Assignment 07

Using the bisection method find the real zero of:

(i) $x \cdot \text{Exp}[x] = 1$
(ii) $\text{Cos}[x] = x$

Print["Item (i)"];
Clear[f, x];
f[x_] := x \cdot \text{Exp}[x] - 1.;
Table[{{x/2, f[x/2]}, {x, -1, 2}}]
Item (i)

\begin{align*}
\left\{ \left\{ \frac{1}{2}, -1.30327 \right\}, \{0, -1.\}, \left\{ \frac{1}{2}, -0.175639 \right\}, \{1, 1.71828 \} \right\}
\end{align*}

Clear[x1, x2, xforfindroot];
x1 = 1/2;
x2 = 1;
xforfindroot = x1;

Do[
    mid = (x1 + x2) / 2.;
    Print["Iteration ", k];
    Print["Approximation to the root = ", mid];
    If[f[x] \times f[mid] = 0, Goto[end]];
    If[f[x1] \times f[mid] < 0, \{x1 = x1, x2 = mid\}, \{x1 = mid, x2 = x2\}];
    , {k, 1, 20}];
Label[end];
If[f[x] \times f[mid] = 0, Print["The root is ", mid]];

Print[];
Print["The actual root is ", \text{FindRoot}[f[x] = 0, \{x, xforfindroot\}]];

Iteration 1
Approximation to the root = 0.75
Iteration 2
Approximation to the root = 0.625
Iteration 3
Approximation to the root = 0.5625
Iteration 4
Approximation to the root = 0.59375
Iteration 5
Approximation to the root = 0.578125
Iteration 6
Approximation to the root = 0.570313
Iteration 7
Approximation to the root = 0.566406
Iteration 8
Approximation to the root = 0.568359
Iteration 9
Approximation to the root = 0.567383
Iteration 10
Approximation to the root = 0.566895
Iteration 11
Approximation to the root = 0.567139
Iteration 12
Approximation to the root = 0.567261
Iteration 13
Approximation to the root = 0.5672
Iteration 14
Approximation to the root = 0.567169
Iteration 15
Approximation to the root = 0.567154
Iteration 16
Approximation to the root = 0.567146
Iteration 17
Approximation to the root = 0.567142
Iteration 18
Approximation to the root = 0.567144
Iteration 19
Approximation to the root = 0.567143
Iteration 20
Approximation to the root = 0.567143
The actual root is \( \{x \to 0.567143\} \)

Print["Item (ii)"]; Clear[f, x];
\n\( f[x_] := \text{Cos}[1.x] - x; \)
Table[\(\{x/2, f[x/2]\}\), \{x, -1, 2\}]

Item (ii)
\[ \left\{ \left\{ -\frac{1}{2}, 1.37758 \right\}, \{0, 1.\}, \left\{ \frac{1}{2}, 0.377583 \right\}, \{1, -0.459698\} \right\} \]

Clear[xforfindroot, x1, x2];
x1 = 1/2;
x2 = 1;
xforfindroot = x1;
Do[
    mid = (x1 + x2) / 2.;
    Print["Iteration ", k];
    Print["Approximation to the root = ", mid];
    If[f[x] \times f[mid] == 0, Goto[end]]; 
    If[f[x1] \times f[mid] < 0, \{x1 = x1, x2 = mid\}, \{x1 = mid, x2 = x2\}];
    , \{k, 1, 20\}];
Label[end];

If[f[x] \times f[mid] == 0, Print["The root is ", mid]]; Print[];
Print["The actual root is ", FindRoot[f[x] == 0, \{x, xforfindroot\}]]

Iteration 1
Approximation to the root = 0.75
Iteration 2
Approximation to the root = 0.625
Iteration 3
Approximation to the root = 0.6875
Iteration 4
Approximation to the root = 0.71875
Iteration 5
Approximation to the root = 0.734375
Iteration 6
Approximation to the root = 0.742188
Iteration 7
Approximation to the root = 0.738281
Iteration 8
Approximation to the root = 0.740234
Iteration 9
Approximation to the root = 0.739258
Iteration 10
Approximation to the root = 0.73877
Iteration 11
Approximation to the root = 0.739014
Iteration 12
Approximation to the root = 0.739136
Iteration 13
Approximation to the root = 0.739075
Iteration 14
Approximation to the root = 0.739105
Iteration 15
Approximation to the root = 0.73909
Iteration 16
Approximation to the root = 0.739082
Iteration 17
Approximation to the root = 0.739086
Iteration 18
Approximation to the root = 0.739084
Iteration 19
Approximation to the root = 0.739085
Iteration 20
Approximation to the root = 0.739085

The actual root is \{x \to 0.739085\}

Using the method of false position, find the zero of:
[hint: you need to find the equation of the line connecting the points (x1,f(x1))
and (x2,f(x2)), as you have done in a previous assignment]
(i) \(\tan[x] = \frac{1}{1+x^2}\) \(0 \leq x < \pi/2\)
(ii) \(\cos[x] = x\) [comparing with item (ii) above for the bisection method, which
method works faster for this case?

