Assignment 07

Using the bisection method find the real zero of:
(i) x Exp[x] =1
(ii) Cos[x] = x

```
Print["Item (i)"];
Clear[f, x];
f[x_] := x Exp[x] - 1.;
Table[{x / 2, f[x / 2]}, {x, -1, 2}]
```

Item (i)

$\left\{\left\{-\frac{1}{2}, -1.30327\right\}, \{0, -1.\}, \left\{\frac{1}{2}, -0.175639\right\}, \{1, 1.71828\}\right\}$

```
Clear[x1, x2, xforfindroot];
x1 = 1 / 2;
x2 = 1;
xforfindroot = x1;

Do[
  mid = (x1 + x2) / 2.;
  Print["Iteration ", k];
  Print["Approximation to the root = ", mid];
  If[f[x] × f[mid] == 0, Goto[end]];
  If[f[x1] × f[mid] < 0, {x1 = x1, x2 = mid}, {x1 = mid, x2 = x2}];
  , {k, 1, 20}];

Label[end];
If[f[x] × f[mid] == 0, Print["The root is ", mid]];

Print[];
Print["The actual root is ", FindRoot[f[x] == 0, {x, xforfindroot}]];
```

```
Iteration 1

Approximation to the root = 0.75

Iteration 2

Approximation to the root = 0.625

Iteration 3
```

```
Approximation to the root = 0.5625
Iteration 4
Approximation to the root = 0.59375
Iteration 5
Approximation to the root = 0.578125
Iteration 6
Approximation to the root = 0.570313
Iteration 7
Approximation to the root = 0.566406
Iteration 8
Approximation to the root = 0.568359
Iteration 9
Approximation to the root = 0.567383
Iteration 10
Approximation to the root = 0.566895
Iteration 11
Approximation to the root = 0.567139
Iteration 12
Approximation to the root = 0.567261
Iteration 13
Approximation to the root = 0.5672
Iteration 14
Approximation to the root = 0.567169
Iteration 15
Approximation to the root = 0.567154
Iteration 16
Approximation to the root = 0.567146
Iteration 17
Approximation to the root = 0.567142
Iteration 18
Approximation to the root = 0.567144
Iteration 19
Approximation to the root = 0.567143
Iteration 20
Approximation to the root = 0.567143
```

The actual root is {x → 0.567143}

```
Print["Item (ii)"];
Clear[f, x];
f[x_] := Cos[1. x] - x;
Table[{x / 2, f[x / 2]}, {x, -1, 2}]
```

Item (ii)

$\left\{\left\{-\frac{1}{2}, 1.37758\right\}, \{0, 1.\}, \left\{\frac{1}{2}, 0.377583\right\}, \{1, -0.459698\}\right\}$

```
Clear[xforfindroot, x1, x2];
x1 = 1 / 2;
x2 = 1;
xforfindroot = x1;
Do[
  mid = (x1 + x2) / 2.;
  Print["Iteration ", k];
  Print["Approximation to the root = ", mid];
  If[f[x] × f[mid] == 0, Goto[end]];
  If[f[x1] × f[mid] < 0, {x1 = x1, x2 = mid}, {x1 = mid, x2 = x2}];
  , {k, 1, 20}];
Label[end];

If[f[x] × f[mid] == 0, Print["The root is ", mid]];

Print[];
Print["The actual root is ", FindRoot[f[x] == 0, {x, xforfindroot}]]
```

Iteration 1

Approximation to the root = 0.75

Iteration 2

Approximation to the root = 0.625

Iteration 3

Approximation to the root = 0.6875

Iteration 4

Approximation to the root = 0.71875

Iteration 5

Approximation to the root = 0.734375

Iteration 6

Approximation to the root = 0.742188

Iteration 7

Approximation to the root = 0.738281

```
Iteration 8
Approximation to the root = 0.740234
Iteration 9
Approximation to the root = 0.739258
Iteration 10
Approximation to the root = 0.73877
Iteration 11
Approximation to the root = 0.739014
Iteration 12
Approximation to the root = 0.739136
Iteration 13
Approximation to the root = 0.739075
Iteration 14
Approximation to the root = 0.739105
Iteration 15
Approximation to the root = 0.73909
Iteration 16
Approximation to the root = 0.739082
Iteration 17
Approximation to the root = 0.739086
Iteration 18
Approximation to the root = 0.739084
Iteration 19
Approximation to the root = 0.739085
Iteration 20
Approximation to the root = 0.739085

The actual root is {x → 0.739085}
```

Using the method of false position, find the zero of:
[hint: you need to find the equation of the line connecting the points (x1,f(x1)) and (x2,f(x2)), as you have done in a previous assignment]
(i) Tan[x] = $\frac{1}{1+x^2}$ $0 <= x < \pi/2$
(ii) Cos[x] = x [comparing with item (ii) above for the bisection method, which

method works faster for this case?]

