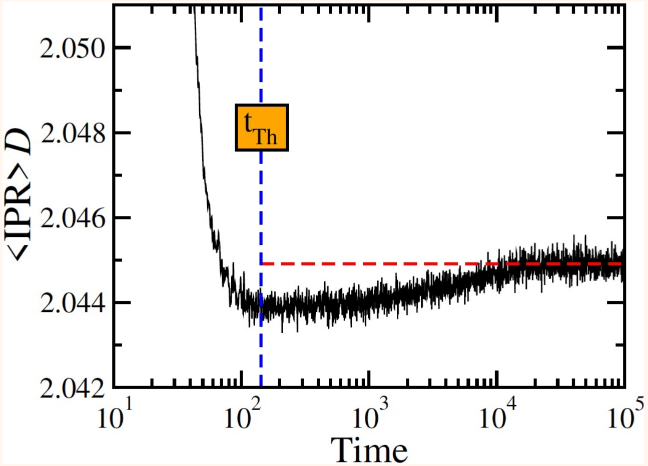


PRB **104**, 085117 (2021)

Lea F. Santos, Yeshiva University

Thermalization Time
Before the Correlation Hole



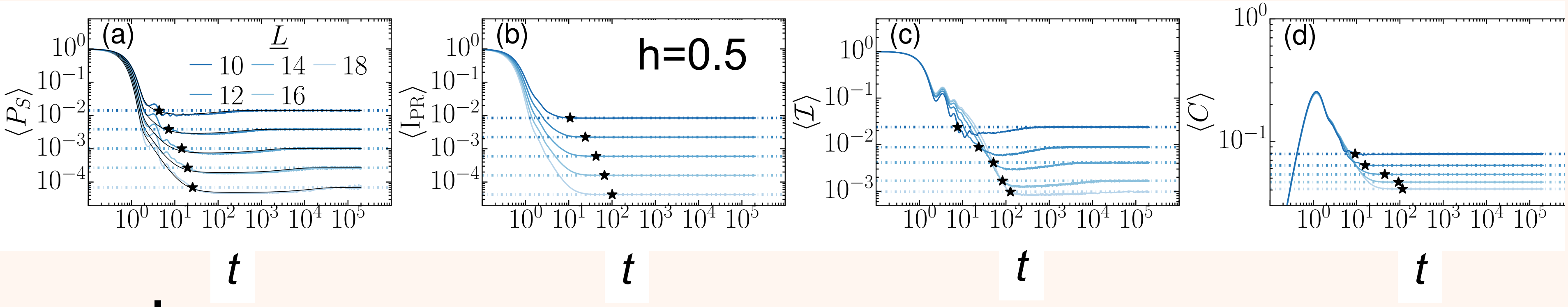


Difficulties:

- No analytical guidance
- Smoothening
- Long times
- Few system sizes

Thermalization time in many-body quantum systems
Lezama, Torres, Bernal, Bar Lev & LFS,
PRB **104**, 085117 (2021)

Thermalization Time
Before the Correlation Hole



$$t^* \propto e^L$$

(deep chaotic region)
h=0.5

$$t^* \propto L^\gamma$$

$$\gamma > 3$$

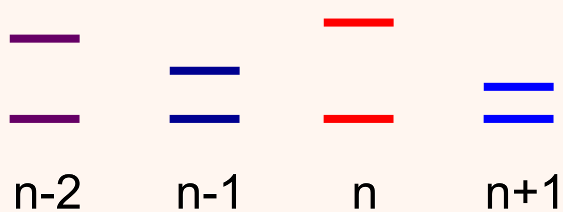
MANY-BODY LOCALIZATION

1D Anderson Localization

$$H = \sum_{n=1}^{L} \frac{\varepsilon_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} \left(\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right)$$

$$H = \sum_{n=1}^{L} \epsilon_n c_n^\dagger c_n - J \sum_{n=1}^{L-1} \left(c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n \right)$$

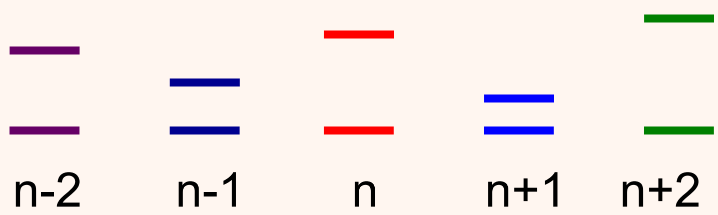
1D Anderson localization



1D disordered Spin-1/2 Model with interaction

$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n=1}^{L} \frac{J}{4}\left(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \sigma_n^z\sigma_{n+1}^z\right)$$

Onsite disorder
h: disorder strength



Random numbers

$$h_n \in [-h, h]$$

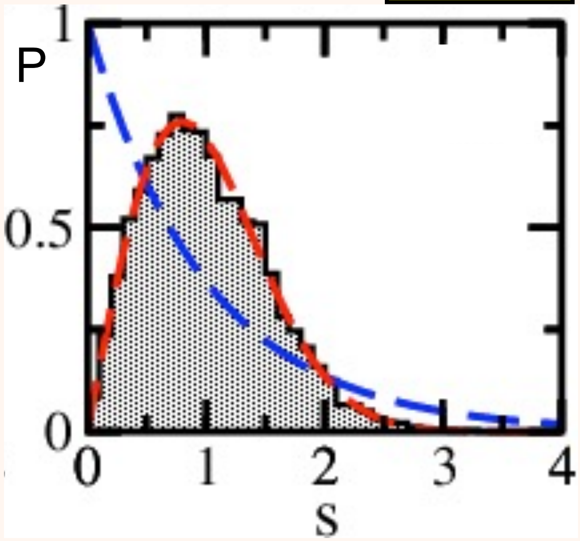
L: number of sites

1D Disordered Spin-1/2 Model

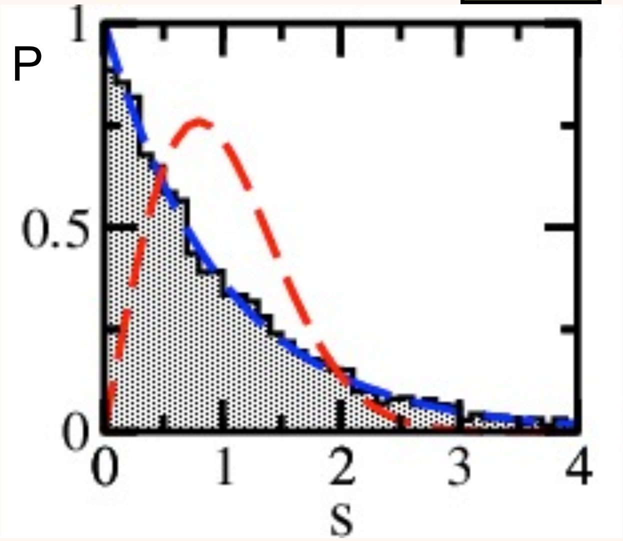
$$H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \sigma_n^z \sigma_{n+1}^z)$$