To find the line
\[ y = a \ x + b \]
passing through point \((x_1, f(x_1))\) and \((x_2, f(x_2))\), we need to solve the system of equations:
\[ f(x_1) = a \ x_1 + b \]
\[ f(x_2) = a \ x_2 + b \]

From the first equation:
\[ b = f(x_1) - a \ x_1, \]
which when substituted into the second equation gives
\[ a = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \]

The zero of the straight line is obtained when \(y=0\).
From \( 0 = a \ x + b \Rightarrow x = - b/a \Rightarrow x = - f(x_1)/a + x_1 \)
we find the next point \(x_3\) for the new interval:
\[ \text{intersec} = x_3 = x_1 - \frac{x_2-x_1}{f(x_2) - f(x_1)} \]

Print "Item (i)";
Clear[f, x];
f[x_] := Tan[1. x] - 1 / (1 + x^2)
Table[{x/2, f[x/2]}, {x, 0, 5}]

Item (i)
\[
\left\{ 0, -1. \right\}, \left\{ \frac{1}{2}, -0.253698 \right\}, \left\{ 1, 1.05741 \right\}, \\
\left\{ \frac{3}{2}, 13.7937 \right\}, \left\{ 2, -2.38504 \right\}, \left\{ \frac{5}{2}, -0.884953 \right\}
\]
Clear[xforfindroot, x1, x2];
x1 = 1/2;
x2 = 1;
xforfindroot = x1;

Do[
  intersec = x1 - f[x1] (x2 - x1) / (f[x2] - f[x1]);
  Print["Iteration ", k];
  Print["Approximation to the root = ", intersec];
  If[f[x1] × f[intersec] = 0, {Print["I found the root!"], Break[]}];
  If[f[x1] × f[intersec] < 0, {x1 = x1, x2 = intersec}, {x1 = intersec, x2 = x2}];
, {k, 1, 11}];

Print[];
Print["The actual root is ", FindRoot[f[x] = 0, {x, xforfindroot}]]
Iteration 1
Approximation to the root = 0.596749
Iteration 2
Approximation to the root = 0.617728
Iteration 3
Approximation to the root = 0.622484
Iteration 4
Approximation to the root = 0.623574
Iteration 5
Approximation to the root = 0.623825
Iteration 6
Approximation to the root = 0.623882
Iteration 7
Approximation to the root = 0.623896
Iteration 8
Approximation to the root = 0.623899
Iteration 9
Approximation to the root = 0.623899
Iteration 10
Approximation to the root = 0.6239
Iteration 11
Approximation to the root = 0.6239

The actual root is \( x \to 0.6239 \)

Print["Item (ii)"]; Clear[f, x]; f[x_] := Cos[1. x] - x; Table[{{x/2, f[x/2]}, {x, -1, 2}}
  Item (ii)
  \[\{\{-1/2, 1.37758\}, \{0, 1\}, \{1/2, 0.377583\}, \{-0.459698\}\} \]
Clear[xforfindroot, x1, x2];
x1 = 1/2;
x2 = 1;
xforfindroot = x1;

Do[
  intersec = x1 - f[x1] (x2 - x1) / (f[x2] - f[x1]);
  Print["Iteration ", k];
  Print["Approximation to the root = ", intersec];
  If[f[x1] × f[intersec] == 0, Goto[end]]; 
  If[f[x1] × f[intersec] < 0, {x1 = x1, x2 = intersec}, {x1 = intersec, x2 = x2}];
  , {k, 1, 11}];
Label[end];

If[f[x] × f[intersec] == 0, Print["The root is ", intersec]];

Print[];
Print["The actual root is ", FindRoot[f[x] == 0, {x, xforfindroot}]]
Iteration 1
Approximation to the root = 0.725482
Iteration 2
Approximation to the root = 0.738399
Iteration 3
Approximation to the root = 0.739051
Iteration 4
Approximation to the root = 0.739083
Iteration 5
Approximation to the root = 0.739085
Iteration 6
Approximation to the root = 0.739085
Iteration 7
Approximation to the root = 0.739085
Iteration 8
Approximation to the root = 0.739085
Iteration 9
Approximation to the root = 0.739085
Iteration 10
Approximation to the root = 0.739085
Iteration 11
Approximation to the root = 0.739085

