Introduction to Modern Physics

Two pillars of modern physics: quantum mechanics (QM) and relativity. Both generalize classical physics, include classical laws as special cases.

1) Relativity extends the range of application of physical laws to high velocities. Universal constant: speed of light $c$.

2) QM extends that range to the region of small dimensions. Universal constant: Planck's constant $\hbar$, $\hbar = h/2\pi$.

We start with the milestones that led to modern quantum mechanics.

Several phenomena in contradiction with classical laws led solving conflicts needed introduction of quantum ideas. Examples of energy discreteness.

Concept introduced by Planck when studying thermal radiation.
Thermal radiation

1. Radiation falls on object
   - Part reflected (more on light-colored bodies)
   - Part absorbed (more on dark bodies)

2. Radiation absorbed → Increase kinetic energy of atoms → Oscillogram/acaularized charges emit

   - Temperature rises
   - Average translational kinetic energy of atoms determines temperature of body
   - Temperature reduces
   - Kinetic energy of oscillations reduces

   Good absorber → Good emitter

When rate of absorption = rate of emission = constant temperature, body is in thermal equilibrium

Electromagnetic radiation emitted called thermal radiation

At ordinary temperatures (<600°C): thermal radiation not visible

- Large λ, short ν
- Object seen by reflected light

600°C - 700°C: Enough energy in visible spectrum, object glows

- Higher T → Body emits more, ν becomes higher

Detailed form of spectrum of thermal radiation emitted depends on composition but for a blackbody it is universal and depends only on the temperature

- Ideal body that absorbs all thermal radiation incident upon it

All blackbodies at the same temperature emit thermal radiation with the same spectrum.
Specific form of a blackbody spectrum CANNOT be understood on the basis of classical arguments.

Special distribution of blackbody radiation:

\[ R_{\nu}(\nu) = \text{spectral radiance} \]
\[ = \frac{\text{energy emitted per unit time, per unit area}}{\text{per unit frequency} \cdot \text{per unit area}} \]
\[ = \frac{J}{\nu^3} \]

\[ R_{\nu}(\nu) \, d\nu = \text{energy emitted per unit time in radiation of frequency in the interval } \nu \text{ to } \nu + d\nu \text{ per unit area of the surface at temperature } T \]

Experiments:

1. Power decreases as \( \nu \to 0, \nu \to \infty \)

2. Frequency at which radiated power is most intense increases with temperature:

   \[ \nu_{\text{max}} \propto T \]

   Wien's displacement law

3. Total power radiated in all frequencies increases with temperature:

   \[ R_T = \int_0^\infty R_{\nu}(\nu) \, d\nu \] (area under the curve)

   \[ R_T = \frac{\text{flux of energy per unit time per unit area}}{\text{intensity}} \]

   \[ R_T = 6 \times 10^{-8} \text{ W/m}^2\text{K}^4 \] (Stefan-Boltzmann constant)

   \[ T = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \]

   Wien's constant

   \[ \lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m K} \]

   Obs. \( \lambda T = c \Rightarrow \lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m K} \)
Question

Assume that stellar surfaces behave like blackbodies.

For the sun $\text{A}_{\text{max}} = 5100 \text{ Å}$, whereas for the North Star $\text{A}_{\text{max}} = 3500 \text{ Å}$.

Find the surface temperature of these stars ($1 \text{ Å} = 10^{-10} \text{ m}$)

\[ \text{A}_{\text{max}} T = 2.898 \times 10^{-2} \text{ m K} \]

\[ T_{\text{sun}} = \frac{2.898 \times 10^{-2}}{5100 \times 10^{-10}} = 5700 \text{ K} \]

\[ T_{\text{north star}} = \frac{2.898 \times 10^{-2}}{3500 \times 10^{-10}} = 8300 \text{ K} \]

Question

Measurement of $\text{A}_{\text{max}}$ from a certain star indicates $T = 3000 \text{ K}$.

If the star radiates 100 times the power radiated by the sun, how big is the star? Sun surface temperature $= 5800 \text{ K}$

\[ R_{\text{star}} = \frac{P_{\text{star}}}{4 \pi \text{A}_{\text{star}} T_{\text{star}}^4} = \frac{100 P_{\odot}}{4 \pi \text{A}_{\odot} T_{\odot}^4} \Rightarrow 4 \pi \text{A}_{\text{star}} = \frac{100 P_{\odot}}{P_{\odot} T_{\odot}^4} = 100 \frac{T_{\odot}^4}{T_{\text{star}}^4} \]

\[ \Rightarrow 4 \pi \text{A}_{\text{star}} = \frac{100}{\text{A}_{\odot}^{4/3}} \Rightarrow A_{\text{star}} = 10 \pi \text{A}_{\odot} \left( \frac{T_{\odot}}{T_{\text{star}}} \right)^2 \]

\[ \pi_{\text{star}} = 10 \left( \frac{5800}{3000} \right)^2 \pi_{\odot} \Rightarrow \pi_{\text{star}} = 37.4 \pi_{\odot} \]
Classical theory of cavity radiation

Example of blackbody: a cavity in a body connected by a small hole to the outside

\[
\text{(lab blackbody)}
\]

Energy density \( P_r(v) \) = energy contained in a unit volume of the cavity at temperature \( T \)

\[
P_r(v) \propto R_r(v)
\]

Energy per unit volume in the frequency interval \( v \) to \( v + dv \)

\[
P_r(v) dv = \bar{E} N(v) dv
\]

