Introduction to Modern Physics

Two pillars of modern physics: quantum mechanics (QM) and relativity

both → generalization of classical physics, include classical laws as special cases

- relativity → extends the range of application of physical laws to high velocities

  universal constant: speed of light $\boxed{c}$

- QM → extends that range to the region of small dimensions

  universal constant: Planck's constant $\boxed{h}$, $\hbar = h/2\pi$

We start with the milestones that led to modern quantum mech.

Several phenomena in contradiction with classical laws

↳ solving conflicts needed introduction of quantum ideas

Ex: discreteness of energy

concept introduced by Planck when studying thermal radiation

Thermal radiation

∘) radiation falls on object
- → part reflected (more on light-colored bodies)
- → part absorbed (more on dark bodies)

∘) radiation absorbed ⇒ increases kinetic energy of atoms ⇒ oscillate ⇒
⇒ oscillating/accelerated charges emit

temperature rises {average translational kinetic energy of atoms determine temperature of body

temperature reduces {reduces kinetic energy of oscillations

good absorber → good emitter

∘) When rate of absorption = rate of emission ⇒ constant temperature
body in thermal equilibrium

↳ electromagnetic radiation emitted called thermal radiation

At ordinary temperatures ($< 600°C$) : thermal radiation NOT visible
large λ, short ν
object seen by reflected light

$600°C - 700°C$: enough energy in visible spectrum, object glows

→ higher T → body emits more, ν becomes higher

Detailed form of the spectrum of the thermal radiation emitted deps. on composition but for a

blackbody it is universal and depends only on the temperature

↳ ideal body that absorbs all thermal radiation incident upon it

→ all blackbodies at the same temperature emit thermal radiation with the same spectrum.

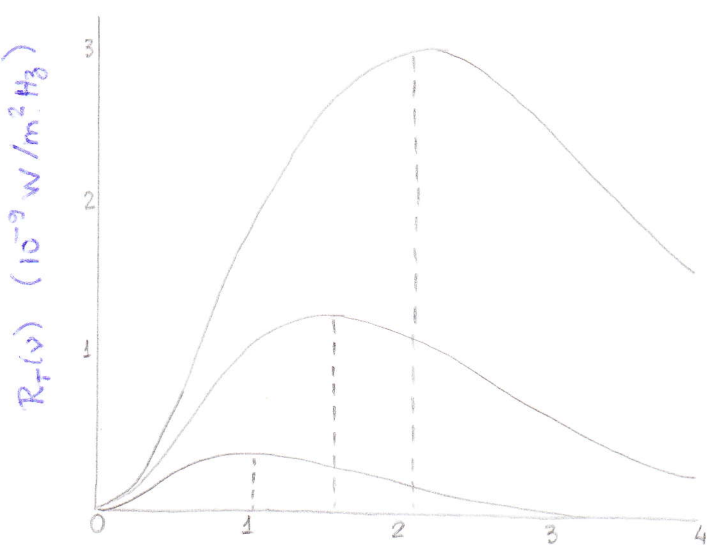
Specific form of a <u>blackbody</u> spectrum <u>CANNOT</u> be understood on the basis of <u>classical</u> arguments

Spectral distribution of black body radiation

$R_T(\nu)$ = spectral radiancy $\left\{ \begin{array}{l} \text{energy emitted per unit time, per unit area} \\ \text{per unit frequency} \rightarrow \frac{J}{s\, m^2\, Hz} = \frac{W}{m^2\, Hz} \end{array} \right.$

$R_T(\nu)\, d\nu$ = $\underbrace{\text{energy emitted per unit time}}_{(\text{power}) \rightarrow W}$ in radiation of frequency in the interval ν to $\nu + d\nu$ per unit area of the surface at temperature T

<u>experiments</u>:



(1) power decreases as $\nu \rightarrow 0$, $\nu \rightarrow \infty$

(2) <u>frequency</u> at which radiated power is most intense increases with <u>temperature</u>

