Relativity

(Tipler: Chapters 1 and 2 , Eisberg: Appendix A)

Newton's first law: "every object continues in its state of rest, or uniform velocity in a straight line, as long as no net force acts on it"

(Law of Inertia)

In any inertial reference frame, a particle moves without any change in velocity if no net force acts on it

(*) Special theory of relativity deals with inertial frames
(Einstein - 1905)
↑
(Annus Mirabilis)

↑
our subject

(frames of reference moving with constant velocity relative to each other)

General theory of relativity deals with accelerated frames and gravity
(Einstein - 1916)

(deformation of space-time caused by large masses)

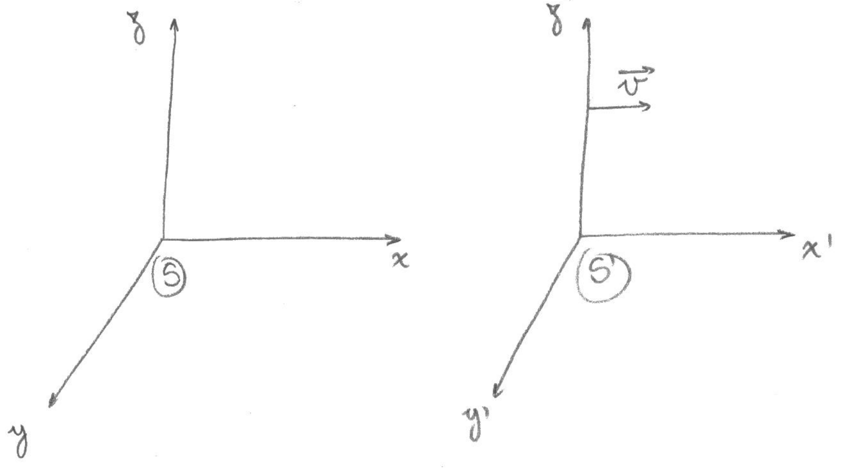
subject of more advanced course

1) How do we transform our description of a system from one frame to another one? (coordinates)

2) What happens to the equations which govern the behavior of the system when we make the transformation?

Classical physics to describe the state of a system we need
a reference frame → use it for coordinates, time derivatives of coord.
Given $\textcircled{m}$ and $\textcircled{\vec{F}}$; Newton's equations calculate the state at any future time

→) assume that the origins of S and S' coincide at $t = t' = 0$



$\overbrace{(x, y, z, t)}^{S}$ or $\overbrace{(x', y', z', t')}^{S'}$ can be equally used to specify the coordinates of the particle at any instant of time

Galilean Transformation

Answer to question ①

Particle in S' seen by S

$$\begin{cases} x = x' + vt \\ y = y' \\ z = z' \\ \boxed{t = t'} \end{cases} \Rightarrow \boxed{u_x = u_x' + v}$$

Particle in S seen by S'

$$\begin{cases} x' = x - vt \\ y' = y \\ z' = z \\ \boxed{t' = t} \end{cases} \Rightarrow \boxed{u_x' = u_x - v}$$

Answer to question ②

$$\left\{ \begin{array}{l} \underline{S} \text{ (Newton's 2nd law)} \\ m\dfrac{d^2x}{dt^2} = F_x \\ m\dfrac{d^2y}{dt^2} = F_y \\ m\dfrac{d^2z}{dt^2} \end{array} \right.$$

$$\left. \begin{array}{l} \dfrac{dx'}{dt} = \dfrac{dx}{dt} - v \Rightarrow \dfrac{d^2x'}{dt^2} = \dfrac{d^2x}{dt^2} \\ \dfrac{dy'}{dt} = \dfrac{dy}{dt} \Rightarrow \dfrac{d^2y'}{dt^2} = \dfrac{d^2y}{dt^2} \\ \dfrac{dz'}{dt} = \dfrac{dz}{dt} \Rightarrow \underbrace{\dfrac{d^2z'}{dt^2} = \dfrac{d^2y}{dt^2}}_{\text{acceleration is the same is S and S'}} \end{array} \right\} \xRightarrow{\text{mass}}$$

is an intrinsic property of the particle

$$\left\{ \begin{array}{l} \cancel{F_{x'} = F_x} \\ \cancel{F_{y'} = F_y} \\ \cancel{F_{z'} = F_z} \end{array} \right.$$

and $F_{x'} = F_x$, $F_{y'} = F_y$, $F_{z'} = F_z$ ⟹

$$F_x = m\frac{d^2x}{dt^2} = m\frac{d^2x'}{dt^2} \quad \text{and since } F_x = F_x' \Rightarrow F_x' = m\frac{d^2x'}{dt^2}$$

$$m\frac{d^2x'}{dt'^2} = F_x' \qquad m\frac{d^2y'}{dt'^2} = F_y' \qquad m\frac{d^2z'}{dt'^2} = F_z'$$