The actual root is \{x \rightarrow 0.739085\}

The false – position method worked faster
Using Newton’s method find the real zero of:
(i) \( \text{ArcTan}[x] = 1 \) for \( x=1 \)
(ii) \( \text{Log}[x] = 3 \) for \( x=10 \)

```
Print["Item (i)"];
Clear[f, xint, df, xforfindroot];
f[x_] := ArcTan[x] - 1.;
xint = 1.;
df = D[f[x], x];
xforfindroot = xint;

Do[
    xint = xint - f[xint] / (df /. x -> xint);
    Print["Iteration ", k];
    Print["Approximation to the root = ", xint];
    , {k, 1, 5}]

Print[];
Print["The actual root is ", FindRoot[f[x] = 0, {x, xforfindroot}]]
```

Item (i)
Iteration 1
Approximation to the root = 1.4292
Iteration 2
Approximation to the root = 1.55006
Iteration 3
Approximation to the root = 1.55738
Iteration 4
Approximation to the root = 1.55741
Iteration 5
Approximation to the root = 1.55741

The actual root is \( x \rightarrow 1.55741 \)
Print["Item (ii)" ];
Clear[f, xint, df, xforfindroot];
f[x_] := Log[x] - 3.;
xint = 1.;
df = D[f[x], x];
xforfindroot = xint;

Do[
xint = xint - f[xint] / (df /. x -> xint);
Print["Iteration ", k];
Print["Approximation to the root = ", xint];
, {k, 1, 5}]

Print[];
Print["The actual root is ", FindRoot[f[x] == 0, {x, xforfindroot}]]

Item (ii)
Iteration 1
Approximation to the root = 4.
Iteration 2
Approximation to the root = 10.4548
Iteration 3
Approximation to the root = 17.2812
Iteration 4
Approximation to the root = 19.88
Iteration 5
Approximation to the root = 20.0845

The actual root is \{x \rightarrow 20.0855\}
Using Newton’s method find the solutions for
\[ f(x,y) = \exp(3x) + 4y \]
\[ g(x,y) = 3y^3 - 2 \ln(x) + 7.31x^2 \]

use as an initial guess \( x_0=1 \) and \( y_0=2 \)

Stop when \(|f| \) and \(|g| \) are smaller than \( 10^{-5} \)

```math
\begin{align*}
  f[x_, y_] &= \text{Exp}[3. x] + 4 y; \\
  g[x_, y_] &= 3. y^3 - 2 \text{Log}[x] + 7.31 x^2; \\
  \text{FindRoot[} \{f[x, y], g[x, y]\}, \{x, 1\}, \{y, 2.\}] \\
  \{x \rightarrow 0.466288, y \rightarrow -1.01265\}
\end{align*}
```
Clear[f, g, dfx, dfy, dgx, dgy, xo, yo, fo, go, fx, fy, gx, gy, DD, Dx, Dy, h, k];

f[x_, y_] = Exp[3. x] + 4 y;
g[x_, y_] = 3. y^3 - 2 Log[x] + 7.31 x^2;

dfx = D[f[x, y], x];
dfy = D[f[x, y], y];
dgx = D[g[x, y], x];
dgy = D[g[x, y], y];

xo = 1;
yo = 2.;

Do[
  fo = f[xo, yo];
  go = g[xo, yo];
  Print["xo and yo are ", {xo, yo}];
  Print["and the functions at those points are ", {f[xo, yo], g[xo, yo]}];
  Print[];

  If[Abs[fo] ≤ 10^-5 && Abs[go] ≤ 10^-5, {Print[
    "I found approximate roots such that |f| and |g| are smaller than 10^-5"],
    Break[]}];

(* Print[{fo,go}];*)
  fx = dfx /. {x → xo, y → yo};
  fy = dfy /. {x → xo, y → yo};
  gx = dgx /. {x → xo, y → yo};
  gy = dgy /. {x → xo, y → yo};
  (* Print[{fo,go,fx,fy,gx,gy}]; *)
  DD = fx gy - gx fy;
  Dx = fo gy - go fy;
  Dy = fx go - gx fo;
  (* Print[{DD,Dx,Dy}]; *)
  h = -Dx / DD;
  k = -Dy / DD;
  xo = xo + h;
  yo = yo + k;
, {kk, 1, 20}]
xo and yo are \{1, 2.\}
and the functions at those points are \{28.0855, 31.31\}

xo and yo are \{0.581906, 1.27684\}
and the functions at those points are \{10.8374, 9.80318\}

xo and yo are \{0.0653926, 0.78722\}
and the functions at those points are \{4.36562, 6.94951\}

xo and yo are \{0.0902284, -0.32685\}
and the functions at those points are \{0.00346275, 4.76558\}

xo and yo are \{0.308874, -0.542676\}
and the functions at those points are \{0.355257, 2.56759\}

I found approximate roots such that |f| and |g| are smaller than $10^{-5}$