$h_n \in [-h, h]$

Chaotic h=0.5J



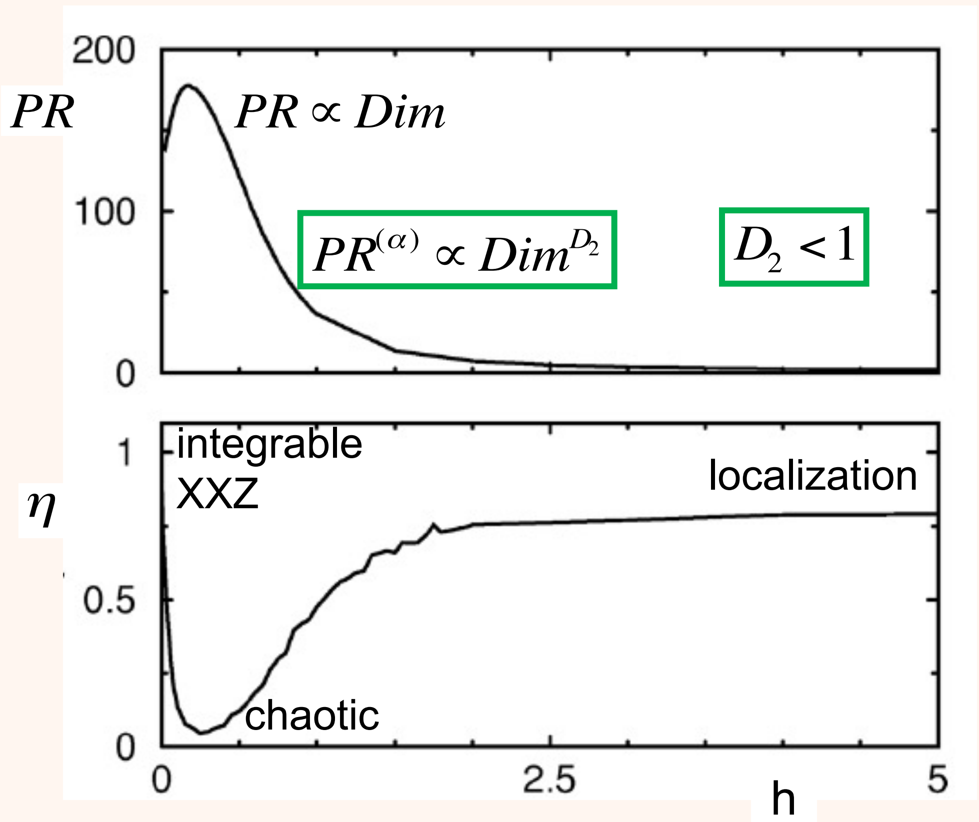
Localized $h>h_c$



LFS, Rigolin, Escobar
Entanglement versus chaos in disordered spin chains
PRA **69**, 042304 (2004)

Integrable-chaos-integrable

$$H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$



Participation Ratio

$$PR^{(\alpha)} \equiv \frac{1}{\sum_{i=1}^{D} | c_i^{(\alpha)} |^4}$$

$$\psi^{(\alpha)} = \sum_{i=1}^{D} c_i^{(\alpha)} \phi_i$$

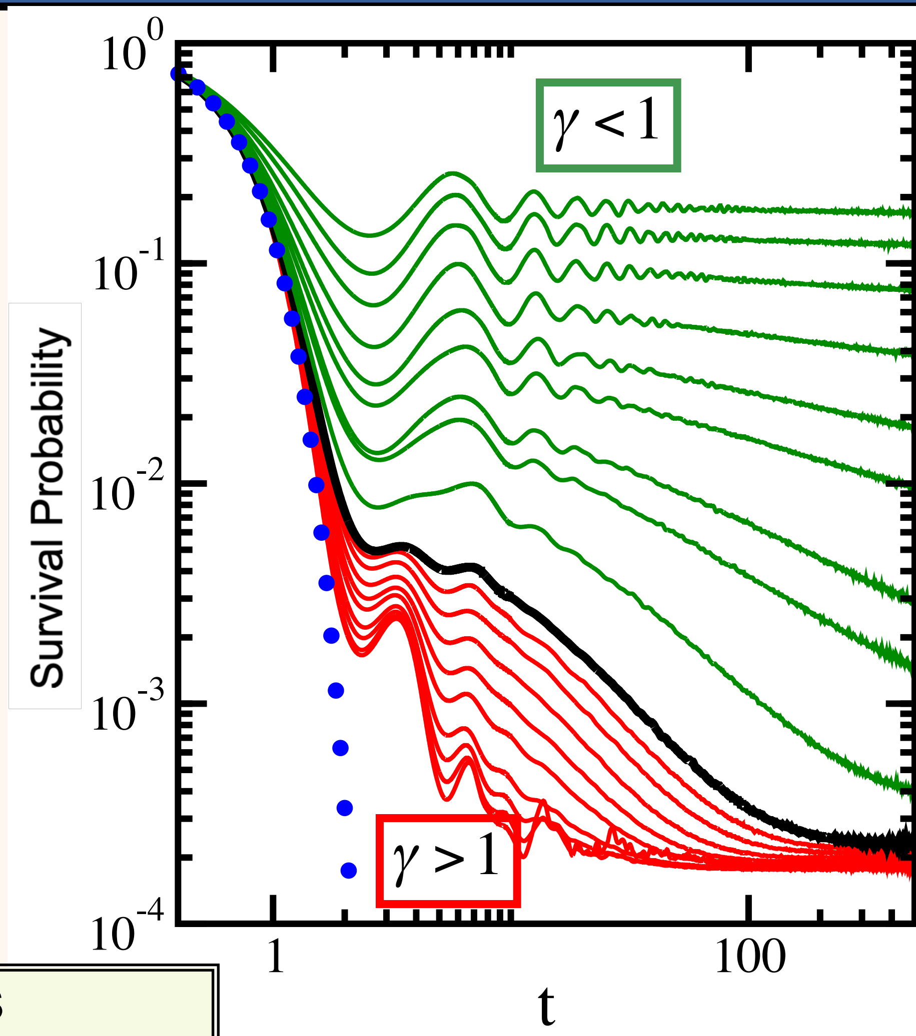
$$\eta = \frac{\int_0^{s_0} [P(s) - P_{WD}(s)] ds}{\int_0^{s_0} [P_P(s) - P_{WD}(s)] ds},$$

PRA **69**, 042304 (2004)
PRB **92**, 01420 (2015)
Ann. Phys. **529**, 1600284 (2017)

