HW 5.4, 5.7, 5.10, 5.17 a b c
5.40 a b
5.42, 5.44 (Tipler)

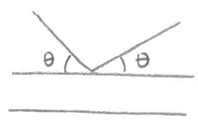
SUMMARY

- **wave-particle duality** also applies to matter

$$\begin{cases} E = h\nu \xrightarrow{\omega = 2\pi\nu} \boxed{E = \hbar\omega} \\ p = h/\lambda \xrightarrow{k = 2\pi/\lambda} \boxed{p = \hbar k} \end{cases}$$

- to detect wave aspects: $\boxed{\lambda \gtrsim a}$ (size of slit)

$\left\{\begin{array}{l} \text{double-slit experiment: interference} \\ \text{Davisson – Germer experiment: diffraction} \end{array}\right.$

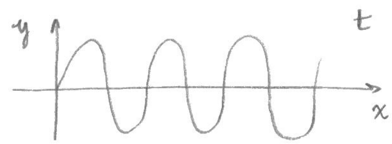


certain θ ⟹ peak in I (constructive interference)

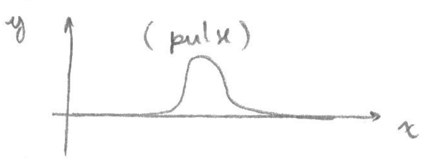
other values of θ' ⟹ NO I (destructive interference)

- **uncertainty principle**

mathematical qualitative justification for



know $\boxed{\lambda}$, $x = ?$



know $\boxed{x}$, $\lambda = ?$

pulse → localized particle

$$y(x,t) = \sum A_n \cos(k_n x - \omega_n t)$$

the more spread k, p, λ ⟹ the more localized is the particle

(**wave packet**)

phase velocity $v_p = \dfrac{\omega}{k}$

group velocity (follows the particle) $\boxed{v_g = \dfrac{d\omega}{dk}}$

(t) $\Delta x\, \Delta k \sim 1$ (property of any wave) (!)

⟹ $\boxed{\Delta x\, \Delta p \geq \hbar/2}$ ← Quantum mechanics (!) →

(x) $\Delta t\, \Delta\omega \sim 1$ (property of any wave) (!)

⟹ $\boxed{\Delta t\, \Delta E \geq \hbar/2}$

Δt: rate of change of an observable

Δt is small, quick change if uncertainty in E is large

Δx, Δp : uncertainty, spread

measurements on identically prepared systems **DO NOT** give identical results

The Uncertainty Principle

(mathematically: $\psi = \sum \cos \Rightarrow \Delta x \, \Delta p$
physically: measurement disturbs the system)

↳ explains principle of complementarity:

If experiment forces [matter / radiation] to reveal its wave character ($\lambda \to p$), it suppresses its particle character (x) (and vice-versa)

(Two-slit experiment)
Feymann's text

The precision of measurement is inherently limited by the measurement process itself such that

$$\boxed{\Delta p_x \Delta x \geq \hbar/2} \qquad (\hbar = h/2\pi)$$

Even with ideal instruments we can never do better than $\Delta p_x \Delta x \geq \hbar/2$

Restriction is not on accuracy to which x and p_x can be measured, but on the product $\Delta p_x \Delta x$ in a simultaneous measurement of both

NOTE: Planck's constant characterizes the quantum result / the limitation on measurements

If $\boxed{\hbar \to 0}$ (classical domain) $\Rightarrow$ no limitations on measurements (classical view)

(correspondence principle)

In the microscopic domain, we cannot determine x and p simultaneously,
cannot specify initial conditions exactly
cannot precisely determine the future behavior of a system

↓

we can only talk about probabilities of events

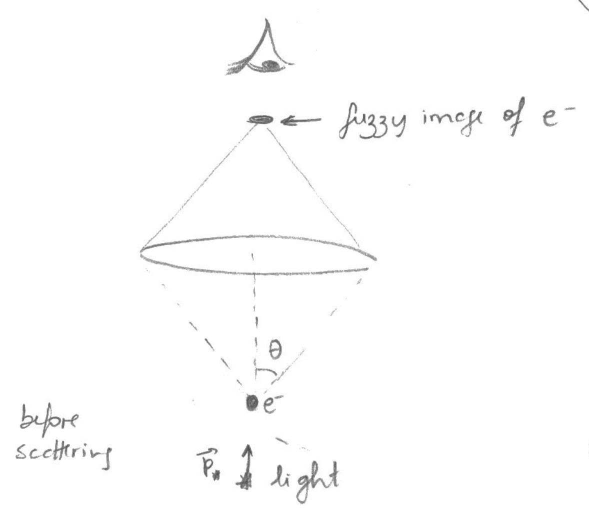
A physical justification for the uncertainty principle

A measurement disturbs the system...

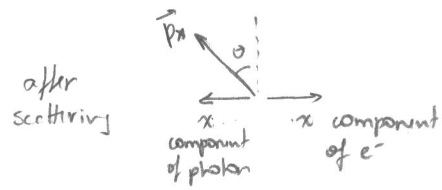
To measure the position of an e^-, we need light (microscope) $\rightarrow$ at least one photon

The photon is scattered by the e^- and comes back to our 'eyes'

$\Rightarrow$ Resolving power of a microscope: $\Delta x = \dfrac{\lambda}{2\sin\theta}$ $\rightarrow$ accuracy to which e^- can be located



•) To reduce Δx, need shorter λ



$\Rightarrow$ Photon initial momentum is p

after colliding with e^-, it is scattered within the angular range 2θ

so the x component of the photon varies from

$$-p\sin\theta \text{ to } p\sin\theta \Rightarrow \underbrace{\Delta p_x = 2p\sin\theta}_{\text{uncertainty after scattering}}$$

but the e^- receives a recoil momentum in x of the same magnitude

$$\Delta p_x = 2p\sin\theta = 2\frac{h}{\lambda}\sin\theta$$

$$\Delta x\,\Delta p = \frac{\lambda}{2\sin\theta}\,2\frac{h}{\lambda}\sin\theta = h \xrightarrow[\text{estimate for}]{\text{good}} \Delta x\,\Delta p \geq \frac{\hbar}{2} = \frac{h}{4\pi}$$

to reduce Δx needs shorter λ $\leftrightarrow$ to reduce Δp needs larger λ

consequence of uncertainty principle

$\hookrightarrow$ <u>zero point energy</u> / quantum fluctuations

particle cannot be at rest even at temperature = 0