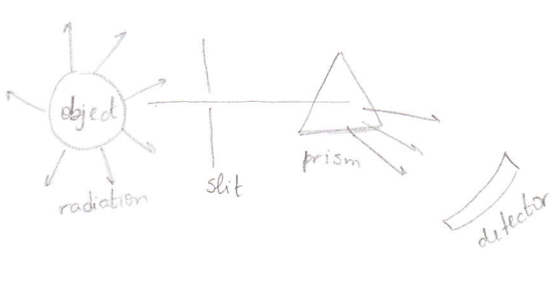
$\boxed{\nu_{max} \propto T}$ Wien's (*) displacement law

(3) total <u>power</u> radiated in all frequencies increases with <u>temperature</u>

$$R_T = \int_0^{\infty} R_T(\nu)\, d\nu \quad \left(\begin{array}{c}\text{area under} \\ \text{the curve}\end{array}\right)$$

radiancy = $\frac{\text{flux of}}{\text{energy}}$ $\boxed{\text{intensity}}$ = energy per unit time per unit area $\left(\frac{J}{m^2 s} = \frac{W}{m^2}\right)$

$\boxed{R_T = \sigma T^4}$ Stefan's law



$\sigma = 5.67 \times 10^{-8}\ W/m^2 K^4$ (Stefan - Boltzmann const)

(*) <u>Obs</u>: $\lambda \nu = c$ $\Rightarrow$ $\boxed{\lambda_{max} T = 2.898 \times 10^{-3}\ m\,K}$

Wien's constant

Question

Assume that stellar surfaces behave like blackbodies.
For the sun $\lambda_{max} = 5100\ \text{Å}$, whereas for the North Star $\lambda_{max} = 3500\ \text{Å}$
Find the surface temperature of these stars
$(1\ \text{Å} = 10^{-10}\ m)$

$$\lambda_{max} T = 2.898 \times 10^{-3}\ m.K$$

$$T_{sun} = \frac{2.898\times10^{-3}}{5100\times10^{-10}} = \boxed{5700\ K} \qquad T_{\substack{North\\star}} = \frac{2.898\times10^{-3}}{3500\times10^{-10}} = \boxed{8300\ K}$$

Question

Measurement of λ_{max} from a certain star indicates $T = 3000\ K$.
If the star radiates 100 times the power radiated by the sun, how big is the star? Sun surface temperature = 5800 K

$$R_{star} = \frac{P_{star}}{area_{star}} = \frac{100\ P_\odot}{4\pi r^2_{star}} = \sigma T^4_{star} \Rightarrow 4\pi r^2_{star} = \frac{100\ P_\odot}{\sigma T^4_{star}}$$

$$R_\odot = \frac{P_\odot}{area_\odot} = \frac{P_\odot}{4\pi r^2_\odot} = \sigma T^4_\odot \Rightarrow P_\odot = \sigma T^4_\odot \left(4\pi r^2_\odot\right)$$

$$\Rightarrow \cancel{4\pi} r^2_{star} = \frac{100\ \cancel{\sigma} T^4_\odot (\cancel{4\pi} r^2_\odot)}{\cancel{\sigma} T^4_{star}} \Rightarrow r_{star} = 10\, r_\odot \left(\frac{T_\odot}{T_{star}}\right)^2$$

$$r_{star} = 10\left(\frac{5800}{3000}\right)^2 r_\odot \Rightarrow \boxed{r_{star} = 37.4\, r_\odot}$$

·) CLASSICAL theory of cavity radiation

Example of blackbody: a cavity in a body connected by a small hole to the outside

(standing waves)

(lab blackbody)

energy density $\rho_T(\nu)$ = energy contained in a unit volume of the cavity at temperature T

$\rho_T(\nu) \propto R_T(\nu)$

→ [u in Tipler]

Energy per unit volume in the frequency interval ν to $\nu + d\nu$

Rayleigh and Jeans

$$\rho_T(\nu)\, d\nu = \frac{\bar{\varepsilon}\, N(\nu)\, d\nu}{V}$$

$N(\nu)\,d\nu$ → number of standing waves in the frequency interval ν to $\nu + d\nu$