Power-law exponent

$t^{-\gamma}$

$PR^{(\alpha)} \propto Dim^{D_2}$



$PR^{(\alpha)} \propto Dim^{D_2}$

h>J

$t^{-\gamma}$

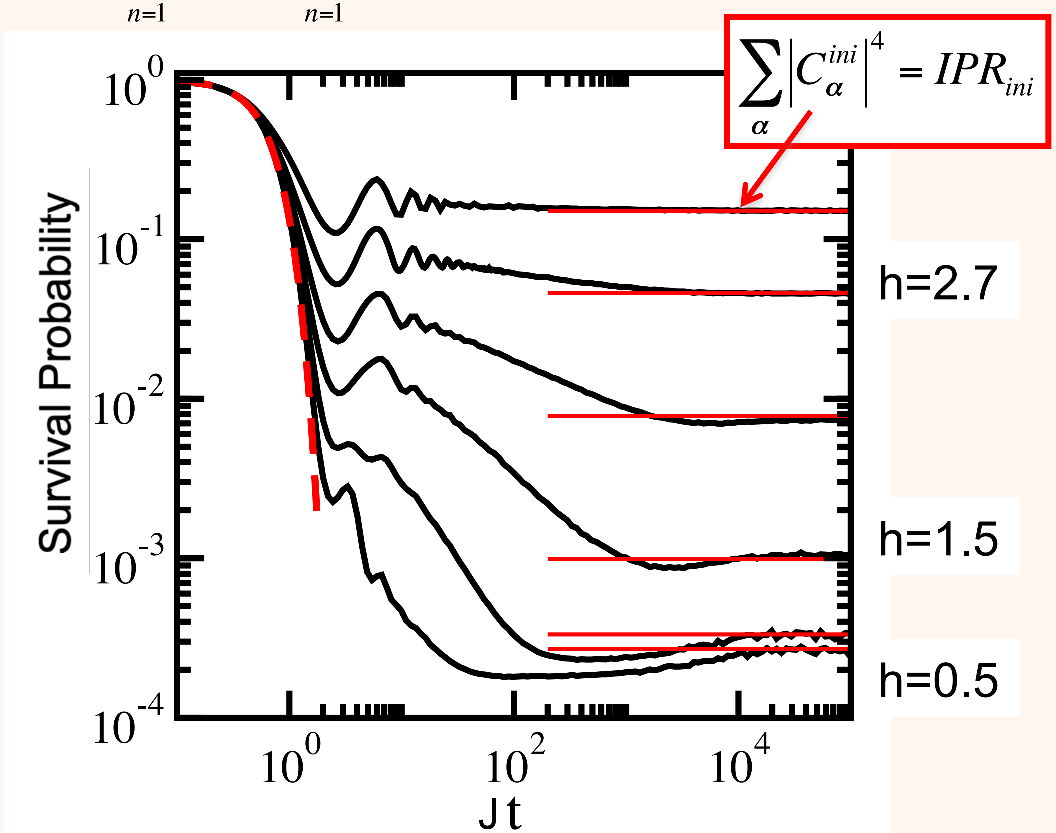
h<J

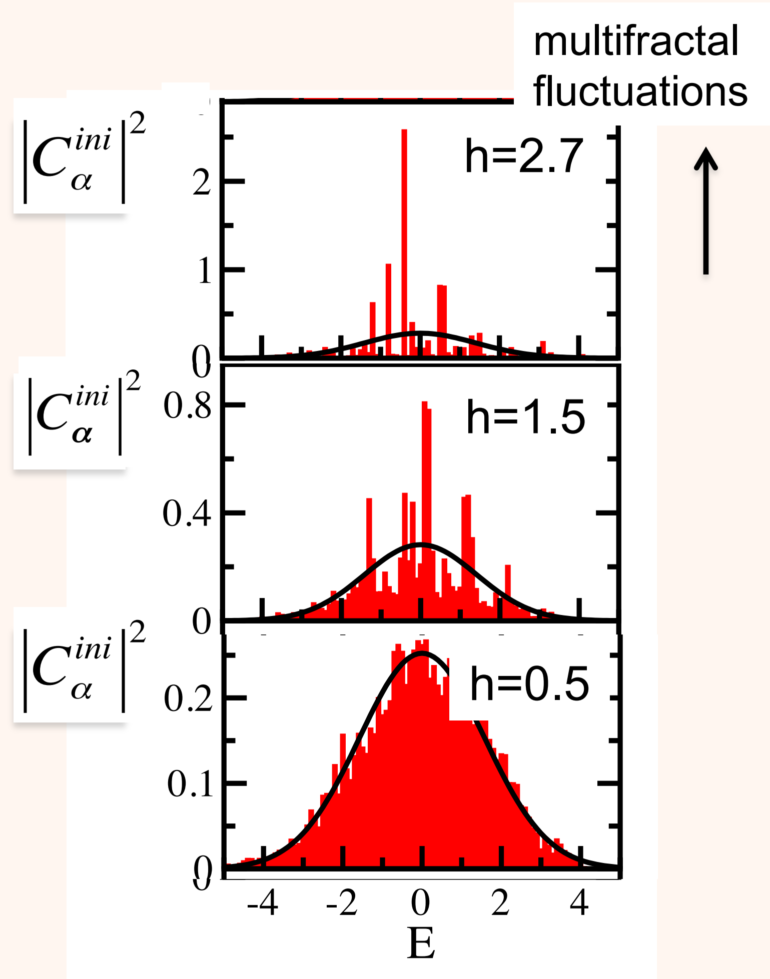
$PR^{(\alpha)} \propto Dim$

Torres & LFS
PRB **92**, 01420 (2015)
Ann. Phys. **529**, 1600284 (2017)

Sparse LDOS
System with strong disorder

$$H_{final} = \sum_{n=1}^{L} h_n S_n^z + \sum_{n=1}^{L} J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + S_n^z S_{n+1}^z)$$





L=16, 8 up spins

Torres & LFS
PRB **92**, 01420 (2015)

Power-law Exponent and D_2

h>J

$$SP(t) = \left| \sum_{\alpha} |C_{\alpha}^{ini}|^2 e^{-iE_{\alpha}t} \right|^2$$

$$\langle SP(t) \rangle = \int G(E) e^{-iEt} dE$$

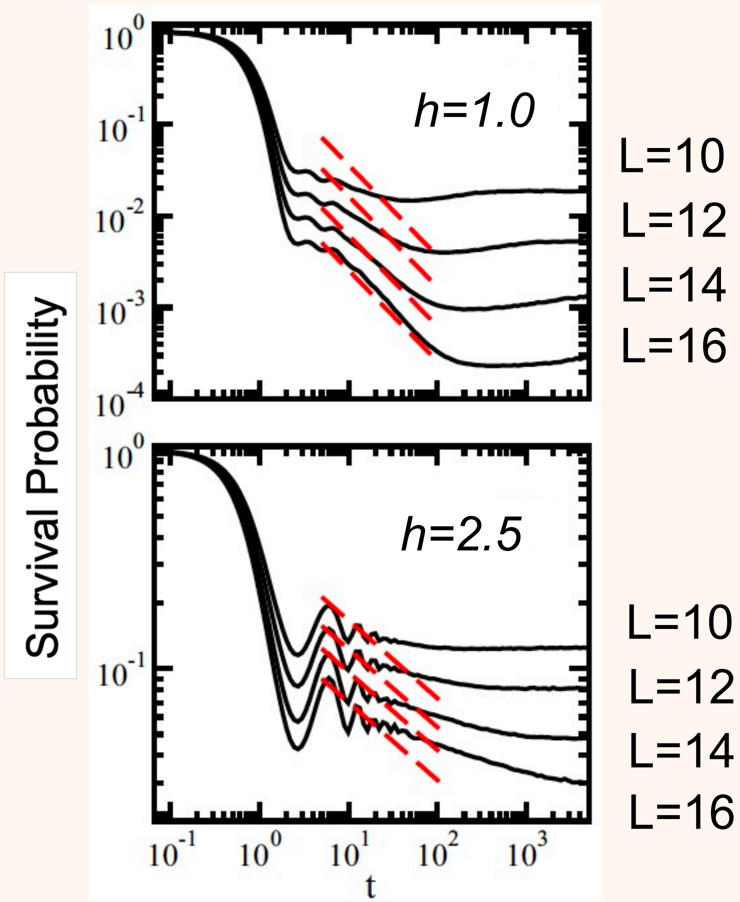
$$G(E) = \left\langle \sum_{\alpha\beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 \delta(E_{\alpha} - E_{\beta} - E) \right\rangle$$

$$G(E \to 0) \propto E^{\gamma - 1}$$

$$\langle SP(t) \rangle = \propto t^{-\gamma}$$

Torres & LFS
PRB **92**, 01420 (2015)

