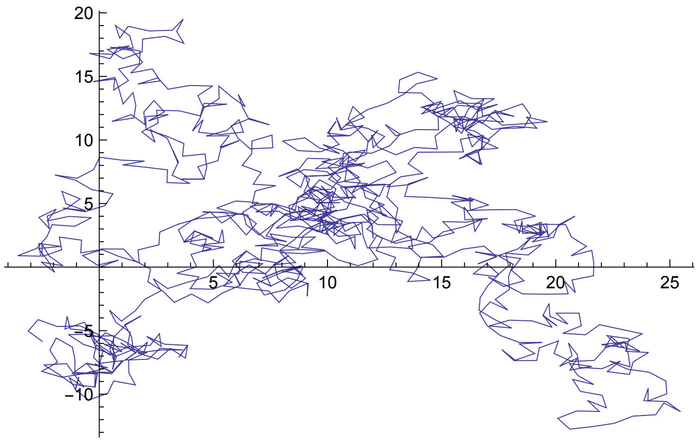
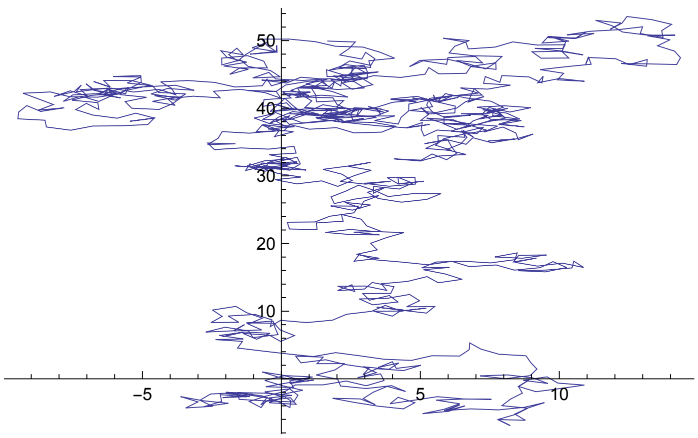
Assignment 10

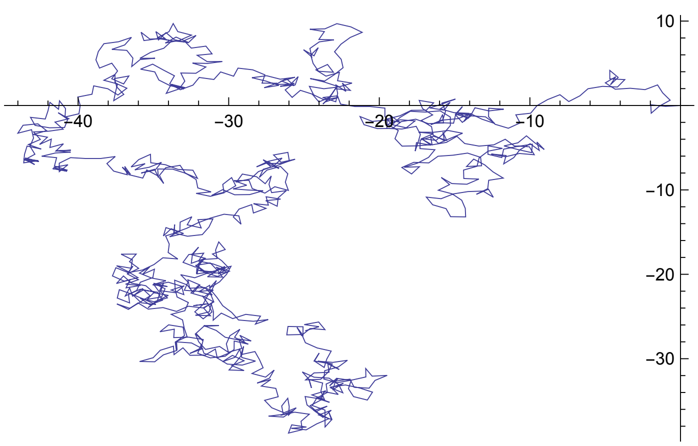
QUESTION

Use N=1000 and show 3 snapshots of a 2D Random Walk

```
Clear[Nt];
Nt = 1000;
Do[
 Clear[RW2];
 RW2 = {{0, 0}};
 Do[
  angle = RandomReal[{0, 360}];
  AppendTo[RW2, {Cos[angle Degree], Sin[angle Degree]}];
  , {k, 1, Nt}];
 tabSum = Table[Sum[RW2[[k]], {k, 1, j}], {j, 1, Nt}];
 Print[ListPlot[tabSum, Joined → True]];
 , {kk, 1, 3}]
```


20
15
10
5
5
10
15
20
25
−5
−10


50
40
30
20
10
−5
5
10


10
−40
−30
−20
−10
−10
−20
−30

QUESTION

Simulate 1000 random walks in a plane, each walk having 25 steps (steps having equal lengths =1). Let each walk start at (0,0) and each step be in a random direction. Compute the average distance from (0,0) after 4, 9, 16 and 25 steps.

```
Clear[Nt, Nrea, ave, RW2, tabSum];
Nt = 25;
Nrea = 1000;
Do[

  RW2 = {};
  Do[
   angle = RandomReal[{0, 360}];
   AppendTo[RW2, {Cos[angle Degree], Sin[angle Degree]}];
   , {k, 1, Nt}];

  Do[
   tabSum[r, nn^2] = Sum[RW2[[k]], {k, 1, nn^2}];
   , {nn, 2, 5}];

  , {r, 1, Nrea}];


Do[
  ave[nn^2] =
   Sum[Sqrt[tabSum[r, nn^2][[1]]^2 + tabSum[r, nn^2][[2]]^2], {r, 1, Nrea}] / Nrea;
  Print[{nn^2, ave[nn^2]}];
  , {nn, 2, 5}];
```

```
{4, 1.75901}
{9, 2.6342}
{16, 3.50203}
{25, 4.34666}
```

