Stability of the Nuclear Atom

Rutherford's model ⇒ at the center of the atom there is a nucleus whose $m_{nucleus} \sim m_{atom}$ and $q_{nucleus} = Ze$

around nucleus : $Z\ e^-$

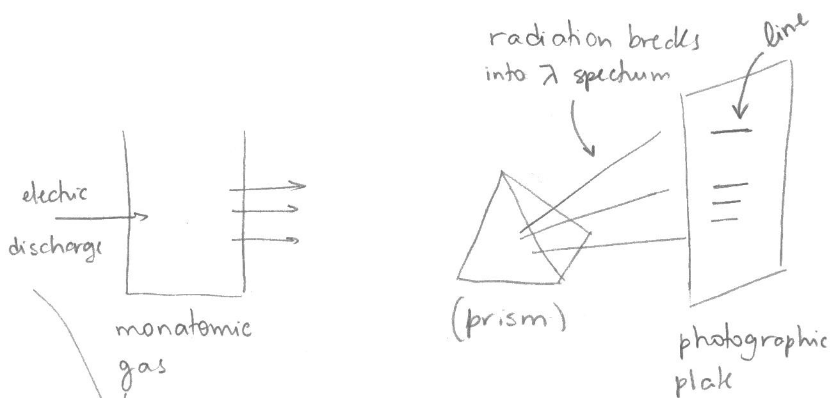
but stability ?

- stationary e^- ⇒ no possible stable arrangement, e^- would fall into nucleus
- circulating as in the solar system ⇒ e^- would lose energy (accelerated e^- ⇒ radiation) and fall into nucleus

ALSO continuous spectrum that would be emitted in the process is not observed, spectrum is discrete

Solution: Bohr's model

Atomic Spectra



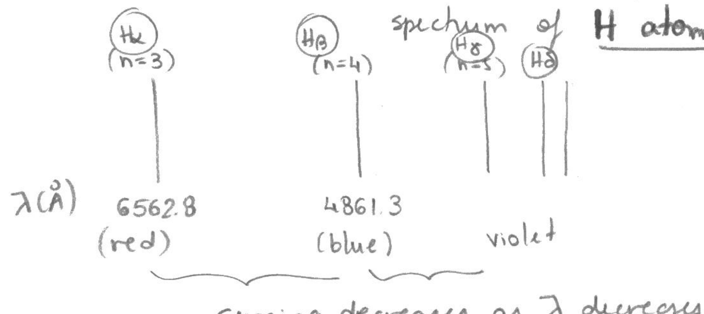
collisions ⇒ some atoms have more energy. When returning to normal energy state → give up excess energy by emitting electromagnetic radiation

electromagnetic radiation emitted by free atoms is concentrated at a number of discrete wavelengths

Each kind of atom has its own characteristic spectrum

SPECTROSCOPY

(1885) Balmer — access to VISIBLE spectrum of H atom



$$\boxed{\lambda = 3646\ \frac{n^2}{n^2-4}}$$

1890 Rydberg: → extended to other elements

Balmer formula written for $1/\lambda$ for H → $K = \frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$ $n = 3, 4, 5$

R_H → Rydberg constant $= 10967757.6 \pm 1.2\ m^{-1}$

See Table 4-1, Eisberg, for Hydrogen series for UV and IR

Spectrum of free atoms → discrete

spectroscopy

stars

Bohr's postulate

↳ explains stability at lines of the spectrum of atoms

1) e^- moves in circular orbit around nucleus due to Coulomb attraction obeying laws of classical mechanics

2) only possible orbits: those for which L is an integral multiple of $\hbar$
$\hbar = h/2\pi$ $(L = n\hbar)$ ← quantum number

3) e^- are constantly accelerating, but when moving in allowed orbit they do not radiate, E is const   (stationary states)

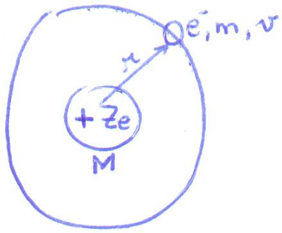
4) Electromagnetic radiation is emitted only when an e^- moves from one allowed orbit to another one. The frequency of the emitted radiation is

$$\nu = \frac{E_i - E_f}{h}$$

Bohr's Model

-) predictions derived from postulates agree with experiments

consider a single e^-



$Z=1$ (hydrogen) - neutral

$Z=2$ (singly ionized helium)

$Z=3$ (doubly ionized lithium)

-) Assume:

- circular orbit
- $M \gg m \Rightarrow$ nucleus is fixed

$$(*) \quad \underbrace{\frac{kZe^2}{r^2}}_{\text{Coulomb}} = \underbrace{m\frac{v^2}{r}}_{ma}$$

$\hookrightarrow$ centripetal acceleration

-) Orbital angular momentum of e: } $L = mvr$

$\downarrow$

quantization condition $L = n\hbar$ $\Rightarrow$ { only some radii exist; only some speeds exist

$mvr = n\hbar$ $\quad n = 1, 2, 3 \ldots$

$\hookrightarrow v = \frac{n\hbar}{mr}$

-) (*) Find RADIUS: $r = \frac{kZe^2}{mv^2} \Rightarrow \cancel{r} = \frac{kZe^2}{\cancel{m} n^2 \hbar^2} m^2 r^2$

$$\boxed{r = \frac{n^2 \hbar^2}{k m Z e^2}}$$

$> Z, < r$

$> n, > r$

$$\boxed{a_0 = \frac{\hbar^2}{mke^2}}$$ (Bohr radius)