Power-law Exponent and D_2



h>J

Power-law exponent coincides with the generalized dimension D_2

$$t^{-\gamma}$$

$$PR^{(\alpha)} \propto Dim^{D_2}$$

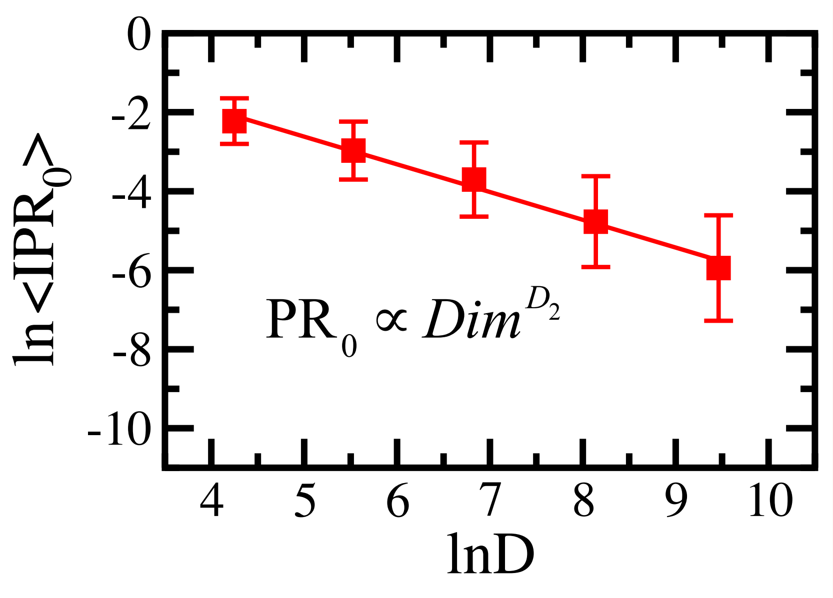
Finite-size effects

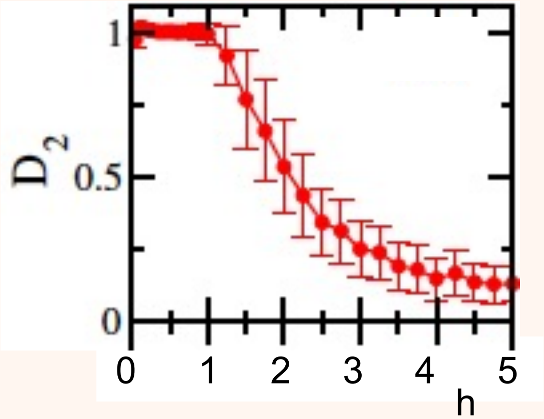
Torres & LFS
PRB **92**, 01420 (2015)

Power-law exponent: correlations

$$PR_0 = \frac{1}{\sum_\alpha \left| C_\alpha^{(0)} \right|^4} \propto (Dim)^{D_2} \Rightarrow D_2 < 1$$

h>J

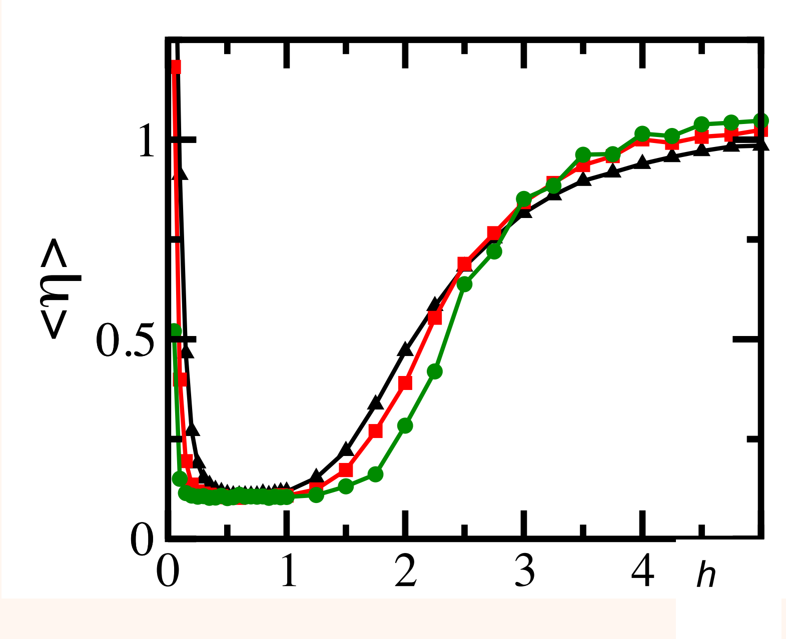




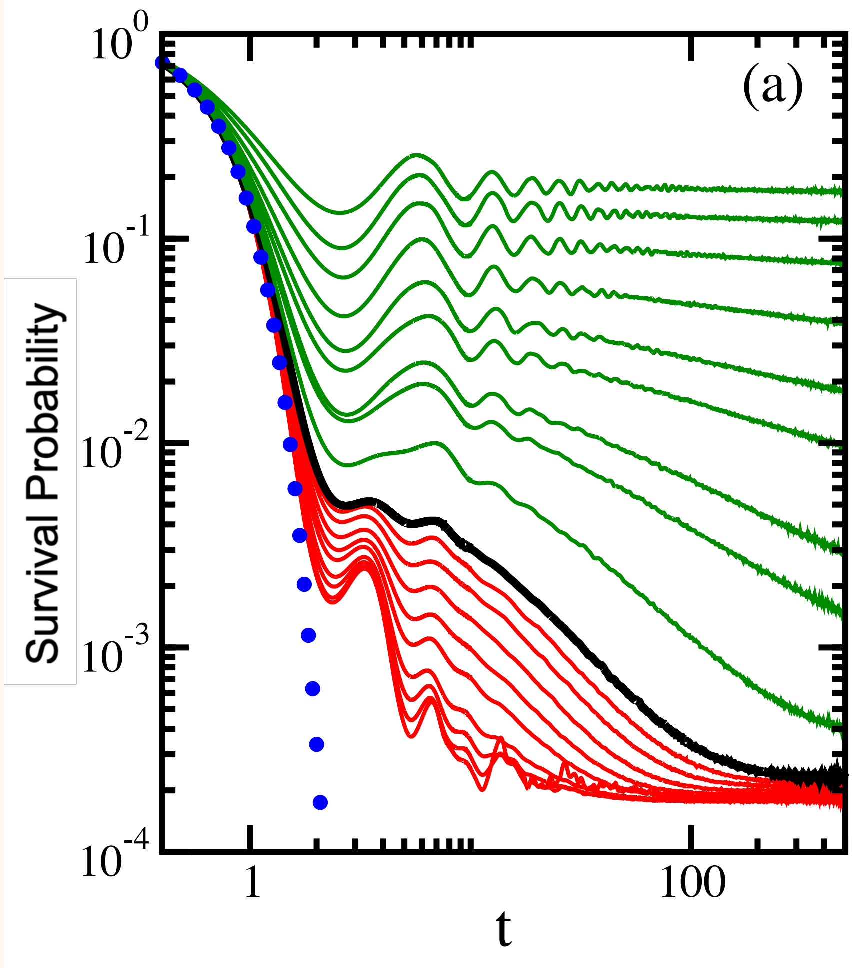
Generalized dimension
Multifractal dimension

Torres & LFS
PRB **92**, 01420 (2015)

Scaling Analysis?



