# Wave like properties of particles

symmetry
world is made of matter and radiation

Louis de Broglie

wave-particle dualism applies also to matter

photon has light wave associated with it that governs its motion

material particle ⟶ matter wave "

To describe particle, we need $\underbrace{E, p}$

↳ connect them wave concepts through $h$

$$\begin{cases} E = h\nu \\ p = h/\lambda \leftarrow \end{cases}$$

$\boxed{\lambda = h/p}$ (de Broglie relation)

↓ wave length of matter wave

Example 3.1 (Eisberg) a) What is the de Broglie wavelength of a baseball (1.0 Kg) moving at speed $v = 10\,m/s$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.6\times10^{-34}\,J.s}{1\,Kg\times10\,m/s} = \overline{\underline{6.6\times10^{-35}\,m}} = 6.6\times10^{-25}\,\mathring{A}$$

b) ... of an $e^-$ where $K = 100\,eV$?

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{6.6\times10^{-34}\,J.s}{\left(2\times9.1\times10^{-31}Kg\times100\,eV\times1.6\times10^{-19}\,\frac{J}{eV}\right)^{1/2}} = \overline{\underline{1.2\times10^{-10}\,m}} = 1.2\,\mathring{A}$$

$$K = \frac{p^2}{2m}$$

o) visible light: $10^{-6} - 10^{-7}\,m$

a: dimension of apparatus, slit

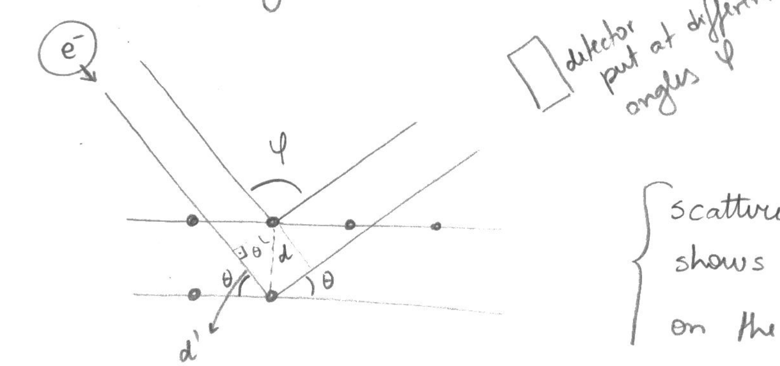
in optics if $a \gg \lambda \rightarrow$ geometrical optics : no diffraction, interference

to observe wavelike aspects we need $\lambda \gtrsim a$

At de Broglie time, they could find $a \sim 1\,\text{Å}$ between atoms in a crystal

Davisson and Germer / Thomson

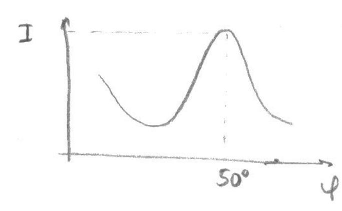
Thomson's son




scattered electron beam shows a peak depending on the energy of e⁻

For $V = 54\,V \longrightarrow$ peak at $\varphi = 50°$

↳ can only be explained as a constructive interference of waves (like Bragg diffraction)




$$\sin\theta = \frac{d'}{d} \Rightarrow \left\{ \begin{array}{l} \Delta = 2d' = 2d\sin\theta \\ \Delta = n\lambda \end{array} \right\} \Rightarrow \underline{n\lambda = 2d\sin\theta}$$

$$\varphi + 2\theta = \pi$$

$$\lambda = 2d\sin\left(\frac{\pi - \varphi}{2}\right)$$

crystal of nickel : $d = 0.91\,\text{Å}$ (know from X-ray scattering)

$$\longrightarrow \quad \lambda = 2 \times 0.91 \sin(65°) = \boxed{1.65\,\text{Å}}$$

$$\longrightarrow \quad \lambda = \frac{h}{p} = \frac{6.6\times10^{-34}\ \text{J.s}}{\sqrt{2mK}} = \boxed{1.65\,\text{Å}}$$

$54\,\text{eV} = 54 \times 1.6 \times 10^{-19}\,\text{J}$

(!)

