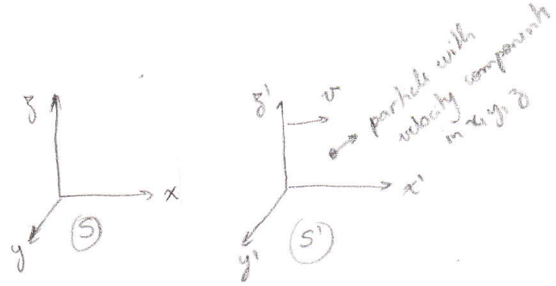
Relativistic Velocity Transformation

$$\begin{cases} x' = \gamma(x - vt) \Rightarrow dx' = \gamma(dx - v\,dt) \\ dy' = dy \\ dz' = dz \\ t' = \gamma\left(t - \frac{v}{c^2}x\right) \Rightarrow dt' = \gamma\left(dt - \frac{v\,dx}{c^2}\right) \end{cases}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



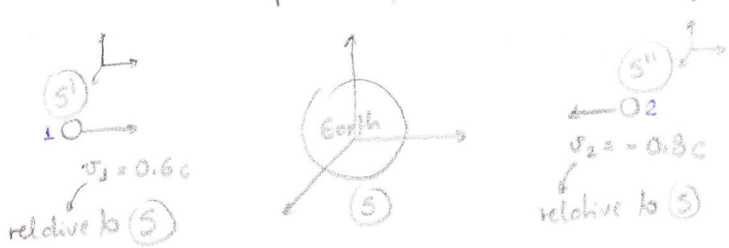
$$u_x' = \frac{dx'}{dt'} = \frac{\gamma(dx - v\,dt)}{\gamma\left(dt - \frac{v}{c^2}dx\right)} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \boxed{\frac{u_x - v}{1 - \frac{v\,u_x}{c^2}}}$$

$$u_y' = \frac{dy'}{dt'} = \frac{dy}{\gamma\left(dt - \frac{v}{c^2}dx\right)} = \frac{\frac{dy}{dt}}{\gamma\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)} = \boxed{\frac{u_y}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}}$$

$$\boxed{u_z' = \frac{u_z}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}}$$

$$(v)\begin{cases} u_x = \dfrac{u_x' + v}{1 + \frac{v\,u_x'}{c^2}} \\ u_y = \dfrac{u_y'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)} \end{cases} \qquad u_z = \frac{u_z'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)}$$

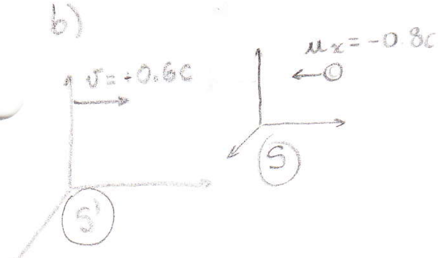
Example 1-4 Two protons approaching Earth. a) What is Earth's velocity relative to each proton? b) What is the velocity of each proton relative to each other?



a) For S': $v_E' = -0.6c$

For S'': $v_E'' = +0.8c$

b)



velocity of ② with respect to S'

$$u_x' = \frac{-0.8c - 0.6c}{1 - \frac{0.6c(-0.8c)}{c^2}}$$

$$u_x' = \frac{-1.4c}{1.48} = \boxed{-0.95c}$$

velocity of ① with respect to S''

$$u_x'' = \frac{0.6c - (-0.8c)}{1 - \frac{(-0.8c)(0.6c)}{c^2}} = \boxed{+0.95c}$$