η=0: chaos



Entropies: log behavior

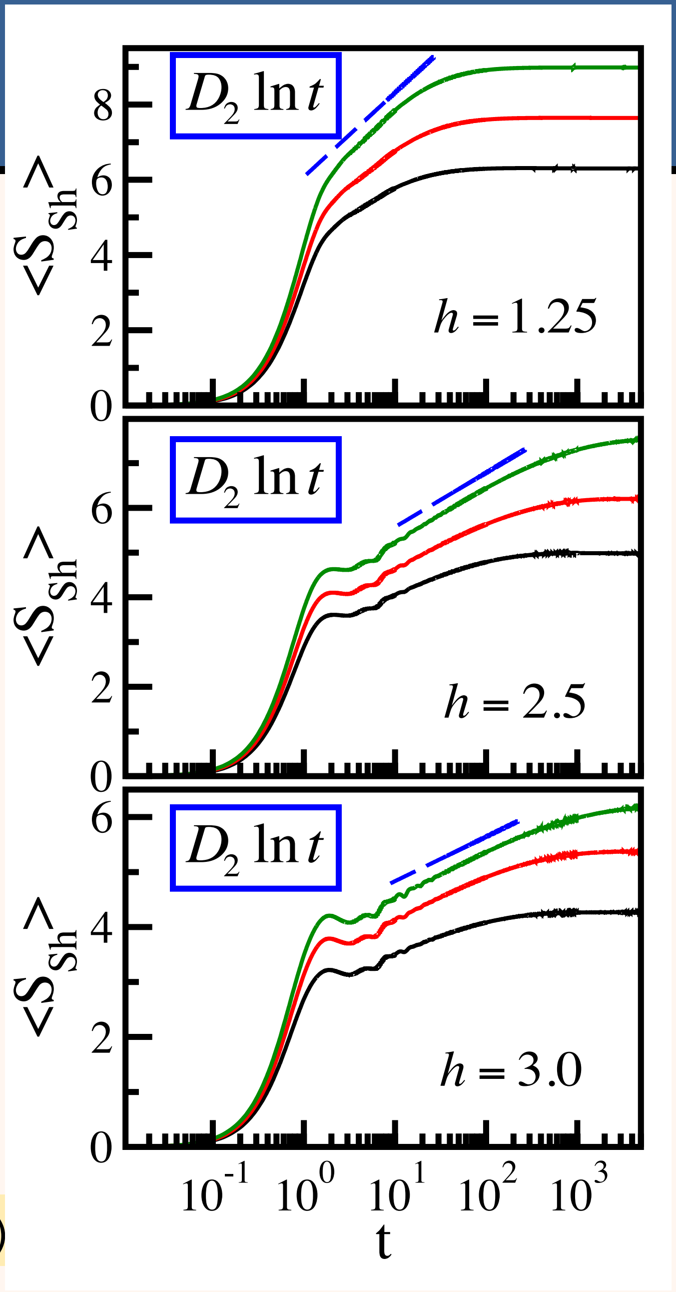
Intermediate level statistics: h>J

Nonergodic delocalized states: $PR^{(\alpha)} \propto Dim^{D_2}$

$D_2 < 1$

$$Sh(t) = -\sum_n W_n(t) \ln W_n(t)$$

$$W_n(t) = \left| \langle \phi_n | e^{-iHt} | \Psi(0) \rangle \right|^2$$

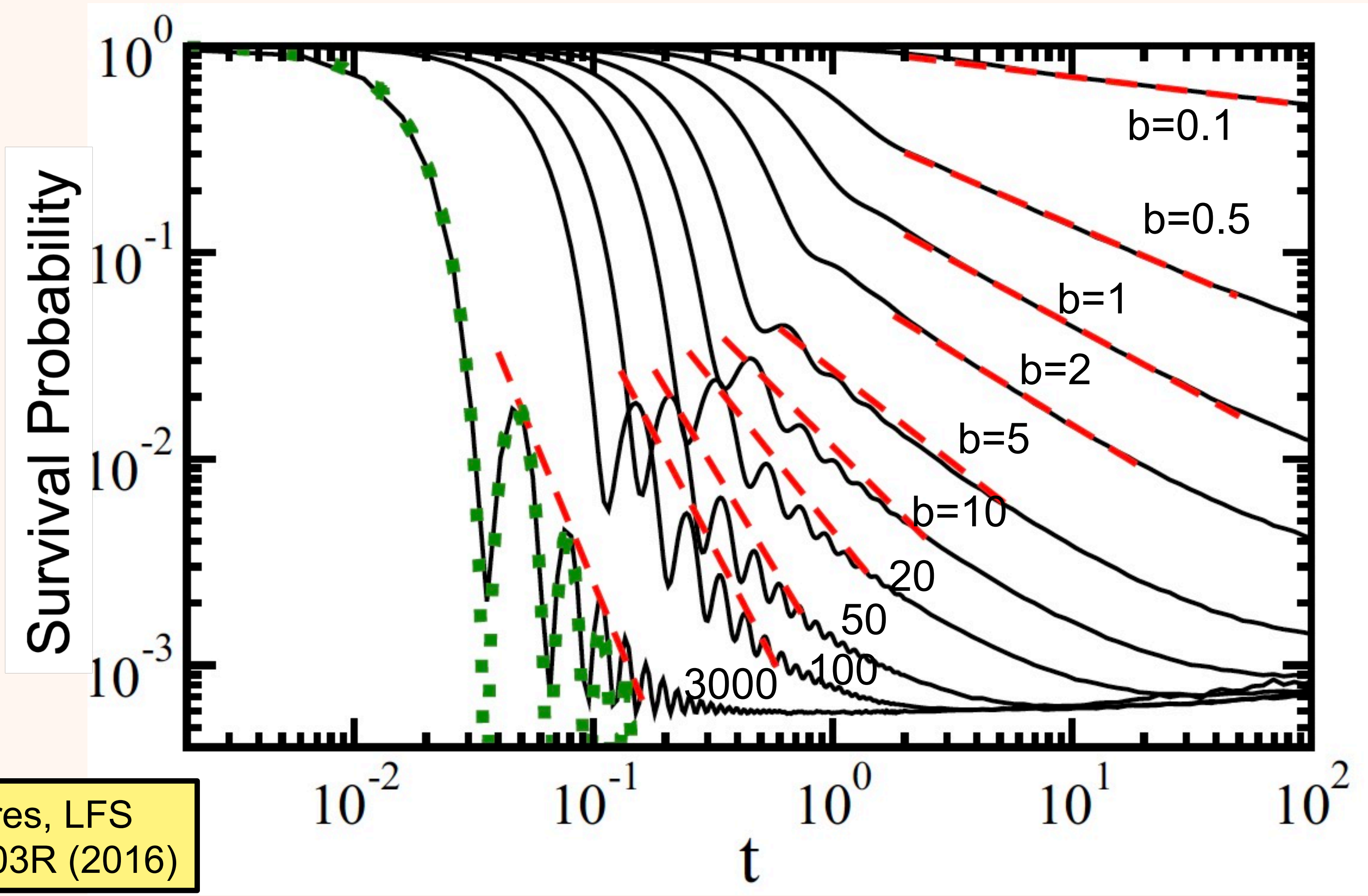


Torres & LFS, Ann. Phys. (2017)