Wave-particle duality

↳ each experiment can only see <u>one</u> aspect

↳ in a given measurement only <u>one</u> model applies

·) entity detected by some interaction with matter } acts like <u>particle</u>, is <u>localized</u>

·) when it moves / propagates: behaves like <u>wave</u>, interference, <u>not localized</u>

<u>Principle</u> of <u>complimentarity</u>: wave and particle models are complimentary (Bohr)

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·) To describe { light waves → guide photons; radiation:

↳ wave for electric field

$$\mathcal{E}(x,t) = A \sin\left[2\pi\left(\frac{x}{\lambda} - \nu t\right)\right]$$

→) satisfies wave equation: $\boxed{\frac{\partial^2 \mathcal{E}}{\partial x^2} = \frac{1}{(\lambda\nu)^2}\frac{\partial^2 \mathcal{E}}{\partial t^2}}$

(intensity) $I = \overline{\mathcal{E}}^2$

$I = N h\nu$ — $N$: average number of photons per unit time crossing unit area perpendicular to direction of propagation

$\overline{\mathcal{E}}^2 \leftrightarrow N$

↳ related to the probability of finding a photon in a unit area

·) To describe matter waves → guide material particles

$$\Psi(x,t) = A \sin\left[2\pi\left[\frac{x}{\lambda} - \nu t\right)\right]$$

wave function

satisfies Schrödinger equation

$\Psi^2$ ~~near~~ probability of finding a particle in a unit volume at the position $x$ at time $t$

1D : $x$

$\Psi(x,t) \rightarrow$ wave function (complex)

$|\Psi(x,t)|^2 \rightarrow$ probability of finding particle at point $x$ at time $t$

$$\int_a^b |\Psi(x,t)|^2 \, dx = \begin{cases} \text{probability of finding the particle} \\ \text{between } a \text{ and } b \text{ at time } t \end{cases}$$

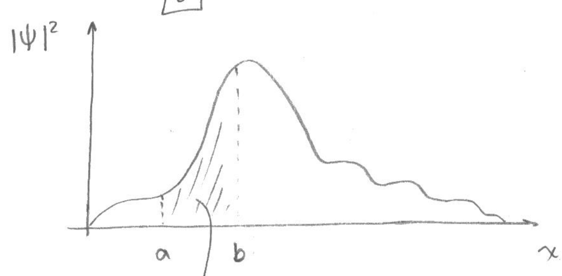
Contrary to classical physics, quantum physics is INTRINSICALLY probabilistic

- Classical physics: if we know the precise initial position AND momentum of a particle, we can predict its motion

- Quantum physics: can never know position AND momentum at the same time more accurately than is allowed by the Heisenberg uncertainty principle

$$\boxed{\Delta p \Delta x \geq \hbar/2}$$

1D $\boxed{t}$




AREA
probability to find particle at $t$ between a and b

To describe $\left\{\begin{array}{l}\text{a localized particle} \\ \text{a pulse}\end{array}\right.$, just $\Psi(x,t) = A \begin{array}{c}\cos\\ \sin\end{array}(kx - \omega t)$ is

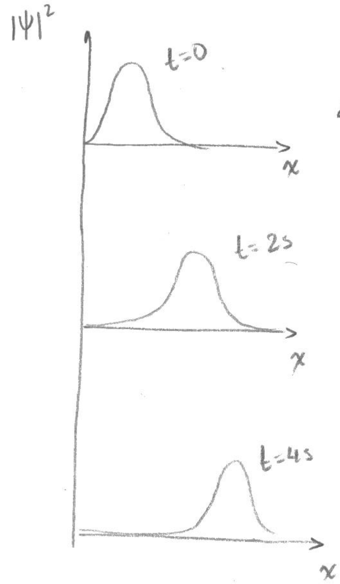
not enough, we need a $\boxed{\text{wave packet}}$

$$\Psi(x,t) = \sum_{n=1}^{\overset{\boxed{\text{infinity}}}{\infty}} A_n \cos(k_n x - \omega_n t)$$

$A \rightarrow$ amplitude

$k = \frac{2\pi}{\lambda} \rightarrow$ wave number

$\omega = 2\pi\nu \rightarrow$ angular frequency




matter wave is in the form of a GROUP

Mathematica

that's what they tried with Mathematica

2 sin's ⇒ enveloppe moving slower

many sin's ⇒ more localized, but other peaks still present. For just one $\boxed{\int}$ instead of $\boxed{\Sigma}$

so waves won't be in phase anymore

→) Let us derive the uncertainty principle relation by combining

$p = h/\lambda$, $E = h\nu$ and properties of waves

Let us start with the sum of $\boxed{\text{two}}$ sinusoidal waves of slightly different $\lambda$ and $\nu$