Newton's law of motion are <u>INVARIANT</u>
for any inertial frame
under a galilean transformation

<u>BUT</u> Maxwell's equations do <u>change</u> their mathematical form under a Galilean transformation

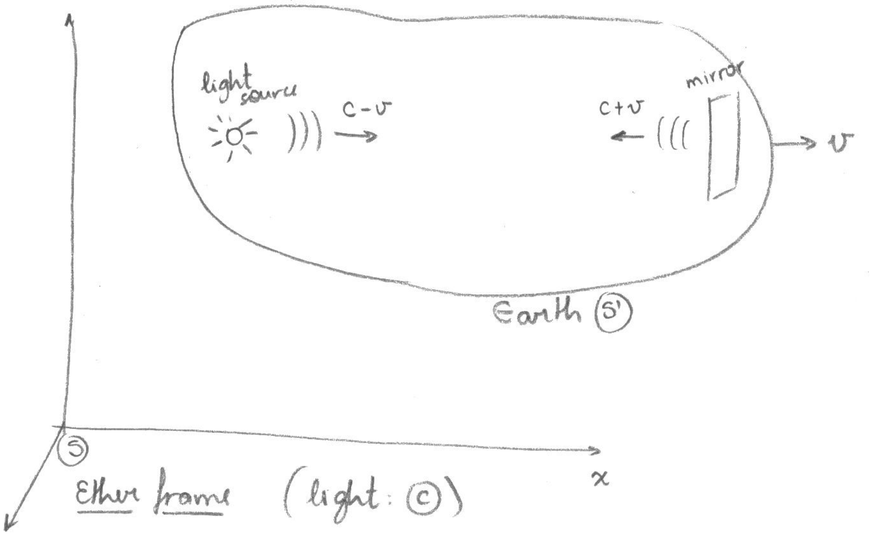
⟶ Maxwell's equations predicted the existence of electromagnetic waves whose speed was ©

XIX century WRONG beliefs

- they must propagate in a medium called (ether)
- there was only one frame, the <u>ether frame</u> where the velocity of light was ©

<u>Michelson - Morley experiment</u>

The motion of Earth through the ether should be detectable

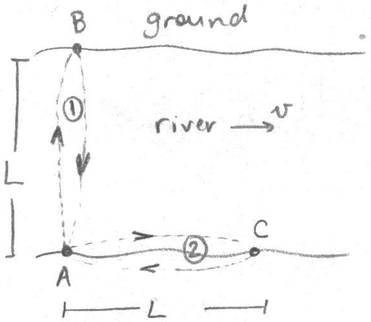


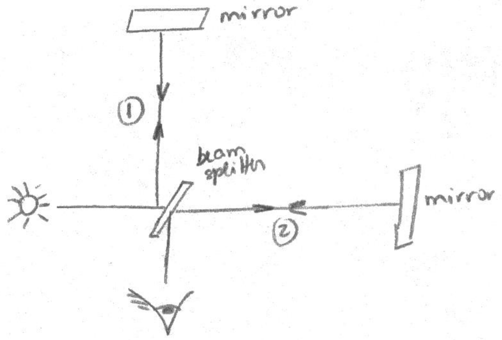
light to the right (+x):
$u'_x = u_x - v \Rightarrow u_x' = c - v$

light to the left (−x):
$\underset{\ominus}{u_x'} = \underset{-c}{u_x} - v \Rightarrow |u_x'| = c + v$

↑

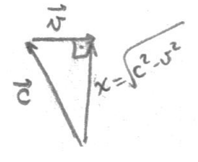
too small to measure directly
so they designed an experiment based on
(interferometry)



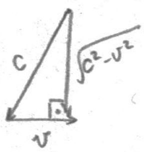


ⓒ is the speed of each boat in still water
c > v. Which boat (① or ②) wins the race?