QUESTION

Using the Gaussian distribution

$$P(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right),$$

verify that
(a) it is normalized;
(b) <x> = μ
(c) variance = σ^2
[Hints: Assume Re[s^2]>0; Use PowerExpand]

```
Print["Normalization"];
Assuming[{Re[s^2] > 0}, PowerExpand[Integrate[
   (1 / Sqrt[2 Pi s^2]) Exp[- (x - mu)^2 / (2 s^2)], {x, -Infinity, Infinity}]]]
Print["Average"];
Assuming[{Re[s^2] > 0}, PowerExpand[Integrate[
   x (1 / Sqrt[2 Pi s^2]) Exp[- (x - mu)^2 / (2 s^2)], {x, -Infinity, Infinity}]]]
Print["Variance"];
Assuming[{Re[s^2] > 0},
 PowerExpand[Integrate[ (x - mu)^2 (1 / Sqrt[2 Pi s^2]) Exp[- (x - mu)^2 / (2 s^2)],
   {x, -Infinity, Infinity}]]]
```

Normalization

1

Average

mu

Variance

s^2

QUESTION

Using derivatives, verify that

$$\rho(x,t) = \frac{1}{\sqrt{4\,\pi D t}} \exp\left(\frac{-x^2}{4\,Dt}\right)$$

is a solution of the diffusion equation

$$\frac{\partial\rho}{\partial t} = D\frac{\partial^2\rho}{\partial x^2}$$

```
Clear[rho, dt, d2x]
rho = (1 / Sqrt[4 Pi dd t]) Exp[-x^2 / (4 dd t)];
dt = Simplify[D[rho, t]]
d2x = dd Simplify[D[D[rho, x], x]]
dt == d2x
```

$$\frac{dd\, e^{-\frac{x^2}{4\,dd\,t}} \left(-2\,dd\,t + x^2\right)}{8\sqrt{\pi}\,(dd\,t)^{5/2}}$$

$$\frac{dd\, e^{-\frac{x^2}{4\,dd\,t}} \left(-2\,dd\,t + x^2\right)}{8\sqrt{\pi}\,(dd\,t)^{5/2}}$$

```
True
```

QUESTION

A normal (Gaussian) distribution corresponds to a distribution of random numbers such that its mean is μ and the standard deviation is σ.

A way to generate random numbers that satisfy such distribution is by writing: RandomReal[NormalDistribution[μ,σ]].

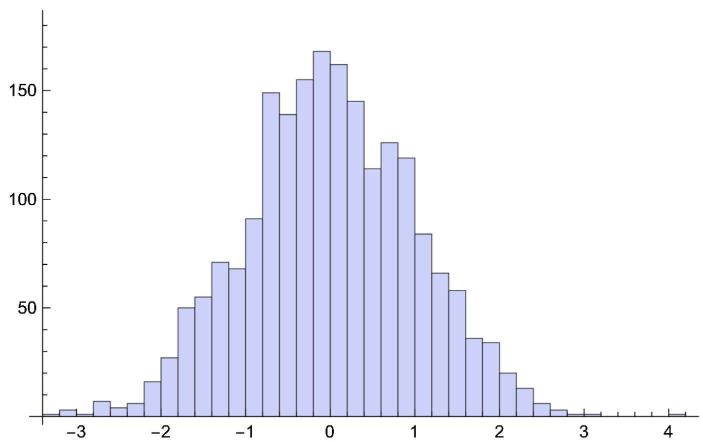
(i) Generate a list with 2000 random numbers from a Gaussian distribution with μ=0 and σ=1. Make a histogram with this list using a command from *Mathematica*.

(ii) With the same list above, make a histogram using only do-loops. Use three different bin sizes= 0.5, 0.2, and 0.1.