$$\Psi(x,t) = \sin(kx - \omega t) + \sin((k + dk)x - (\omega + d\omega)t)$$

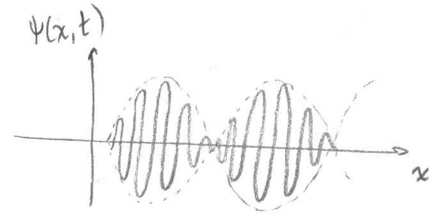
$$\sin A + \sin B = 2\cos\left[(A-B)/2\right] \sin\left[(A+B)/2\right] = 2\left(\cos\frac{A}{2}\cos\frac{B}{2} + \sin\frac{A}{2}\sin\frac{B}{2}\right)\left(\sin\frac{A}{2}\cos\frac{B}{2} + \sin\frac{B}{2}\cos\frac{A}{2}\right)$$

$$= \sin A\cos^2\frac{B}{2} + \cos^2\frac{A}{2}\sin B + \sin^2\frac{A}{2}\sin B + \sin A\sin^2\frac{B}{2}$$

sin A    sin B

$$\Psi(x,t) = 2\cos\left(\frac{dk}{2}x - \frac{d\omega}{2}t\right)\sin\left(\left(k+\frac{dk}{2}\right)x - \left(\omega+\frac{d\omega}{2}\right)t\right)$$

since $d\omega \ll \omega$ and $dk \ll k$




$$\boxed{\Psi(x,t) = \underbrace{2\cos\left(\frac{dk}{2}x - \frac{d\omega}{2}t\right)}_{\substack{\text{enveloppe,} \\ \text{it modulates } \sin(kx-\omega t)}}\sin(kx-\omega t)}$$

In mathematica - past HW

$p_1 = \sin[1.x]$

$p_2 = \sin[1.1\,x]$

$\left\{ \begin{array}{l} A=?\ ,\ t=? \\ dk=? \end{array} \right.$

$pe = 2\cos[0.1/2\,x]$ ← what is it?

Suppose a particle moving at speed $v$. what is the velocity of the associated matter wave?

↳ The associated wave is the $\left\{\begin{array}{c}\text{packet} \\ \text{GROUP}\end{array}\right\}$, which also moves with $v$

⟶) group velocity: given by the enveloppe

$$\boxed{v_g = \frac{d\omega}{dk}}$$

careful(!) $\left\{ \begin{array}{l} k = 2\pi/\lambda \\ \dfrac{dk}{d\lambda} = -\dfrac{2\pi}{\lambda^2} \end{array} \right.$

$E = h\nu$ and $\omega = 2\pi\nu \Rightarrow \boxed{E = \hbar\omega}$

$p = h/\lambda$ and $k = 2\pi/\lambda \Rightarrow \boxed{p = \hbar k}$

$$\boxed{v_g = \frac{dE}{dp}}$$

$$v_g = \frac{dE}{dp} \overset{E = p^2/2m}{=} \frac{2p}{2m} = v$$

⟶) velocity of individual wave $\sin(kx-\omega t)$ is smaller

$$v_w = \lambda\nu = \frac{E}{p} = \frac{1/2\, m v^2}{m v} = \frac{v}{2}$$

Therefore, to describe a localized particle we use a wave packet with group velocity $v_g = \frac{d\omega}{dk}$ equal to the velocity of the particle

→ Sum of $\boxed{\text{more}}$ waves

$$\boxed{t=0}$$

$$\psi(x) = \begin{cases} \tfrac{1}{4}\cos[2\pi\, 9x] \\ + \tfrac{1}{3}\cos[2\pi\, 10x] \\ + \tfrac{1}{2}\cos[2\pi\, 11x] \\ + 1\cos[2\pi\, 12x] \\ + \tfrac{1}{2}\cos[2\pi\, 13x] \\ + \tfrac{1}{3}\cos[2\pi\, 14x] \\ + \tfrac{1}{4}\cos[2\pi\, 15x] \end{cases} \qquad \psi'(x) = \begin{cases} \tfrac{1}{4}\cos[9x] \\ + \tfrac{1}{3}\cos[10x] \\ + \tfrac{1}{2}\cos[11x] \\ + 1\cos[12x] \\ + \tfrac{1}{2}\cos[13x] \\ + \tfrac{1}{3}\cos[14x] \\ + \tfrac{1}{4}\cos[15x] \end{cases}$$

$\Delta x$ = width at half of the main peak

$$\begin{cases} \Delta x \approx 0.05 \\ \\ \Delta k \approx 4\pi \end{cases} \qquad \begin{cases} \Delta x \approx 0.2 \\ \\ \Delta k = 1 \end{cases}$$

$\Delta k$ = width at half of the peak for


<u>Conclusion</u>: larger $\Delta k \Rightarrow$ narrower $\Delta x$

$\Delta x$ is inversely proportional to $\Delta k$

We need Fourier integral to prove that $\Delta x\, \Delta k \geq 1/2 \xrightarrow{p = \hbar k} \boxed{\Delta x\, \Delta p \geq \hbar/2}$