Example 1-3 Two cosmic-ray muons are recorded by detectors in the lab, one at t_a at x_a and the other at t_b at x_b.

ref. frame (S)

what is the time interval between those two events in (S') which moves at speed v relative to S?

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$\underbrace{t_b' - t_a'}_{} = \gamma\left(t_b - t_a\right) - \frac{\gamma v}{c^2}\left(x_b - x_a\right)$$

depends on time interval in (S) AND spatial separation of clocks in (S)

→) PROPER TIME interval } $\underbrace{(t_b - t_a)}_{\tau}$ when $x_b = x_a$

PROPER TIME: time interval measured in a frame where the events occur in the same place

For S': •) PROPER TIME interval is the minimum time interval between those events

it occurs when $v = 0 \Rightarrow \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 1 \Rightarrow (t_b' - t_a') = (t_b - t_a)$

•) in general: $\gamma \geq 1$ and $\gamma \to \infty$ when $v \to c$

so $(t_b' - t_a') = \gamma(t_b - t_a) \geq (t_b - t_a)$

time interval is LONGER by a factor (γ) in a frame moving relative to the frame of the proper time

TIME DILATION

→) For which reference frame S'' would these events be simultaneous?

$$t_b'' - t_a'' = 0 \Rightarrow \gamma(t_b - t_a) = \frac{\gamma v}{c^2}(x_b - x_a)$$

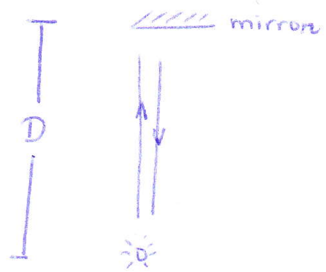
$$\boxed{\frac{v}{c} = \frac{(t_b - t_a)\,c}{(x_b - x_a)}}$$

TIME DILATION

S' moves at v with respect to S

At $t=0$, they coincide
$\downarrow$
Observer O' at rest in S' triggers a flash gun

In S'



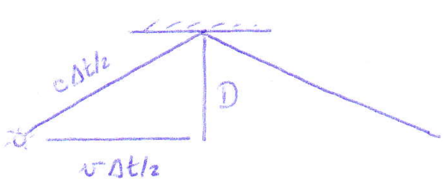
time interval between emission and reception

$$\Delta t' = \frac{2D}{c} \qquad \Delta t' = \tau$$

(proper time)

$\rightarrow$ light travels at c in S and S'

In S



$$c^2\left(\frac{\Delta t}{2}\right)^2 = D^2 + v^2\left(\frac{\Delta t}{2}\right)^2$$

$$\Delta t^2 = \frac{4D^2}{c^2 - v^2} = \frac{4D^2}{c^2(1 - v^2/c^2)}$$

$$\boxed{\Delta t = \gamma \Delta t'}$$

$\gamma \geq 1 \Rightarrow$ Time dilation

$\Rightarrow$ light travels farther in S

$\Rightarrow$ it takes longer in S to reach the mirror and return

Length Contraction

HW

Tipler
- 1-13a
- 1-19
- 1-23a
- 1-24
- 1-28

PROPER length

length of an object measured in the reference frame in which the object is at rest

What is L' as seen from an observer in S'?

$$L' = x'_b - x'_a = \gamma\left((x_b - x_a) - v\,\underbrace{(t_b - t_a)}\right)$$

no information about Δt
in S, the object is at rest, so we can measure its extremes x_b and x_a at any time

$$L = x_b - x_a = \gamma\left(\underbrace{(x'_b - x'_a)}_{L'} + v\,\underbrace{(t'_b - t'_a)}\right)$$

we have to measure x'_b and x'_a at the same time, because the object is moving, so $t'_b - t'_a = 0$

$$\boxed{L' = \frac{1}{\gamma} L}$$

length of the object is smaller when measured in a frame with respect to which it is moving

Muon Decay

Muons decay after 2.2 μs in a reference frame where they are at rest

If muons are detected at 9.000 m above the sea level, are they expected to be observed at sea level?
They move at 0.998 c toward the surface of Earth

$$v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = 0.998c \times 2.2 \times 10^{-6} = \underline{660\,m}$$

so it might seem as the muons should decay before reaching sea level

BUT observations show that nearly all muons detected at 9.000m reach sea level

Events $\left\{ \begin{array}{l} \text{muon formed } t'_A \\ \text{muon decayed } t'_B \end{array} \right\}$ in $\boxed{\text{muon}}$ ref. frame $t'_B - t'_A = \tau$ proper lifetime

CAREFUL muon lifetime is 2.2 μs in its own reference frame

For the Earth reference frame,
the muon lifetime is increased according to time dilation

⟹ Earth ref. frame

$$\Delta t = \gamma\, \Delta t' \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \simeq 15.8$$

$\Delta t \simeq 35\ \mu s$

↳ which explains why muons DON'T decay

⟹ Muon ref. frame

↳ muons see atmosphere moving

atmosphere is at rest with respect to $\boxed{\text{Earth}}$. 9.000m is its proper thickness L

L' contracts

$$L' = L/\gamma = \frac{9.000}{15.8} \approx \boxed{570\,m}$$

↳ so the muons have no reason to decay
they have not travelled 660m yet

Example 1-8

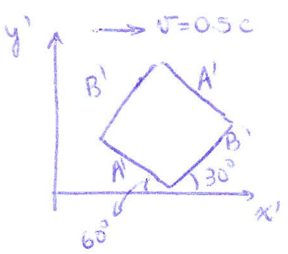
Elephants have a gestation period of 21 months.
A freshly impregnated elephant is sent toward space at $v = 0.75c$
If we monitor radio transmissions from the spaceship,
how long after launch should we find out about the
newborn calf?