$$t_{\textcircled{1}} = t_{A\to B} + t_{B\to A} = \frac{L}{\sqrt{c^2-v^2}} + \frac{L}{\sqrt{c^2-v^2}} = \frac{2L}{c\sqrt{1-v^2/c^2}} = \frac{2L}{c}\underbrace{\left(1-\frac{v^2}{c^2}\right)^{-1/2}}_{\text{Taylor expansion}} = \boxed{\frac{2L}{c}\left(1+\frac{1}{2}\frac{v^2}{c^2}+\dots\right)}$$



A → B



$$(1-x)^{-1/2} \approx 1 + \left(\frac{-1}{2}(1-x)^{-3/2}(-1)\right)\Big|_{x\to 0} x + \dots$$

$$= 1 + x + \dots$$

$$x^2+v^2=c^2$$

$$x=\sqrt{c^2-v^2}$$

$$t_{\textcircled{2}} = t_{A\to C} + t_{C\to A} = \frac{L}{c+v} + \frac{L}{c-v} = \frac{L(c-\cancel{v}+c+\cancel{v})}{c^2-v^2} = \frac{2L}{c^2\underbrace{\left(1-\frac{v^2}{c^2}\right)}} \simeq \boxed{\frac{2L}{c}\left(1+\frac{v^2}{c^2}+\dots\right)}$$

$$\Big(1-\underbrace{v^2/c^2}_{x}\Big)^{-1} \underset{\text{Taylor}}{\simeq} 1 + \left(-1\left(1-\frac{v^2}{c^2}\right)^{-2}(-1)\right)\Big|_{x\to 0}\frac{v^2}{c^2} + \dots$$

$$t_{\textcircled{1}} \simeq \frac{2L}{c} + \frac{Lv^2}{c^3} \quad , \quad t_{\textcircled{2}} \simeq \frac{2L}{c} + 2L\frac{v^2}{c^3}$$

→ boat ① would win the race

→ Michelson's interferometer is based on the same idea, where the time difference translates into <u>phase</u> difference and consequent interference pattern

Shift expected from Michelson-Morley experiment was <u>never</u> detected

(!) There is <u>no ether</u>, there is (no) <u>special frame</u> in which light speed is ⓒ

electromagnetic waves <u>propagate in the vacuum</u>

Einstein's postulate

we need to modify Galilean transformation

① The laws of physics are the same in all inertial reference frames

↳ all inertial frames are equivalent for all phenomena (mech. or electromag)

② The speed of light in the vacuum is ⓒ for any inertial ref. frame

↳ all observers measure the same value ⓒ for the speed of light, independent of the motion of the source

Galilean transf ⇒ $t = t'$ ⇒ there is the same time scale at all places in any frame – <u>universal</u> time scale

Events and observers

is recorded in <u>space</u> and <u>time</u>

- An event occurs at some point in <u>space</u> and at some <u>instant</u> in time

- Events are described by observers who belong to particular inertial frames

Simultaneity (defining time scale in a single frame)

- <u>simultaneous</u> events: the arrival of the train <u>and</u> hand of watch points at 7

  ↑ nearby ↑

- how to determine simultaneity of events at separated locations?

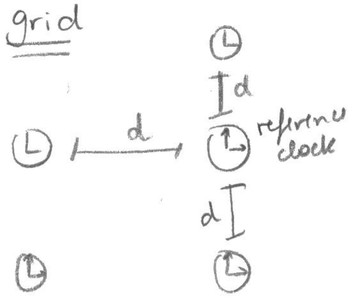
  ↳ need to <u>synchronize</u> clocks

  ↳ which involves <u>transmission</u> of signals

  ↳ with <u>finite</u> speed $v \le c$

→) We CAN synchronize clocks in one reference frame

In one frame



- reference clock sends a flash out light to other
- d is known and also ©

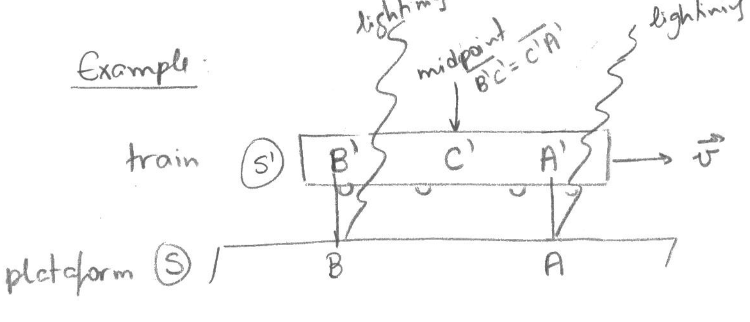
An event occurring at t_1 and x_1 is simulteneous with an event at t_2 and x_2 if light emitted from each arrives simultaneously at the midpoint between x_1 and x_2

↳ this definition mixes time and space

Relativity of simultaneity

Two spatially separated events simultaneous in one reference frame are not, in general, simultaneous in another inertial frame moving relative to the first.