$\bar{\varepsilon}$ → average energy of each standing wave in cavity

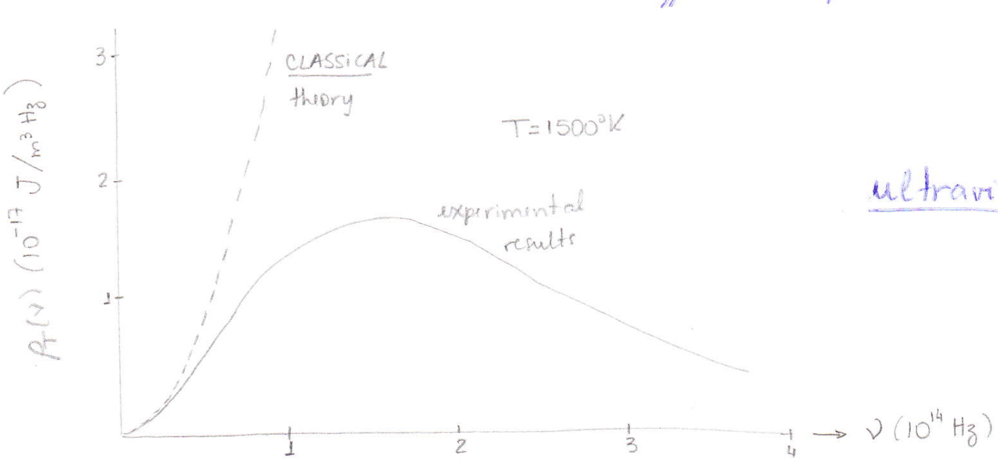
V → volume

$$N(\nu)\,d\nu = \frac{8\pi V}{c^3}\, \nu^2\, d\nu$$ ← derivation from geometrical arguments (see Eisberg's book)

Classical equipartition law $\bar{\varepsilon} = KT$

$$\boxed{\rho_T(\nu)\, d\nu = \frac{8\pi\, \nu^2 KT}{c^3}\, d\nu}$$

Rayleigh-Jeans formula for black body radiation



ultraviolet catastrophe

Classical equipartition law

Boltzmann distribution (appendix C - Eisberg's book)

$P(\mathcal{E}) = A e^{-\mathcal{E}/KT}$

$\int_0^\infty P(\mathcal{E})\, d\mathcal{E} = 1 \rightarrow \boxed{A = \frac{1}{KT}}$

$$P(\mathcal{E}) = \frac{e^{-\mathcal{E}/KT}}{KT}$$

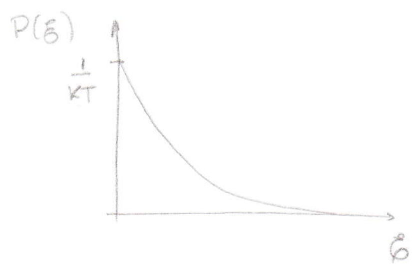
$e^{-\mathcal{E}/KT}$ → fraction of oscillators with energy $\mathcal{E}$

KT → normalization → guarantees that $\boxed{\int_0^\infty P(\mathcal{E})\, d\mathcal{E} = 1}$

(standing wave, ...)

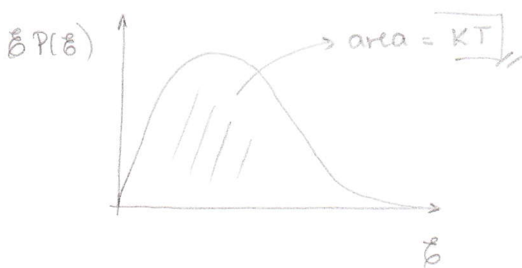
$P(\mathcal{E})\, d\mathcal{E}$: probability of finding an entity of the system with energy in $[\mathcal{E}, \mathcal{E} + d\mathcal{E}]$