To find the line
$y = a\,x + b$
passing through point (x1, f(x1)) and (x2, f(x2)), we need to solve the system of equations:
$f(x1) = a\,x1 + b$
$f(x2) = a\,x2 + b$

From the first equation:
$b = f(x1) - a\,x1$,
which when substituted into the second equation gives
$a = \frac{f(x2) - f(x1)}{x2 - x1}$

The zero of the straight line is obtained when y=0.
From $0 = a\,x + b \Longrightarrow x = -b/a \Longrightarrow x = -f(x1)/a + x1$
we find the next point x3 for the new interval:
$\text{intersec} = x3 = x1 - f(x1)\frac{x2 - x1}{f(x2) - f(x1)}$

```
Print["Item (i)"];
Clear[f, x];
f[x_] := Tan[1. x] - 1 / (1 + x^2)
Table[{x / 2, f[x / 2]}, {x, 0, 5}]
```

```
Item (i)
```

$\left\{\{0, -1.\}, \left\{\frac{1}{2}, -0.253698\right\}, \{1, 1.05741\}, \left\{\frac{3}{2}, 13.7937\right\}, \{2, -2.38504\}, \left\{\frac{5}{2}, -0.884953\right\}\right\}$

```
Clear[xforfindroot, x1, x2];
x1 = 1 / 2;
x2 = 1;
xforfindroot = x1;

Do[
  intersec = x1 - f[x1] (x2 - x1) / (f[x2] - f[x1]);
  Print["Iteration ", k];
  Print["Approximation to the root = ", intersec];
  If[f[x1] × f[intersec] == 0, {Print["I found the root!"], Break[]}];
  If[f[x1] × f[intersec] < 0, {x1 = x1, x2 = intersec}, {x1 = intersec, x2 = x2}];
  , {k, 1, 11}];


Print[];
Print["The actual root is ", FindRoot[f[x] == 0, {x, xforfindroot}]]
```

```
Iteration 1
Approximation to the root = 0.596749
Iteration 2
Approximation to the root = 0.617728
Iteration 3
Approximation to the root = 0.622484
Iteration 4
Approximation to the root = 0.623574
Iteration 5
Approximation to the root = 0.623825
Iteration 6
Approximation to the root = 0.623882
Iteration 7
Approximation to the root = 0.623896
Iteration 8
Approximation to the root = 0.623899
Iteration 9
Approximation to the root = 0.623899
Iteration 10
Approximation to the root = 0.6239
Iteration 11
Approximation to the root = 0.6239

The actual root is {x → 0.6239}
```

```
Print["Item (ii)"];
Clear[f, x];
f[x_] := Cos[1. x] - x;
Table[{x / 2, f[x / 2]}, {x, -1, 2}]
```

Item (ii)

$\left\{\left\{-\frac{1}{2}, 1.37758\right\}, \{0, 1.\}, \left\{\frac{1}{2}, 0.377583\right\}, \{1, -0.459698\}\right\}$

```
Clear[xforfindroot, x1, x2];
x1 = 1 / 2;
x2 = 1;
xforfindroot = x1;

Do[
  intersec = x1 - f[x1] (x2 - x1) / (f[x2] - f[x1]);
  Print["Iteration ", k];
  Print["Approximation to the root = ", intersec];
  If[f[x1] × f[intersec] == 0, Goto[end]];
  If[f[x1] × f[intersec] < 0, {x1 = x1, x2 = intersec}, {x1 = intersec, x2 = x2}];
  , {k, 1, 11}];
Label[end];

If[f[x] × f[intersec] == 0, Print["The root is ", intersec]];

Print[];
Print["The actual root is ", FindRoot[f[x] == 0, {x, xforfindroot}]]
```

```
Iteration 1
Approximation to the root = 0.725482
Iteration 2
Approximation to the root = 0.738399
Iteration 3
Approximation to the root = 0.739051
Iteration 4
Approximation to the root = 0.739083
Iteration 5
Approximation to the root = 0.739085
Iteration 6
Approximation to the root = 0.739085
Iteration 7
Approximation to the root = 0.739085
Iteration 8
Approximation to the root = 0.739085
Iteration 9
Approximation to the root = 0.739085
Iteration 10
Approximation to the root = 0.739085
Iteration 11
Approximation to the root = 0.739085

The actual root is {x → 0.739085}
```