(iii) For the bin size=0.2, normalize the histogram so that the area underneath is 1.
Use the data corresponding to the middle point of the bins and fit it with a Gaussian. What do you get for <x> and the variance?
What is the relative error between your fit and σ=1?
Plot both curves together: the normalized histogram and the Gaussian fit.

```
Print["Item (i)"];
Clear[lis];
lis = Table[RandomReal[NormalDistribution[0, 1.]], {k, 1, 2000}];
Histogram[lis]
```

Item (i)

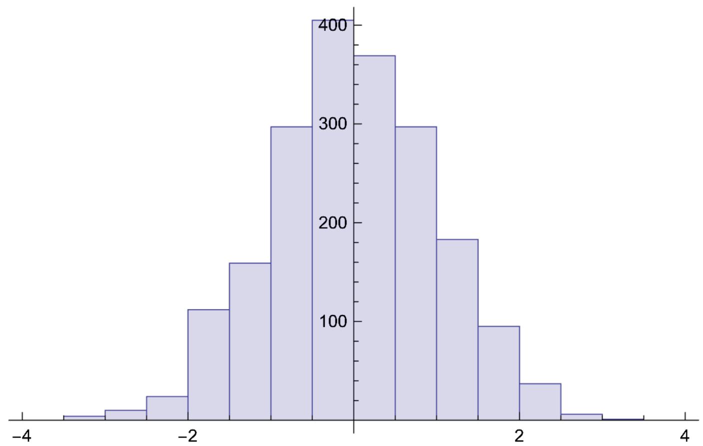



```
Print["Item (ii)"];
Print["bin=0.5"];
Clear[Nt, bin, TotBin, edges];
Nt = 2000;
bin = 0.5;
TotBin = (Floor[Max[lis]] + 1 - Floor[Min[lis]]) / bin;
edges = Table[Floor[Min[lis]] + bin (j - 1), {j, 1, TotBin + 1}];

Do[
  Num[j] = 0;
  Do[
   If[edges⟦j⟧ ≤ lis⟦k⟧ < edges⟦j + 1⟧, Num[j] = Num[j] + 1];
   , {k, 1, Nt}];
  , {j, 1, TotBin}];

Clear[hist];
hist = Table[
```

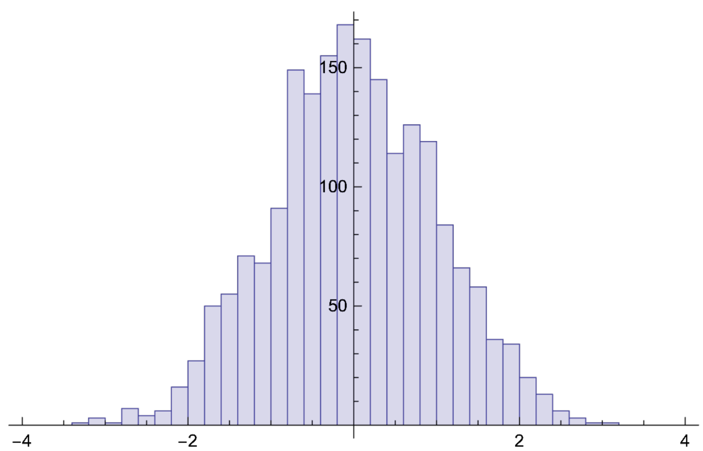
```
    {{edges⟦j⟧, Num[j]}, {edges⟦j + 1⟧, Num[j]}, {edges⟦j + 1⟧, 0}}, {j, 1, TotBin}];
ListPlot[Flatten[hist, 1], Joined → True, PlotRange → All, Filling → Axis]

Print[];
Print["bin=0.2"];
Clear[Nt, bin, TotBin, edges];
Nt = 2000;
bin = 0.2;
TotBin = (Floor[Max[lis]] + 1 - Floor[Min[lis]]) / bin;
edges = Table[Floor[Min[lis]] + bin (j - 1), {j, 1, TotBin + 1}];

Do[
  Num[j] = 0;
  Do[
   If[edges⟦j⟧ ≤ lis⟦k⟧ < edges⟦j + 1⟧, Num[j] = Num[j] + 1];
   , {k, 1, Nt}];
  , {j, 1, TotBin}];

Clear[hist];
hist = Table[
    {{edges⟦j⟧, Num[j]}, {edges⟦j + 1⟧, Num[j]}, {edges⟦j + 1⟧, 0}}, {j, 1, TotBin}];
ListPlot[Flatten[hist, 1], Joined → True, PlotRange → All, Filling → Axis]

Print[];
Print["bin=0.1"];
Clear[Nt, bin, TotBin, edges];
Nt = 2000;
bin = 0.1;
TotBin = (Floor[Max[lis]] + 1 - Floor[Min[lis]]) / bin;
edges = Table[Floor[Min[lis]] + bin (j - 1), {j, 1, TotBin + 1}];

Do[
  Num[j] = 0;
  Do[
   If[edges⟦j⟧ ≤ lis⟦k⟧ < edges⟦j + 1⟧, Num[j] = Num[j] + 1];
   , {k, 1, Nt}];
  , {j, 1, TotBin}];

Clear[hist];
hist = Table[
    {{edges⟦j⟧, Num[j]}, {edges⟦j + 1⟧, Num[j]}, {edges⟦j + 1⟧, 0}}, {j, 1, TotBin}];
ListPlot[Flatten[hist, 1], Joined → True, PlotRange → All, Filling → Axis]
```

