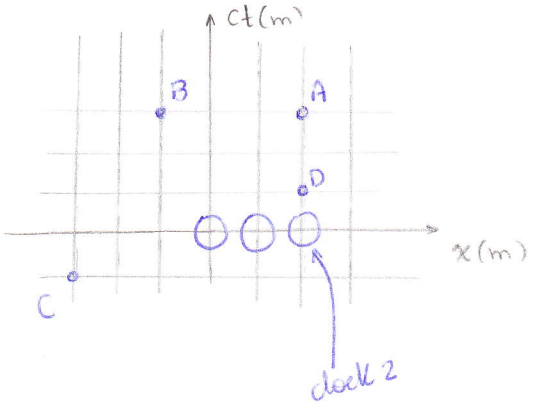
Spacetime Diagram

Events are fundamental

→ t vs x graphs

Let's ignore $y'=y$ and $z'=z$



Reference frame S

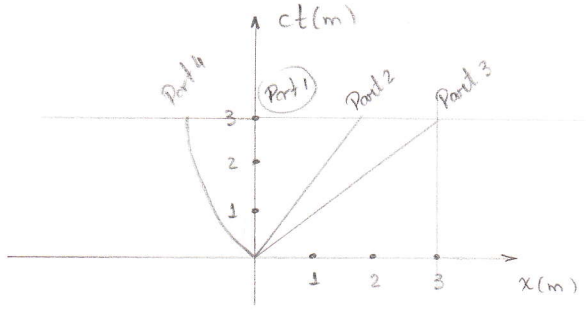
- Events A and D occur at the same place at ≠ times

  $t_A - t_D$ → proper time, as measured by clock 2

- Events A and B are simultaneous

- Event C happened before the present (we consider $ct = ct' = 0$ to be the present

worldline: trajectory a particle on the spacetime diagram



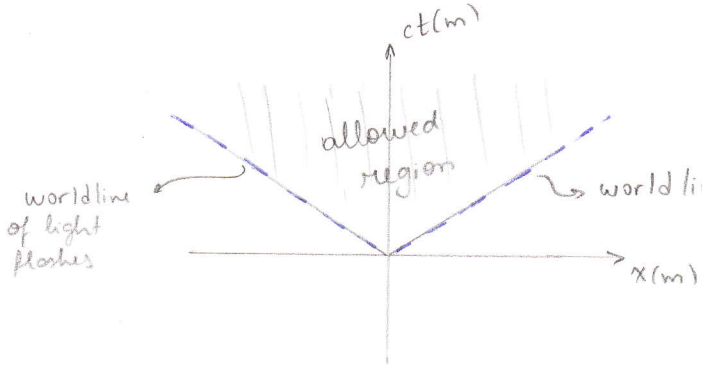
→ Particle 1 is at rest

→ Particle 2 moves at $u < c$

$u = \Delta x / \Delta t$

→ Particle 3 moves at (c) (light pulse)

→ Particle 4 slows down



nothing moves faster than light

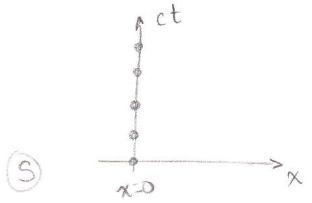
S' moves in the +x direction of S at v

They coincide at $t = t' = 0$ $(x = x' = 0)$

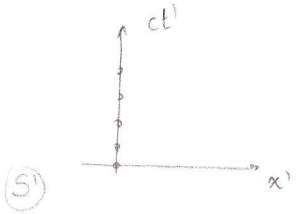
How does S' appear in the spacetime diagram of S ?

To find ct' axis in the spacetime diagram of S

→) Worldline of $x = 0$ in S is ct axis



Worldline of $x' = 0$ in S' is ct' axis



$\beta = \frac{v}{c} < 1$

→) The slope of ct' may be found from Lorentz transformation

$$x' = \gamma(x - vt)$$

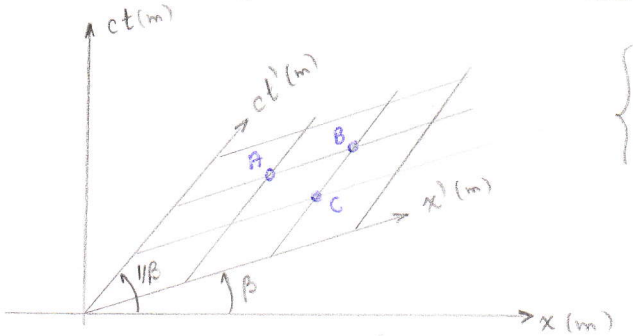
$$v = \frac{x}{t} = \frac{xc}{ct} \Rightarrow ct = \frac{x}{v/c} \Rightarrow \boxed{ct = \frac{1}{\beta} x}$$

→) To find x' axis in the spacetime diagram of S

x' axis is the axis for all points where $ct' = 0$

use Lorentz transformation

$$t' = \gamma\left(t - \frac{v}{c^2} x\right) \Rightarrow t = \frac{v}{c^2} x \Rightarrow \boxed{ct = \beta x}$$



Events:

A and B: simultaneous in S'

B and C: same place in S'

Example 1-7

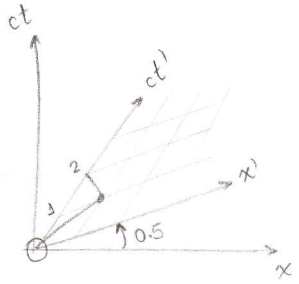
Two events occur at the same point x'_0 at times t'_1 and t'_2 in S', which moves with speed v relative to S. What is the spatial separation of these events measured in S ?

$$\Delta x = \gamma (\Delta x' + v \Delta t') = \gamma v \tau$$

→) Using the figure: $\beta = 0.5$ (slope of x' axis)

↳ $\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.15$

$\tau = \frac{2}{c}$



$$\Delta x = \gamma \frac{v}{c} c\tau = \gamma \beta c\tau = (1.15)(0.5)(2)$$

$$\Delta x = 1.15 \text{ m}$$