The false – position method worked faster

Using Newton’s method find the real zero of:
(i) ArcTan[x] =1   for x=1
(ii) Log[x] = 3   for x=10

```
Print["Item (i)"];
Clear[f, xint, df, xforfindroot];
f[x_] := ArcTan[x] - 1.;
xint = 1.;
df = D[f[x], x];
xforfindroot = xint;

Do[
 xint = xint - f[xint] / (df /. x → xint);
 Print["Iteration ", k];
 Print["Approximation to the root = ", xint];
 , {k, 1, 5}]

Print[];
Print["The actual root is ", FindRoot[f[x] == 0, {x, xforfindroot}]]
```

```
Item (i)

Iteration 1

Approximation to the root = 1.4292

Iteration 2

Approximation to the root = 1.55006

Iteration 3

Approximation to the root = 1.55738

Iteration 4

Approximation to the root = 1.55741

Iteration 5

Approximation to the root = 1.55741


The actual root is {x → 1.55741}
```

```
Print["Item (ii)"];
Clear[f, xint, df, xforfindroot];
f[x_] := Log[x] - 3.;
xint = 1.;
df = D[f[x], x];
xforfindroot = xint;

Do[
 xint = xint - f[xint] / (df /. x → xint);
 Print["Iteration ", k];
 Print["Approximation to the root = ", xint];
 , {k, 1, 5}]

Print[];
Print["The actual root is ", FindRoot[f[x] == 0, {x, xforfindroot}]]
```

```
Item (ii)
Iteration 1
Approximation to the root = 4.
Iteration 2
Approximation to the root = 10.4548
Iteration 3
Approximation to the root = 17.2812
Iteration 4
Approximation to the root = 19.88
Iteration 5
Approximation to the root = 20.0845

The actual root is {x → 20.0855}
```

Using Newton's method find the solutions for
f(x,y) = exp(3x)+4y
g(x,y) = 3y^3-2 ln(x) + 7.31 x^2

use as an initial guess xo=1 and yo=2

Stop when |f| and |g| are smaller than 10^(-5)

```
f[x_, y_] = Exp[3. x] + 4 y;
g[x_, y_] = 3. y^3 - 2 Log[x] + 7.31 x^2;
FindRoot[{f[x, y], g[x, y]}, {x, 1}, {y, 2.}]
```

```
{x → 0.466288, y → -1.01265}
```

```
Clear[f, g, dfx, dfy, dgx, dgy, xo, yo, fo, go, fx, fy, gx, gy, DD, Dx, Dy, h, k];

f[x_, y_] = Exp[3. x] + 4 y;
g[x_, y_] = 3. y^3 - 2 Log[x] + 7.31 x^2;

dfx = D[f[x, y], x] ;
dfy = D[f[x, y], y] ;
dgx = D[g[x, y], x] ;
dgy = D[g[x, y], y] ;

xo = 1;
yo = 2.;

Do[
 fo = f[xo, yo];
 go = g[xo, yo];
 Print["xo and yo are ", {xo, yo}];
 Print["and the functions at those points are ", {f[xo, yo], g[xo, yo]}];
 Print[];

 If[Abs[fo] ≤ 10^(-5) && Abs[go] ≤ 10^(-5), {Print[
    "I found approximate roots such that |f| and |g| are smaller than 10^(-5)"],
   Break[]}];

 (* Print[{fo,go}];*)
 fx = dfx /. {x → xo, y → yo} ;
 fy = dfy /. {x → xo, y → yo} ;
 gx = dgx /. {x → xo, y → yo} ;
 gy = dgy /. {x → xo, y → yo} ;
 (* Print[{fo,go,fx,fy,gx,gy}]; *)
 DD = fx gy - gx fy;
 Dx = fo gy - go fy;
 Dy = fx go - gx fo;
 (* Print[{DD,Dx,Dy}]; *)
 h = -Dx / DD;
 k = -Dy / DD;
 xo = xo + h;
 yo = yo + k;
 , {kk, 1, 20}]
```

```
xo and yo are {1, 2.}
and the functions at those points are {28.0855, 31.31}

xo and yo are {0.581906, 1.27684}
and the functions at those points are {10.8374, 9.80318}

xo and yo are {0.0653926, 0.78722}
and the functions at those points are {4.36562, 6.94951}

xo and yo are {0.0902284, -0.32685}
and the functions at those points are {0.00346275, 4.76558}

xo and yo are {0.308874, -0.542676}
and the functions at those points are {0.355257, 2.56759}

xo and yo are {0.642968, -1.26442}
and the functions at those points are {1.82427, -2.15919}

xo and yo are {0.514669, -1.05828}
and the functions at those points are {0.450199, -0.290913}

xo and yo are {0.469798, -1.01322}
and the functions at those points are {0.0405907, 0.00372875}

xo and yo are {0.466302, -1.01264}
and the functions at those points are {0.000224309, 0.000141854}

xo and yo are {0.466288, -1.01265}
```

and the functions at those points are $\{3.9533\times10^{-9}, 1.41164\times10^{-9}\}$

```
I found approximate roots such that |f| and |g| are smaller than 10^(-5)
```