↳ system with large number of entities in thermal equilibrium at T



→ large prob. for $\mathcal{E} = 0$

average energy: $\bar{\mathcal{E}} = \int_0^\infty \mathcal{E}\, P(\mathcal{E})\, d\mathcal{E}$



$$\int_0^\infty \frac{\mathcal{E}\, e^{-\mathcal{E}/KT}}{KT}\, d\mathcal{E} =$$

by parts

$u = \mathcal{E}$ $\quad dv = e^{-\mathcal{E}/KT}\, d\mathcal{E}$

$du = d\mathcal{E}$ $\quad v = e^{-\mathcal{E}/KT}(-KT)$

$$= \frac{1}{KT}\left[\cancel{\mathcal{E}\, e^{-\mathcal{E}/KT}(-KT)\Big|_0^\infty} - \int_0^\infty e^{-\mathcal{E}/KT}(-KT)\, d\mathcal{E} \right] =$$

$$= \int_0^\infty e^{-\mathcal{E}/KT}\, d\mathcal{E} = e^{-\mathcal{E}/KT}(-KT)\Big|_0^\infty = \boxed{KT}$$

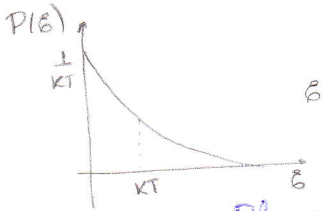
↳ Integration over ALL possible energies

↳ here $\mathcal{E}$ is a CONTINUOUS variable

c) Planck's theory of cavity radiation $\rho_T(\nu)\,d\nu = \dfrac{8\pi\nu^2\,\bar{\mathcal{E}}}{c^3}\,d\nu$

[redacted]

$$\left.\begin{aligned}\bar{\mathcal{E}} &\xrightarrow[\nu\to 0]{} KT\\ \bar{\mathcal{E}} &\xrightarrow[\nu\to\infty]{} 0\end{aligned}\right\}$$ to avoid discrepancy with experimental results, energy needs to be a function of ν



$\mathcal{E} \gg KT$ little contribution to $\bar{\mathcal{E}}$

$\boxed{\mathcal{E} = nh\nu}$ DISCRETE

Planck's constant

determined by best fit of theory with experiment

$\boxed{h = 6.63\times10^{-34}\ \text{J s}}$

$\boxed{\mathcal{E} = nh\nu}$ $n = 0, 1, 2, 3\ldots$ $\rightarrow$ energy is discrete / QUANTIZED

$\bar{\mathcal{E}} = \sum \mathcal{E} A e^{-\mathcal{E}/KT}$ (A: normalization)

$\sum A e^{-\mathcal{E}/KT} = 1$

$\hookrightarrow A = \dfrac{1}{\sum e^{-nh\nu/KT}}$

$$\bar{\mathcal{E}} = \frac{\sum_{n=0}^{\infty} \frac{nh\nu}{KT} e^{-nh\nu/KT}}{\sum_{n=0}^{\infty} e^{-nh\nu/KT}}\,(KT)$$

One way to solve

$$\sum_{n=0}^{\infty} e^{-nx} = 1 + e^{-x} + e^{-2x} + \ldots = \frac{1}{(1-e^{-x})}$$

$$\sum_{n=0}^{\infty} n x e^{-nx} = x e^{-x} + 2x e^{-2x} + 3x e^{-3x} + \ldots$$

$$= x e^{-x} + x e^{-2x} + x e^{-3x} + x e^{-4x} + \ldots + x e^{-2x} + x e^{-3x} + x e^{-4x} + \ldots + x e^{-3x} + x e^{-4x} + \ldots \;\ldots$$

$$= x e^{-x}(1 + e^{-x} + e^{-2x} + \ldots) + x e^{-2x}(1 + e^{-x} + \ldots) + x e^{-3x}(1 + \ldots)$$

$$= \frac{1}{(1-e^{-x})}\, x e^{-x}(1 + e^{-x} + e^{-2x} + \ldots) = \frac{x e^{-x}}{(1-e^{-x})^2}$$

$$\boxed{\frac{\sum n x e^{-nx}}{\sum e^{-nx}} = \frac{x e^{-x}}{(1-e^{-x})}}$$

Another way to solve (Eisberg)