Survival Probability: PBRM

$\langle H_{nn}^2 \rangle = 2$ $\quad 1/[1+|(n-m)/b|^2]$

D=3432

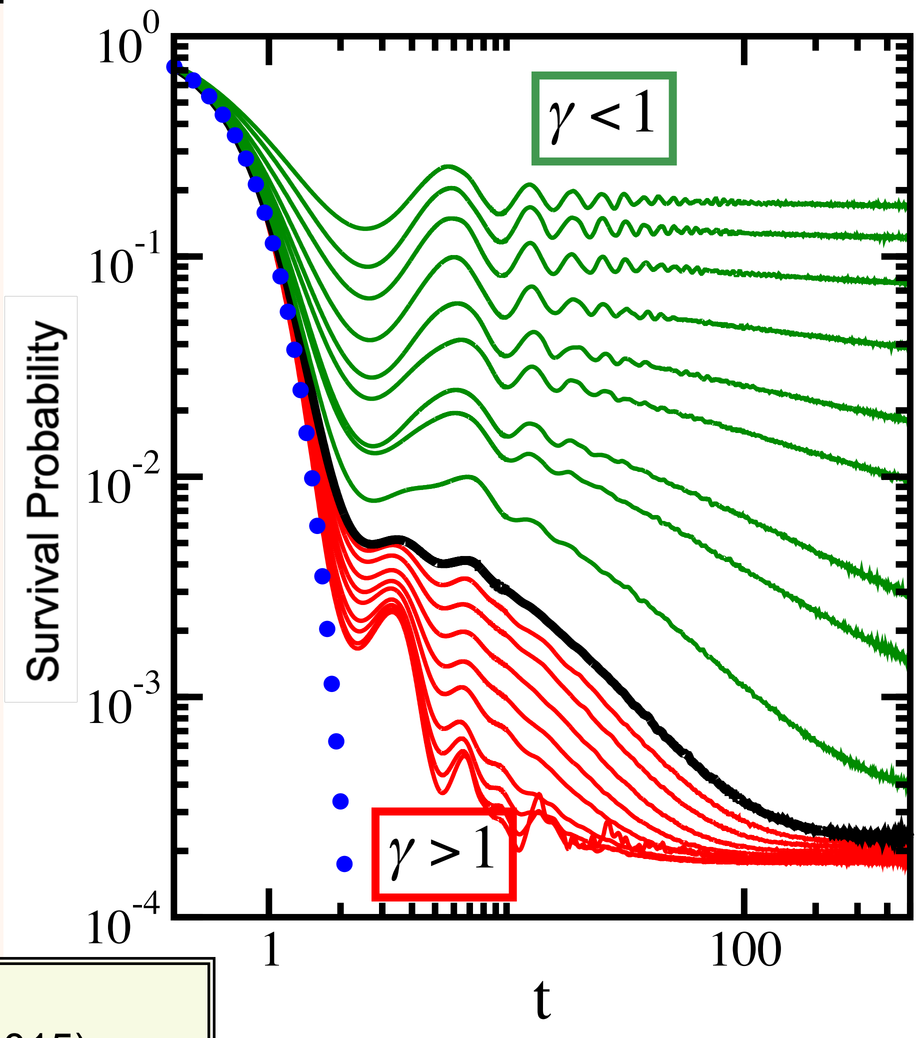


Távora, Torres, LFS
PRA **94**, 041603R (2016)

Power-law exponent

$$t^{-\gamma}$$

$$PR^{(\alpha)} \propto Dim^{D_2}$$



$$PR^{(\alpha)} \propto Dim^{D_2}$$

h>J

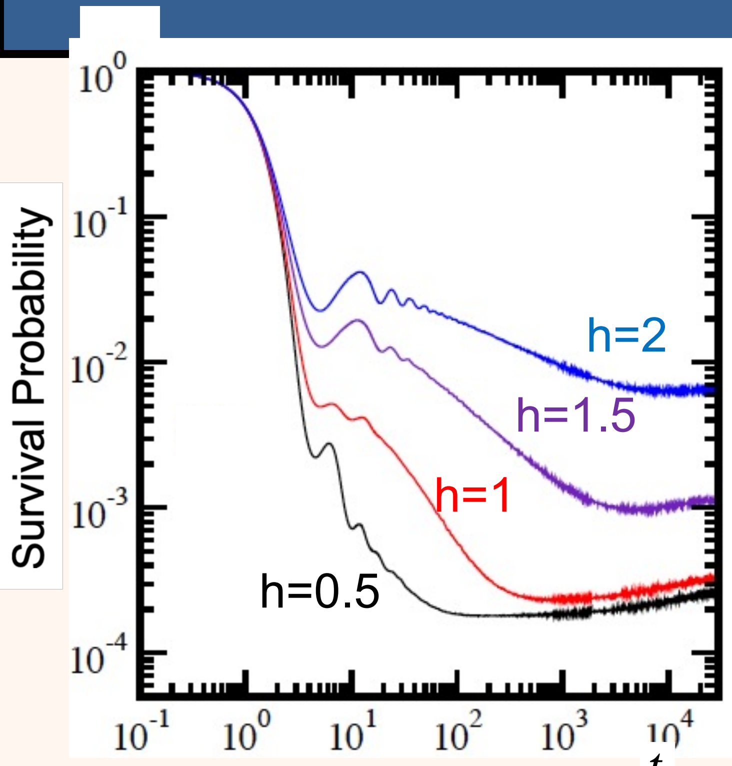
$$t^{-\gamma}$$

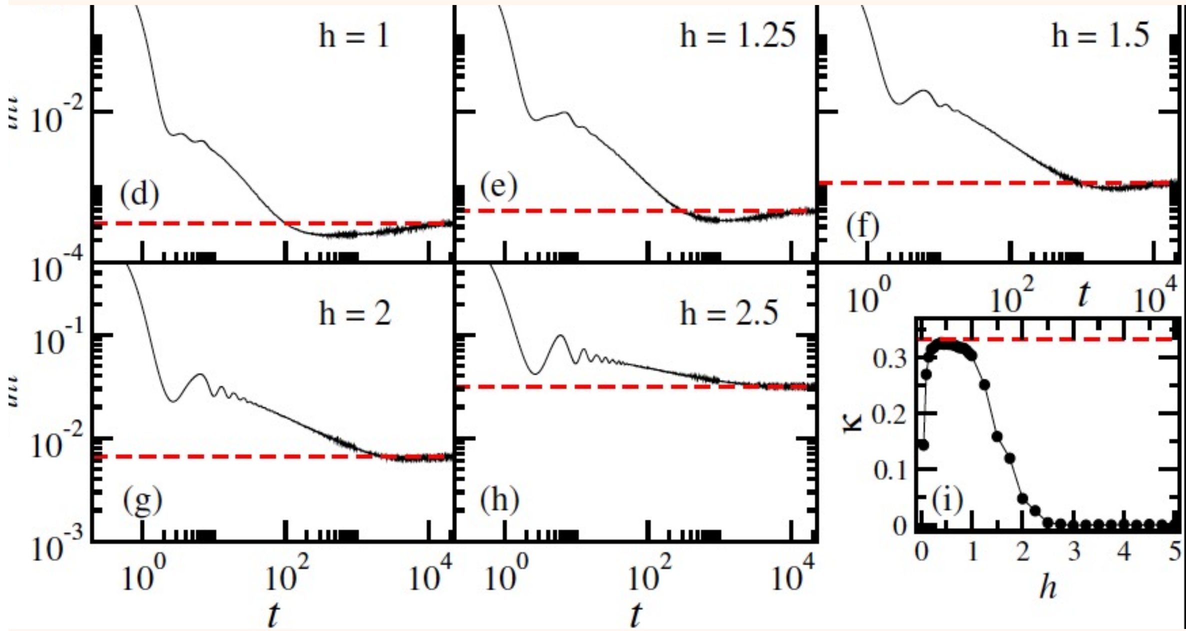
h<J

$$PR^{(\alpha)} \propto Dim$$

Torres & LFS
PRB **92**, 01420 (2015)
Ann. Phys. **529**, 1600284 (2017)

Correlation Hole and Disorder Strength





The hole becomes narrower and shallower.

Time for the minimum of the hole (T_{Th}) gets postponed as the disorder (h) increases.

Fixed system size L=16

Torres & LFS
Philos. Trans. A **65** (2017)

Thouless Time and Disorder Strength

Thouless dimensionless conductance $\frac{t_R}{t_{Th}} \propto \frac{E_{th}}{MLS}$

SURVIVAL PROBABILITY

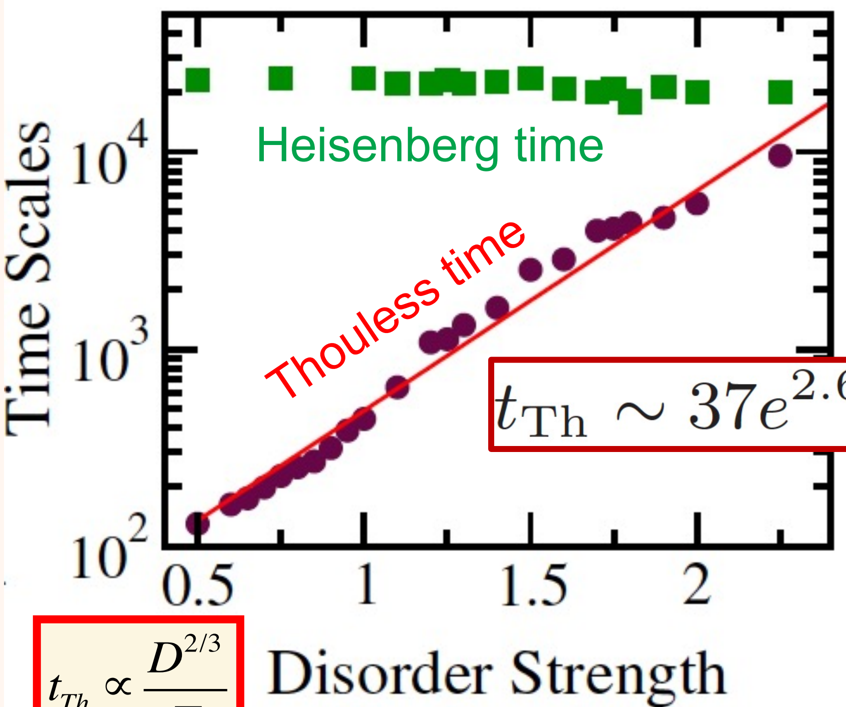
$t_R \propto \frac{D}{\Gamma\sqrt{\delta}}$