$= 0.529\ \mathring{A} \leftarrow$ Hydrogen atom in the ground state

$n = 1, 2, 3 \ldots$

e^- has minimum energy

-) Find speed of e^-: $\boxed{v = \frac{kZe^2}{n\hbar}}$ $\quad n = 1, 2, 3 \ldots$

$\Rightarrow$ n cannot be zero

$> Z, \Rightarrow > v$

$> n \Rightarrow < v$

from $\frac{kZe^2}{r^2} = m\frac{v^2}{r}$ (Coulomb = centripetal a)

$K = \frac{1}{2}mv^2 = \frac{kZe^2}{2r}$

$U = -\frac{kZe^2}{r}$ ← from $U = -\int_r^\infty \frac{kZe^2}{r^2}\,dr = -\frac{kZe^2}{r}$

since Coulomb force is attractive, we need to do work to move e^- from r to ∞ against this force

→) <u>TOTAL</u> energy of e^-

$$E = K + U = -\frac{kZe^2}{2r}$$

$$r = \frac{n^2\hbar^2}{kmZe^2}$$

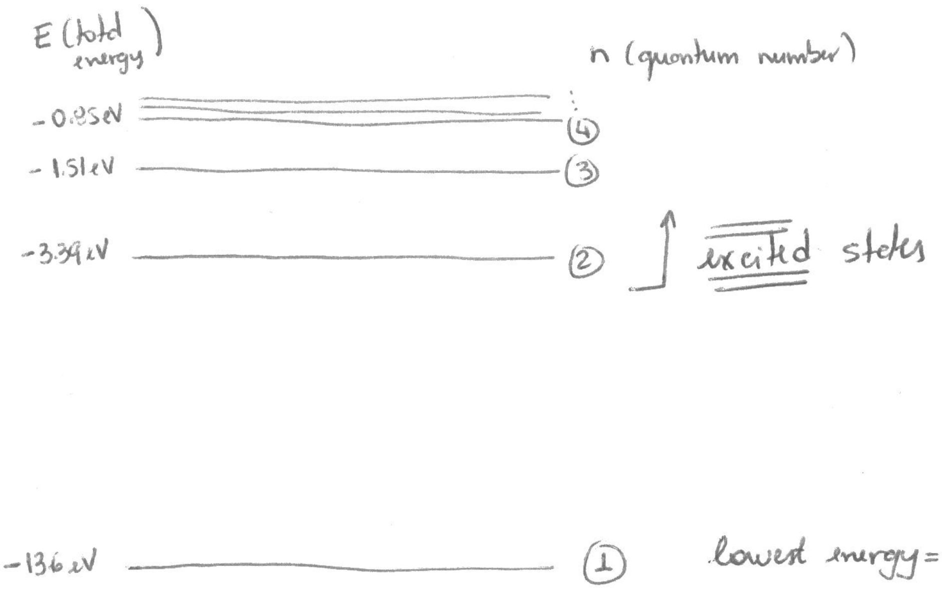
need energy to remove e^-

$$E_0 = \frac{k^2me^4}{2\hbar^2}$$

$$\boxed{E = -\frac{k^2m(Ze^2)^2}{2\hbar^2}\frac{1}{n^2}} \qquad n = 1, 2, 3, \ldots$$

$$\boxed{= -\frac{Z^2E_0}{n^2}}$$

↳ quantization of L leads to <u>quantization</u> of total <u>energy</u> of e^-



lowest energy = <u>ground</u> state (most stable state)

most stable state, most negative

need the largest energy to remove e^-

E_0 { the <u>binding</u> energy of H is -13.6eV
or <u>ionization</u> energy of H is $+13.6$eV

→) <u>Frequency</u> of radiation emitted when e^- makes a transition from state n_i to n_f

$$\nu = \frac{E_i - E_f}{h} = -\frac{k^2m(Ze^2)^2}{h\,2\hbar^2}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = \boxed{+\frac{k^2m(Ze^2)^2}{2h\hbar^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

(E_i: higher energy; E_f: lower energy; $h = \hbar 2\pi$)

Electron makes a transition from state n_i to n_f

$$\nu = \frac{E_i - E_f}{h} = \boxed{\frac{E_0 Z^2}{h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

$K = 1/\lambda = \nu/c$

$$K = \frac{1}{\lambda} = \frac{E_0 Z^2}{hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

→ $\underbrace{\text{Bohr's prediction}}_{R_\infty}$ for the $\underbrace{\text{Rydberg constant}}_{R_H}$

$n_f = 1$ Lyman series

$n_f = 2$ Balmer series : $K = R_\infty \left(1/2^2 - 1/n^2\right)$

⋮

→) Correction for Finite Nuclear Mass (Bohr's assumption that nucleus is fixed → M is infinite)

→ substitute m by μ. μ = reduced mass

$$\text{TOTAL } K = \underbrace{\frac{p^2}{2M}}_{\text{nucleus}} + \underbrace{\frac{p^2}{2m}}_{\text{electron}} = \frac{M+m}{2mM} p^2 = \frac{p^2}{2\mu}$$

$P_{nucleus} = P_{electron}$ $\underbrace{\text{(atom at rest)}}_{P_{atom} = 0}$

new part

$$\boxed{\mu = \frac{mM}{m+M}}$$

$$R = \frac{k^2 \mu e^4}{4\pi c \hbar^3} = R_\infty \left(\frac{1}{1 + m/M}\right)$$