Item (ii)

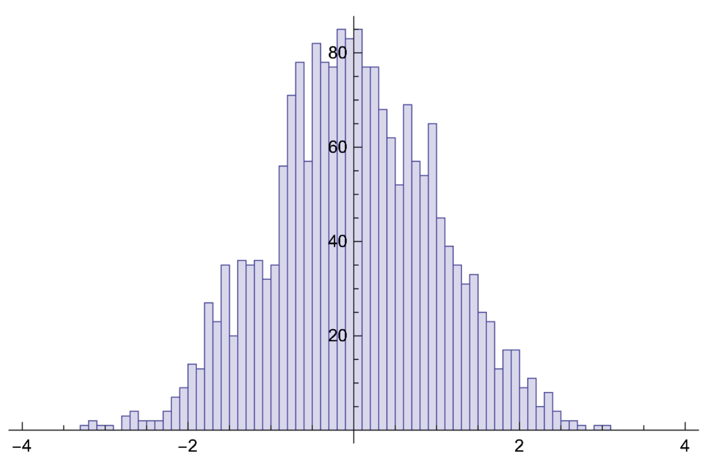
bin=0.5



bin=0.2

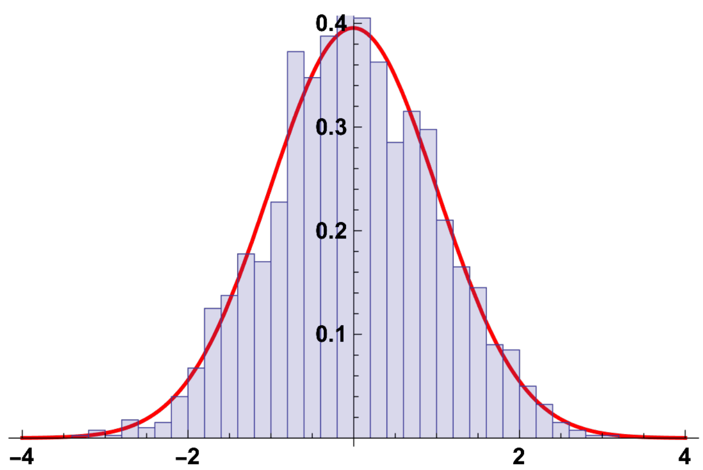


bin=0.1



```
Print["Item (iii)"];
Clear[Nt, bin, TotBin, edges];
Nt = 2000;
bin = 0.2;
TotBin = (Floor[Max[lis]] + 1 - Floor[Min[lis]]) / bin;
edges = Table[Floor[Min[lis]] + bin (j - 1), {j, 1, TotBin + 1}];

Do[
  Num[j] = 0;
  Do[
   If[edges[[j]] ≤ lis[[k]] < edges[[j + 1]], Num[j] = Num[j] + 1];
   , {k, 1, Nt}];
  , {j, 1, TotBin}];

Clear[Ntot];
Ntot = Sum[Num[j], {j, 1, TotBin}];

Clear[hist, l02];
hist = Table[{{edges[[j]], Num[j] / (bin Ntot)},
    {edges[[j + 1]], Num[j] / (bin Ntot)}, {edges[[j + 1]], 0}}, {j, 1, TotBin}];
l02 = ListPlot[Flatten[hist, 1], Joined → True, PlotRange → All, Filling → Axis];

Clear[dat, fat, Pfit];
dat = Table[{(edges[[j]] + edges[[j + 1]]) / 2., Num[j] / (bin Ntot)}, {j, 1, TotBin}];

fat = FindFit[dat, (1 / (Sqrt[2. Pi] c)) Exp[- (x - b) ^2 / (2 c ^2)], {b, c}, x];
Print["<x> = ", b /. fat, " and the variance = ", c /. fat];
Print["Relative error for the variance = ", 100 Abs[(c /. fat) - 1] / 1., "%"];
Pfit = Plot[(1 / (Sqrt[2. Pi] c)) Exp[- (x - b) ^2 / (2 c ^2)] /. fat,
   {x, Floor[Min[lis]], Floor[Max[lis]]}, PlotStyle → {Thick, Red},
   LabelStyle → Directive[Black, Bold, Medium]];

Show[{Pfit, l02}]
```

```
Item (iii)
<x> = -0.00475756 and the variance = 1.00859
Relative error for the variance = 0.858521%
```


0.4
0.3
0.2
0.1
-4
-2
2
4