Note:

$$\frac{\sum_{n=0}^{\infty} n x e^{-nx}}{\sum_{n=0}^{\infty} e^{-nx}} = -x \frac{d}{dx} \ln \sum_{n=0}^{\infty} e^{-nx}$$

since

$$= -x \frac{\frac{d}{dx}\sum_{n=0}^{\infty} e^{-nx}}{\sum_{n=0}^{\infty} e^{-nx}} = -\frac{\sum_{n=0}^{\infty} x \frac{d}{dx}(e^{-nx})}{\sum_{n=0}^{\infty} e^{-nx}} = \frac{\sum n x e^{-nx}}{\sum e^{-nx}}$$

$$-x \frac{d}{dx} \ln \sum_{n=0}^{\infty} e^{-nx} = -x \frac{d}{x} \ln\left(\frac{1}{1-e^{-x}}\right)$$

$$= -x\,(1-e^{-x})\,\frac{(-1)}{(1-e^{-x})^2}\,e^{-x} = \boxed{\frac{x e^{-x}}{(1-e^{-x})}}$$

$$\Rightarrow \bar{\mathcal{E}} = KT\left(\frac{h\nu}{KT}\right)\frac{e^{-h\nu/KT}}{(1-e^{-h\nu/KT})}$$

$x = h\nu/KT$

$$\boxed{\bar{\mathcal{E}} = \frac{h\nu}{e^{h\nu/KT}-1}}$$ //

$$\bar{\varepsilon}(\nu) = \frac{h\nu}{e^{h\nu/kT}-1} \quad \begin{cases} \nu \to 0 \Rightarrow \bar{\varepsilon} \to \dfrac{h\nu}{1+\frac{h\nu}{kT}+\dots-1} \to \boxed{kT} \\ \nu \to \infty \Rightarrow \bar{\varepsilon} \to h\nu\, e^{-h\nu/kT} \to \boxed{0} \end{cases}$$

$$\boxed{\rho_T(\nu)\,d\nu = \frac{8\pi\nu^2}{c^3}\,\frac{h\nu}{e^{h\nu/kT}-1}\,d\nu}$$

Planck's blackbody spectrum

Planck's blackbody spectrum in terms of λ

$$\boxed{c = \lambda\nu} \Rightarrow \nu = \frac{c}{\lambda} \Rightarrow \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2} \Rightarrow \boxed{d\nu = -\frac{c}{\lambda^2}\,d\lambda}$$

$$\rho_T(\lambda)\,d\lambda = \ominus\,\rho_T(\nu)\,d\nu \qquad \begin{cases} d\lambda,\ d\nu \text{ have opposite sign} \\ \text{increase in } \nu \to \text{decrease in } \lambda \end{cases}$$

$$\rho_T(\lambda)\,d\lambda = \frac{c}{\lambda^2}\,\rho_T(\nu)\,d\lambda = \frac{\cancel{c}}{\lambda^2}\,\frac{8\pi}{\cancel{c^3}}\,\frac{c^2}{\lambda^2}\,\frac{hc/\lambda}{e^{hc/\lambda kT}-1}\,d\lambda$$

$$\boxed{\rho_T(\lambda)\,d\lambda = \frac{8\pi hc}{\lambda^5}\,\frac{d\lambda}{e^{hc/\lambda kT}-1}}$$

HW

Tipler: 3-12, 13, 14, 15, 16, 17, 20

Question: Show that the total energy density in a blackbody cavity is proportional to T^4 in accordance with Stefan-Boltzmann law

$$U = \int_0^\infty \rho(\nu)\,d\nu = \frac{8\pi h}{c^3}\int_0^\infty \frac{\nu^3}{e^{h\nu/kT}-1}\,d\nu \qquad x = \frac{h\nu}{kT} \qquad dx = \frac{h}{kT}\,d\nu$$

$$U = \frac{8\pi h}{c^3}\left(\frac{kT}{h}\right)^3\int_0^\infty \frac{x^3}{e^x-1}\left(\frac{kT}{h}\right)dx$$

$$U = \frac{8\pi}{c^3h^3}(kT)^4\underbrace{\int_0^\infty \frac{x^3}{e^x-1}\,dx}_{(\text{dimensionless}) = \pi^4/15} \Rightarrow \boxed{U \propto T^4}$$

use $U = \dfrac{8\pi^5 k^4 T^4}{15 h^3 c^3}$ and $R = \dfrac{c}{4}U$ to find σ in terms of h, c, k