Rayleigh and Jeans

\[
N(v) dv = \frac{8 \pi v^2}{c^3} dv
\]

derivation from geometrical arguments

(Su Eising's book)

Classical equipartition law \( \bar{E} = kT \)

\[
P_r(v) dv = \frac{8 \pi v^2 kT}{c^3} dv
\]

Rayleigh-Jeans formula for blackbody radiation

\[
\begin{align*}
P_r(v) & \propto \frac{1}{v^3} \\
& \propto \frac{1}{v^2}
\end{align*}
\]

\[
\begin{align*}
T &= 1500^\circ K \\
\text{Experimental results}
\end{align*}
\]

ultraviolet catastrophe
Classical equipartition law

\( P(\varepsilon) = A e^{-\varepsilon/kT} \)

fraction of oscillators with energy \( \varepsilon \)

\[ P(\varepsilon) = \frac{e^{-\varepsilon/kT}}{kT} \]

normalization guarantees that \( \int_0^\infty P(\varepsilon) \, d\varepsilon = 1 \)

Standing wave, ...

\( P(\varepsilon) \, d\varepsilon \): probability of finding an entity of the system with energy in \([\varepsilon, \varepsilon + d\varepsilon]\)

\( \Rightarrow \) system with large number of entities in thermal equilibrium at \( T \)

\( \Rightarrow \) large prob. for \( \varepsilon = 0 \)

Average energy

\( \bar{\varepsilon} = \int_0^\infty \varepsilon \, P(\varepsilon) \, d\varepsilon \)

\( \delta P(\varepsilon) \)

Area = \( kT \)

\[ \int_0^\infty \frac{\varepsilon \, e^{-\varepsilon/kT}}{kT} \, d\varepsilon = \]

by parts

\[ m = \varepsilon \quad \text{dv} = e^{-\varepsilon/kT} \, d\varepsilon \]
\[ \text{du} = d\varepsilon \quad v = e^{-\varepsilon/kT} (-kT) \]

\[ = \frac{1}{kT} \left[ \varepsilon \, e^{-\varepsilon/kT} (-kT) \right]_0^\infty - \int_0^\infty e^{-\varepsilon/kT} (-kT) \, d\varepsilon \]

\[ = \frac{1}{kT} \left[ \varepsilon \, e^{-\varepsilon/kT} (-kT) \right]_0^\infty - \frac{kT}{0} = \frac{kT}{0} \]

Integration over ALL possible energies

\( \bar{\varepsilon} \) is a **continuous variable**
Planck's theory of cavity radiation: $P(\gamma) \, d\gamma = \frac{8\pi \hbar^2 \gamma^3}{e^{\frac{\hbar \gamma}{kT}} - 1} \, d\gamma$

- Planck's constant $\hbar = 6.63 \times 10^{-34}$ J s

\[ \overline{\varepsilon} = \hbar \nu \] \[ \nu = 0, 1, 2, 3, \ldots \] energy is discrete / quantized

\[ \overline{\varepsilon} = \sum \frac{\hbar \nu}{kT} e^{-\frac{\hbar \nu}{kT}} \] to avoid discrepancy with experimental results, energy must be a function of $\nu$

Another way to solve (Eisberg)

\[ \sum \frac{\hbar \nu}{kT} e^{-\frac{\hbar \nu}{kT}} = \frac{1}{(1-e^{-\frac{\hbar \nu}{kT}})} \]

Note:

\[ \sum \frac{\hbar \nu}{kT} e^{-\frac{\hbar \nu}{kT}} = -x \cdot \frac{d}{dx} \sum \frac{\gamma}{e^\gamma} \]
Planck's blackbody spectrum

\[ P_\tau (v) \, dv = \frac{8 \pi \nu^2}{c^3} \frac{\nu^3}{e^{\frac{\nu}{kT}} - 1} \]

Planck's blackbody spectrum in terms of \( \lambda \)

\[ c = \lambda \nu \Rightarrow \frac{c}{\lambda} = \frac{d\lambda}{dv} = -\frac{c}{\lambda^2} \Rightarrow dv = -\frac{c}{\lambda^2} \, d\lambda \]

\[ P_\tau (\lambda) \, d\lambda = \Theta \ P_\tau (v) \, dv \]

\( d\lambda, \, dv \) have opposite sign

\( \text{increase in } v \Rightarrow \text{decrease in } \lambda \)

\[ P_\tau (\lambda) \, d\lambda = \frac{c}{\lambda^2} P_\tau (v) \, dv = \frac{c}{\lambda^2} \frac{8 \pi}{c^3} \frac{\nu^2}{\lambda^2} \frac{h c / \lambda}{e^{hc / kT} - 1} \]

Question: Show that the total energy density in a blackbody cavity is proportional to \( T^4 \) in accordance with Stefan-Boltzmann law.

\[ U = \int_0^\infty P(v) \, dv = \frac{8 \pi}{c^3} \int_0^\infty \frac{\nu^3}{e^{\nu/kT} - 1} \, d\nu \]

\[ U = \frac{8 \pi}{c^3} \frac{(kT)^3}{h} \int_0^\infty \frac{\nu^3}{e^{\nu/kT} - 1} \, d\nu \]

\[ U = \frac{8 \pi}{c^3} \frac{(kT)^4}{h^2} \int_0^\infty \frac{\nu^2}{e^{\nu/kT} - 1} \, d\nu \]

\[ U \propto T^4 \]

\[ U = \frac{8 \pi}{15} \frac{k^4 T^4}{h^2 c^2} \]

\[ U \text{ in terms of } h, c, k \]

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