Example:



- lightning bolts strike A and B simultaneously
- they reach C simultaneously

} events are simultaneous in Ⓢ

- train is moving: signal from A' will reach C' before the signal from B'
  ↳ events are not simultaneous in (S')

→) we CANNOT synchronize clocks at different frames / time is not absolute

Lorentz transformation

↳ Einstein's bold move

Galilean transf. had not been questioned

↳ this would imply in changes in Newton's equations!

We want to make a change → this modifies the classical relation $\boxed{t = t'}$

$$x = \textcircled{\gamma}\,(\underline{\underline{x'}} + \underline{\underline{v t'}}) \quad , \quad x' = \textcircled{\gamma}\,(x - vt) \overset{\circledast}{\Rightarrow}$$

$$x = \gamma\,(\gamma x - \gamma v t + v t')$$

$$\gamma v t' = (1-\gamma^2)\,x + \gamma^2 v t$$

$$\textcircled{**}\ \boxed{t' = \gamma t + \frac{(1-\gamma^2)}{\gamma}\,\frac{x}{v}}\quad \textcircled{!}$$

so that Maxwell's equations are <u>invariant</u>

and light propagates at speed ⓒ in any inertial frame

→) At $t = t' = 0$ a flash of light starts at the origin of S and S'

S (spherical wave) S'

$$x^2 + y^2 + z^2 = c^2 t^2$$

$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

$$\circledast \swarrow \textcircled{**}$$

$$\gamma^2 (x - vt)^2 + y^2 + z^2 = c^2\,\gamma^2 \left(t + \frac{1-\gamma^2}{\gamma^2}\,\frac{x}{v}\right)^2$$

$$\underbrace{\left(\gamma^2 - c^2\,\frac{(1-\gamma^2)^2}{\gamma^2 v^2}\right)}_{1} x^2 + \underbrace{\left(-2\gamma^2 v t - c^2\,2\,\frac{(1-\gamma^2)}{v}\,t\right)}_{=0} x + y^2 + z^2 = c^2 t^2 \underbrace{\left(\gamma^2 - \gamma^2\,\frac{v^2}{c^2}\right)}_{1}$$

$$\gamma^2 \left(1 - \frac{v^2}{c^2}\right) = 1 \Rightarrow \boxed{\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \quad \boxed{\beta = \frac{v}{c}}$$

$$\bullet)\ t' = \gamma t + \frac{(1-\gamma^2)}{\gamma}\frac{x}{v} = \gamma\left(t + \underbrace{\frac{(1-\gamma^2)}{\gamma^2}}\frac{x}{v}\right)$$

$$\left(1 - \frac{1}{1-v^2/c^2}\right)\left(1-v^2/c^2\right) = \frac{\cancel{1} - v^2/c^2 - \cancel{1}}{\ } = -\frac{v^2}{c^2}$$

$$\Rightarrow t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

to find t: $x = \gamma(x'+vt')$ into $x' = \gamma(x-vt)$

$$x' = \gamma^2 x' + \gamma^2 v t' - \gamma v t$$

$$t = \frac{(\gamma^2-1)x'}{\gamma v} + \frac{\gamma^{\cancel{2}}\cancel{v}t'}{\cancel{\gamma}\cancel{v}} \Rightarrow t = \gamma t' + \boxed{\frac{\gamma^2-1}{\gamma v}}x'$$

$$\frac{\cancel{1} - \cancel{1} + v^2/c^2\ \sqrt{1-v^2/c^2}}{(1-v^2/c^2)\ v} = \frac{v}{c^2}\frac{1}{\sqrt{1-v^2/c^2}}$$

$$\Rightarrow t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

S

$$\begin{cases} x = \gamma(x'+vt') \\ y = y' \\ z = z' \\ t = \gamma\left(t' + \frac{v}{c^2}x'\right) \end{cases}$$

S'

$$\begin{cases} x' = \gamma(x-vt) \\ y' = y \\ z' = z \\ t' = \gamma\left(t - \frac{v}{c^2}x\right) \end{cases}$$