SUMMARY

1) wave-particle duality also applies to matter

\[
\begin{align*}
E &= h\nu \\
p &= h/\lambda \\
\end{align*}
\]

2) to detect wave aspects: \( \lambda > \alpha \)

\[\text{double-slit experiment: interference}\]

\[\text{Davisson-Germer experiment: diffraction}\]

within \( \theta = \text{peak in}\) (constructive interference)

\[\text{outside}\quad \theta > \text{NO}\] (destructive interference)

\[\text{uncertainty principle}\]

\[\begin{align*}
\begin{align*}
|\lambda|, |\nabla| &= \text{known} \\
|\nabla|, |\lambda| &= \text{known} \\
\end{align*}
\end{align*}\]

\[\begin{align*}
\text{wave packet}\quad \text{phased velocity}\quad \text{group velocity}\quad \text{(follows the particle)}
\end{align*}\]

\[\begin{align*}
\Delta x, \Delta p &\sim h/2 \\
\Delta t, \Delta \omega &\sim h/2
\end{align*}\]

\(\Delta x, \Delta p: \text{uncertainty, spread}\)

\(\text{measurements on identically prepared systems DO NOT give identical results}\)
The Uncertainty Principle

(mathematically: $\psi = \sum c_n \psi_n \Rightarrow \Delta x \Delta p$)

(physically: measurement describes the system)

To explain principle of complementarity:

If experiment forces [matter] to reveal its wave character, it suppresses its particle character (and vice-versa)

(Two-slit experiment)

Feynman's text

The precision of measurement is inherently limited by the measurement process itself such that

$$\Delta p \Delta x \geq \frac{\hbar}{2}$$

Even with ideal instruments we can never do better than $\Delta p \Delta x \geq \frac{\hbar}{2}$

Restriction is not on accuracy to which $x$ and $p$ can be measured, but on the product $\Delta p \Delta x$ in a simultaneous measurement of both.

NOTE:

Planck's constant characterizes the quantum result / the limitation on measurement.

If $\hbar \rightarrow 0$ (classical domain) $\Rightarrow$ no limitations on measurements (classical)

(corresponding principle)

In the microscopic domain, we cannot determine $x$ and $p$ simultaneously, cannot specify initial conditions exactly, cannot precisely determine the future behavior of a system.

We can only talk about probabilities of events.
A physical justification for the uncertainty principle

A measurement disturbs the system...

To measure the position of an e⁻, we need light, at least one photon.

The photon is scattered by the e⁻ and comes back to our 'eyes'.

→ Resolving power of a microscope: \( \Delta x = \frac{\lambda}{2 \sin \Theta} \)

accuracy to which e⁻ can be localized

\( \Theta = 90^\circ \)

→ To reduce \( \Delta x \), need shorter \( \lambda \)

before scattering

after scattering

\( \Theta = 90^\circ \)

\( \Theta = 90^\circ \)

→ Photon initial momentum is \( \mathbf{p} \)

after colliding with e⁻, it is scattered within the angular range \( 2\Theta \)

so the x component of the photon varies from

\(-p \sin \Theta \) to \( p \sin \Theta \) \( \Rightarrow \Delta p = 2p \sin \Theta \)

uncertainty after scattering

but the e⁻ receives a recoil momentum in \( x \) of the same magnitude

\( \Delta p_x = 2p \sin \Theta = \frac{2 \ h \ \sin \Theta}{\lambda} \)

\( \Delta x \Delta p = \frac{\lambda}{2 \sin \Theta} = \frac{2 \ h \ \sin \Theta}{\lambda} \)

\( \frac{\Delta x \Delta p}{\hbar} = \frac{1}{2} \)

\( \frac{\Delta x}{\hbar} = \frac{1}{2} \)

consequence of uncertainty principle

\( \Rightarrow \) zero point energy / quantum fluctuations

perhaps cannot be at rest (at zero kelvin)

\( \Theta = 90^\circ \)

\( \Theta = 90^\circ \)