Example 1-9

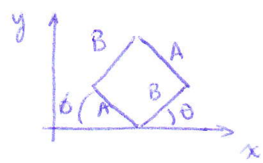
A stick that has a proper length of 1m moves in a
direction parallel to its length with speed v relative to you.
The length of the stick as measured by you is 0.914m.
What is the speed v?

Example 1-10

Consider the square in the $x'y'$ plane of S' with one side
making a $30°$ angle with the x' axis. If S' moves with $\beta = 0.5$
relative to S, what is the shape and orientation
of the figure in S?



$A = ?$ $B = ?$ $\theta = ?$ $\phi = ?$



S' moves in the x direction of S
there is no motion in the y direction $\Rightarrow$ contraction may happen in x
not in y

Example 1-8

S': reference frame of elephant
$\tau = 21$ months (proper gestation period)

S: reference frame of Earth
time dilation
↳ $\Delta t = \gamma \tau$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.75^2}} \simeq 1.51$$

(1) $\Delta t = 21 \times 1.51 \simeq \boxed{31.7 \text{ months}}$

(2) radio signal is sent to Earth
↳ speed (c)

elephant doesn't move in its ref frame

it has to travel a distance $\Delta x = \gamma (\cancel{\Delta x'} + v \Delta t')$
↳ proper time

$$\Delta x = \gamma v \tau$$

$$\Delta x = \gamma (\beta c) \tau = 1.51 (0.75)(21) c$$

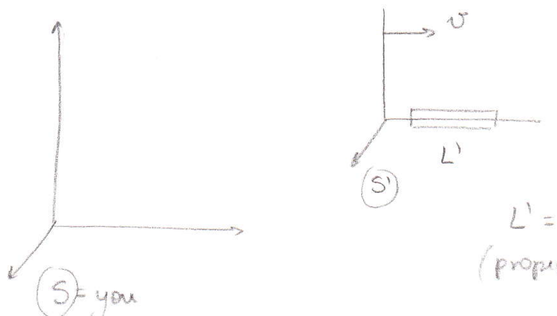
$$\boxed{\Delta x = 23.8 c}$$

(3) time for radio signal to reach Earth

$$\Delta t_s = \frac{\Delta x}{c} = \frac{23.8 c}{c} = \boxed{23.8 \text{ months}}$$

(4) total time for the news: $\Delta t + \Delta t_s = 31.7 + 23.8 = \boxed{55.5 \text{ months}}$

Example 1-9



(stick at rest in S')

$L' = 1m$
(proper length)

S = you

$L = 0.914m$

$$L = \frac{L'}{\gamma} = 1m\sqrt{1 - v^2/c^2} \Rightarrow 1 - v^2/c^2 = (0.914)^2$$

$$\boxed{v = 0.406c}$$

Example 1-10

(square at rest in S') $\Rightarrow$ contraction may happen for S

$$\left\{\begin{array}{ll} A_x = \frac{A_x'}{\gamma} & \Rightarrow A_x = \frac{A' \cos 60^\circ}{\gamma} \\ A_y = A_y' & \Rightarrow A_y = A' \sin 60^\circ \end{array}\right\} A = \sqrt{A_x^2 + A_y^2} = A' \sqrt{\frac{\cos^2 60^\circ}{\gamma^2} + \sin^2 60^\circ}$$

$$\boxed{A = 0.968 A'}$$

$$\left\{\begin{array}{ll} B_x = B_x'/\gamma & \Rightarrow B_x = B' \cos 30^\circ/\gamma \\ B_y = B_y' & \Rightarrow B_y = B' \sin 30^\circ \end{array}\right\} B = \sqrt{B_x^2 + B_y^2} = B' \sqrt{\cos^2 30^\circ/\gamma^2 + \sin^2 30^\circ}$$

$$\boxed{B = 0.901 B'}$$

$$\tan\theta = B_y/B_x = \frac{\cancel{B'} \sin 30^\circ}{\cancel{B'} \cos 30^\circ/\gamma} \Rightarrow \boxed{\theta = 33.7^\circ}$$

$$\tan\phi = A_y/A_x = \frac{\cancel{A'} \sin 60^\circ}{\cancel{A'} \cos 60^\circ/\gamma} \Rightarrow \boxed{\phi = 63.4^\circ}$$