Space-time Diagram

Events are fundamental

Let's ignore \( y' = y \) and \( z' = z \)

1. Events A and D occur at the same place
   at \( \pm \) times
   \( t_A - t_D \rightarrow \) proper time, as measured by
   clock 2

2. Events A and B are simultaneous

3. Event C happened before the present
   (we consider \( ct = ct' = 0 \) to be the present)

**Worldline**: trajectory a particle on the space-time diagram

- Particle 1 is at rest
- Particle 2 moves at \( u < c \)
  \[ u = \frac{dx}{dt} \]
- Particle 3 moves at \( c \) (light pulse)
- Particle 4 slows down

**Worldline of light flashes**: nothing moves faster than light
$S'$ moves in the $+x$ direction of $S$ at $v$.

They coincide at $t=t'=0$ ($x=x'=0$).

How does $S'$ appear in the spacetime diagram of $S$?

1. The worldline of $x=0$ in $S$ is the $ct$ axis.
2. The worldline of $x'=0$ in $S'$ is the $ct'$ axis.

To find the $ct$ axis in the spacetime diagram of $S$:

\[ x' = \gamma(x - vt) \]

\[ \frac{v}{t} = \frac{x}{ct} \Rightarrow ct = \frac{x}{\sqrt{1 - \beta^2}} \]

To find the $x'$ axis in the spacetime diagram of $S$:

$x'$ axis is the axis for all points where $ct' = 0$.

Use Lorentz transformation:

\[ t' = \gamma \left( t - \frac{\gamma}{ct'} x \right) \Rightarrow t = \frac{\gamma}{c^2} x = \frac{1}{\beta} x \]

Event:

\[ \{ A \text{ and } B: \text{ simultaneous in } S' \} \]

\[ \{ B \text{ and } C: \text{ same place in } S' \} \]
Example 1-7

Two events occur at the same point \( x_0 \) at times \( t_1 \) and \( t_2 \) in \( s_1 \), which moves with speed \( v \) relative to \( s \). What is the spatial separation of these events measured in \( s \)?

\[
\Delta x = \beta \left( \Delta x_1 + v \Delta t_1 \right) = \Delta x_0 \beta
\]

- \( \Delta x_0 = 5 \) (\( \Delta x_1, \Delta t_1 \) axis)

\[
\Delta x = 5 \left( \frac{5}{c} \right) = 5 \beta \left( \frac{5}{c} \right) = 1.15 \left( \frac{5}{c} \right)
\]

\[
\Delta x = 1.15 \text{ m}
\]