Delocalized (chaotic)

$$\frac{t_R}{t_{Th}} \propto e^{(L\ln 2)/3}$$

Toward localization

$$\frac{t_R}{t_{Th}} \to 1$$

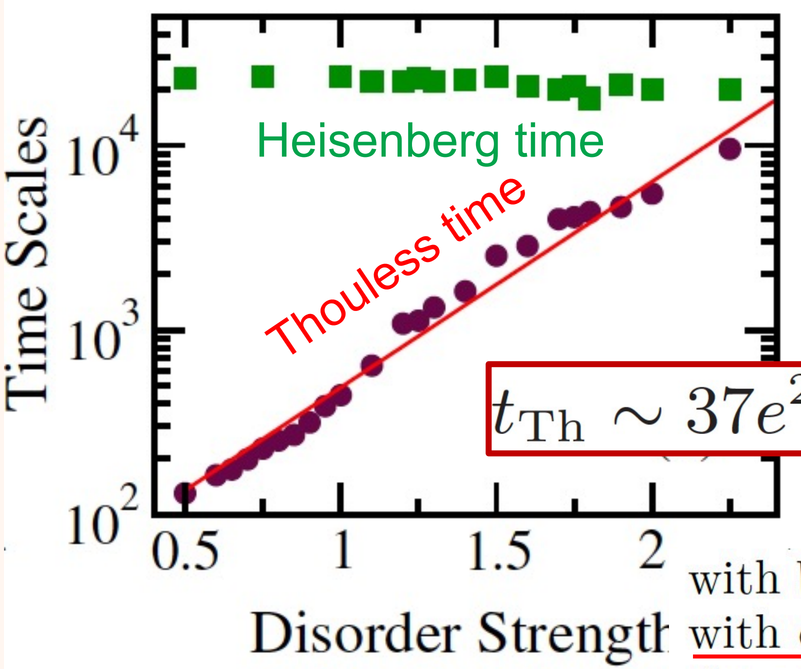


$t_{Th} \propto \frac{D^{2/3}}{\Gamma}$

Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Thouless Time and Disorder Strength

Schiulaz, Torres-Herrera & LFS,
PRB **99**, 174313 (2019)
arXiv:1807.07577



Quantum chaos challenges many-body localization

Jan Šuntajs,[1] Janez Bonča,[2,1] Tomaž Prosen,[2] and Lev Vidmar[1,2]

Šuntajs, Bonča, Prosen, Vidmar
PRE **102**, 062144 (2020)
arXiv:1905.06345

with W/G_0, which suggests an exponential growth of t_{Th} with disorder W. We express these two observations by the ansatz for t_{Th} in which the L and W dependences are decoupled,

$$t_{\mathrm{Th}} = t_0 \, e^{W/\Omega} L^2 \,, \tag{4}$$

and Ω is a constant. While the results in Figs. 3(a)

Thouless Time and Disorder Strength

Thouless dimensionless conductance $\frac{t_R}{t_{Th}} \propto \frac{E_{th}}{MLS}$

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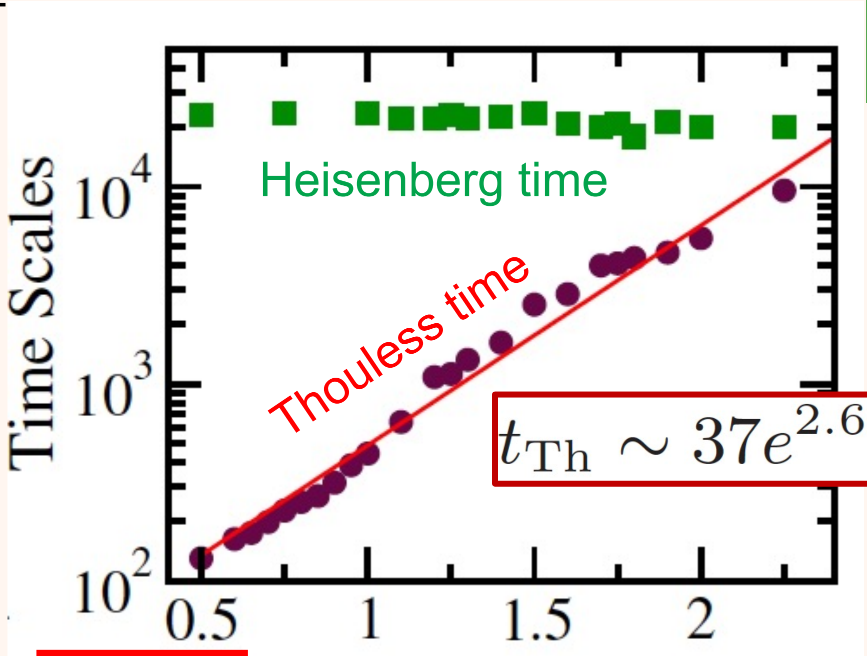

$t_{Th} \propto \frac{D^{2/3}}{\Gamma}$

Toward localization

$$\frac{t_R}{t_{Th}} \to 1$$

Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Thouless Time and Disorder Strength

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PRE **102**, 062144 (2020)
arXiv:1905.06345

Thouless dimensionless conductance

Delocalized (chaotic)

$$\frac{t_R}{t_{Th}} \propto e^{(L\ln 2)/3}$$

Toward localization

$$\frac{t_R}{t_{Th}} \to 1$$

Quantum chaos challenges many-body localization

In this work we introduce an indicator of the ergodicity breaking transition in finite many-body systems,

$$g = \log_{10}(t_{\rm H}/t_{\rm Th}), \qquad (1)$$

which is proportional to the logarithm of the dimensionless conductance $t_{\rm Th}/t_{\rm H}$ and hence carries similarities with the dimensionless conductance in studies of Anderson localization. The ergodicity indicator g interpolates between the quantum ergodic regime $t_{\rm Th}/t_{\rm H} \to 0$ $[g \to \infty]$ and the nonergodic regime $t_{\rm Th}/t_{\rm H} \to \infty$ $[g \to -\infty]$ in the thermodynamic limit. Our main result is that g exhibits a scaling solution across the ergodicity breaking transition in certain disordered spin chains at $t_{\rm Th} \approx t_{\rm H}$. The scaling solution exhibits clear signatures of a crossing point that can be readily detected in rather small systems. The finite-size analysis indicates robustness of quantum chaos upon increasing the disorder; however, limitations to relatively small lattice sizes also prevent us from making conclusions about the fate of the critical regime in the thermodynamic limit.

Spin Autocorrelation Function

$$I(t) = \frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\sigma_k^z e^{iHt}\sigma_k^z e^{-iHt}|\Psi(0)\rangle$$

Local in space
Nonlocal in time
Similar to the density imbalance (experimental)

Schiulaz, Torres-Herrera & LFS,
PRB **99**, 174313 (2019)

Spin Autocorrelation Function and Disorder Strength

Thouless dimensionless conductance $\frac{t_R}{t_{Th}} \propto \frac{E_{th}}{MLS}$

Delocalized (chaotic)

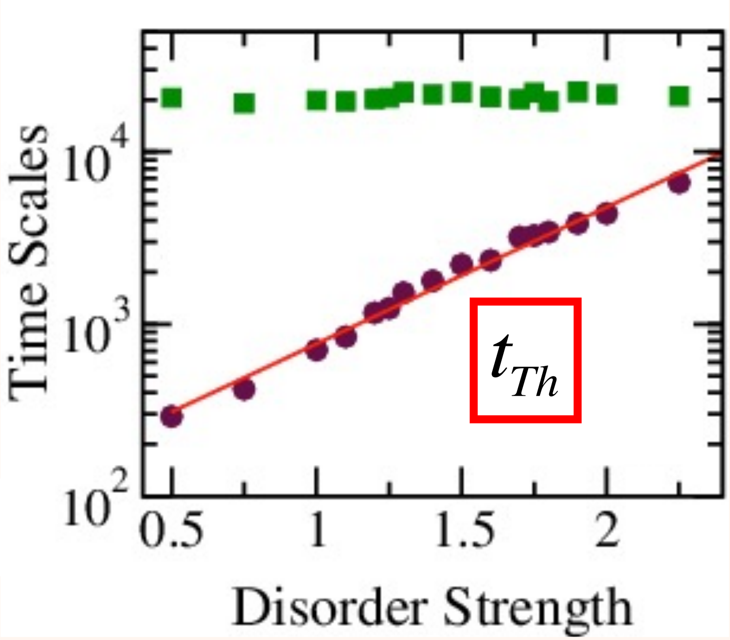
$$\frac{t_R}{t_{Th}} \propto e^{(L\ln 2)/3}$$

Toward localization

$$\frac{t_R}{t_{Th}} \rightarrow 1$$

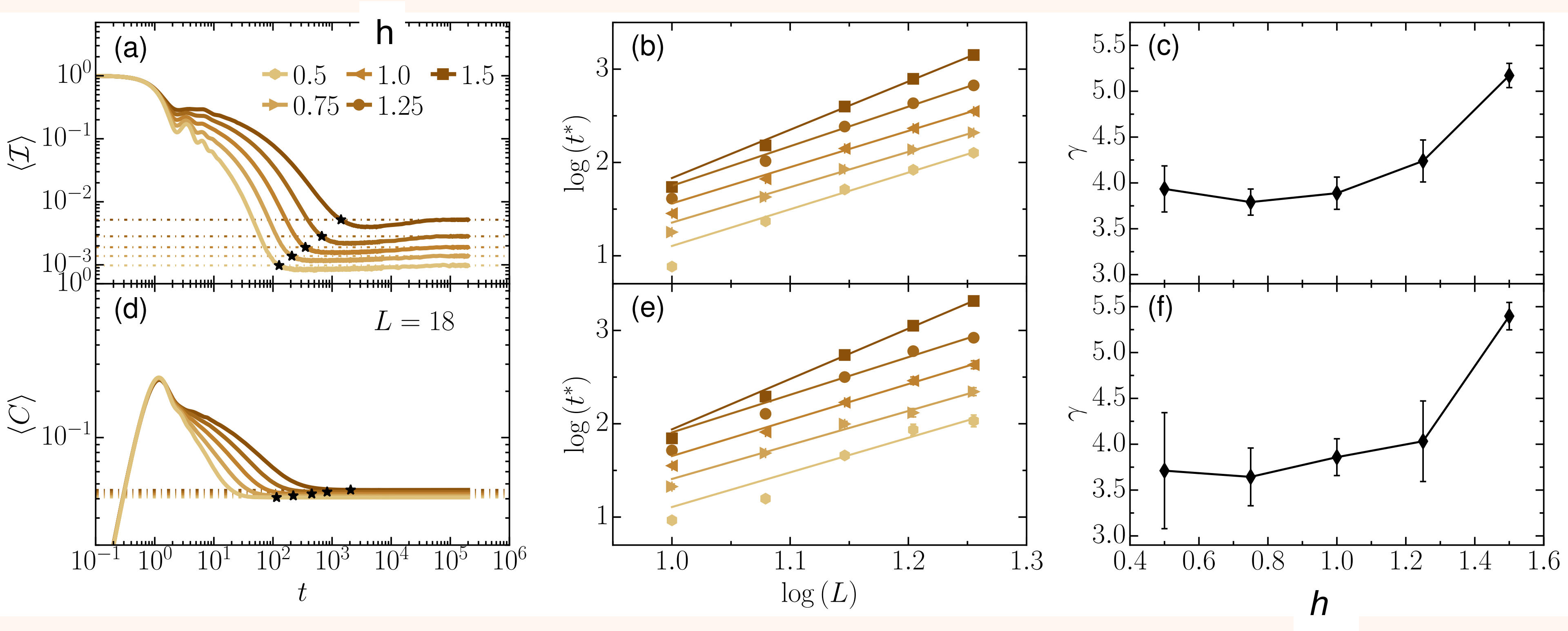
SPIN AUTOCORRELATION

$$t_R \propto \frac{D}{\Gamma\sqrt{\delta}}$$



Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Thermalization Time for the Local Quantities



Fixed L=18

$$t^* \propto L^\gamma$$

$$\gamma > 3$$

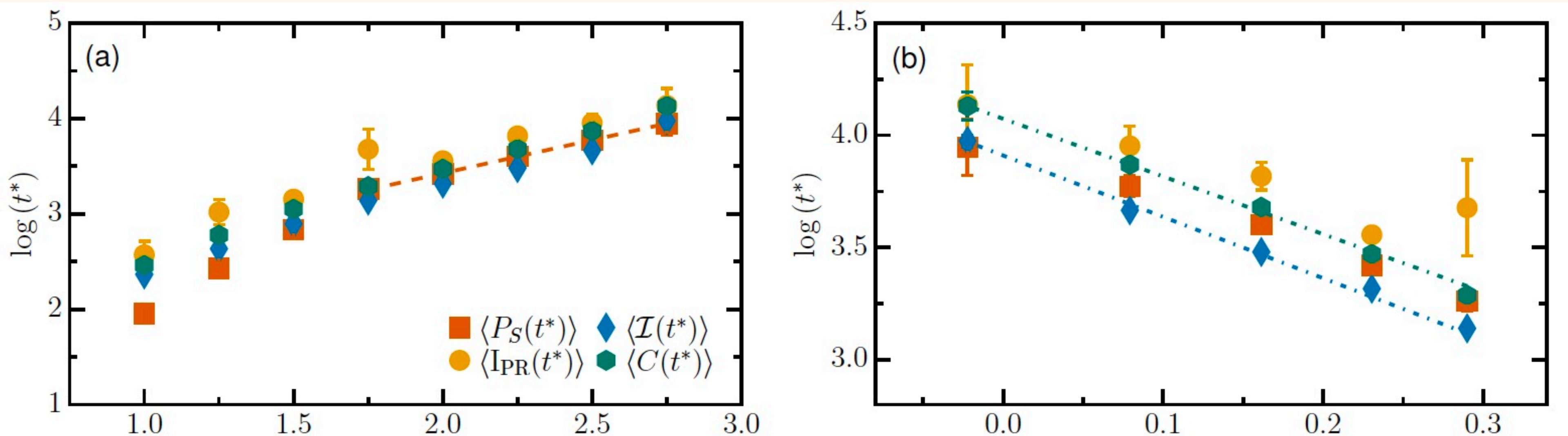
Lea F. Santos, Yeshiva University

PRB **104**, 085117 (2021)

2nd Int School on Adv Quant Mech

Crossing Time vs Disorder Strength

Fixed L=18



h

Log(h-h$_c$)

Survival Probability:

$$t^* \propto e^h$$

Local Quantities:

$$t^* \propto |h - h_c|^{-\beta}$$

Thermalization time in many-body quantum systems
Lezama, Torres, Bernal, Bar Lev & LFS,
PRB **104**, 085117 (2021)