Assignment 06

Solve the equation
$x^4 - 16\, x^3 + 61\, x^2 - 22\, x - 12 = 0$,
exactly and numerically

```
Solve[x^4 - 16 x^3 + 61 x^2 - 22 x - 12 == 0, x]
NSolve[x^4 - 16 x^3 + 61 x^2 - 22 x - 12. == 0, x]
```

$\{\{x \to 3 - \sqrt{5}\}, \{x \to 3 + \sqrt{5}\}, \{x \to 5 - 2\sqrt{7}\}, \{x \to 5 + 2\sqrt{7}\}\}$

```
{{x → -0.291503}, {x → 0.763932}, {x → 5.23607}, {x → 10.2915}}
```

Find the sum of the squares of the roots of
$x^6 - 21\, x^5 + 175\, x^4 - 735\, x^3 + 1624\, x^2 - 1764\, x + 720 = 0$

```
Clear[sol]
sol =
  x /. NSolve[x^6 - 21 x^5 + 175 x^4 - 735 x^3 + 1624 x^2 - 1764 x + 720 == 0, x];
Sum[sol[[k]]^2, {k, 1, 6}]
91.
```

A theorem from algebra says that if
$p(x) = a_0 + a_1\, x + a_2\, x^2 + \ldots + a_{n-1}\, x^{n-1} + a_n\, x^n$,
the sum of the roots of the equation p(x)=0 is $\frac{-a_{n-1}}{a_n}$ and their product is $(-1)^n \frac{a_0}{a_n}$
Verify this for the equation
$20\, x^7 + 32\, x^6 - 221\, x^5 - 118\, x^4 + 725\, x^3 - 18\, x^2 - 726\, x + 252 = 0$

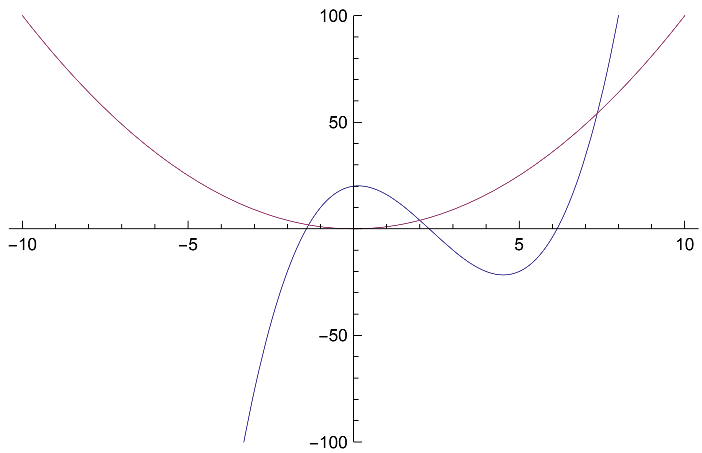
```
Clear[sol]
sol = x /.
   Solve[20 x^7 + 32 x^6 - 221 x^5 - 118 x^4 + 725 x^3 - 18 x^2 - 726 x + 252 == 0, x];
Print["The sum is ", Sum[sol[[k]], {k, 1, 7}], " which coincides with ", -32 / 20]
Print["The product is ", Product[sol[[k]], {k, 1, 7}],
 " which coincides with ", (-1)^7 × 252 / 20]
```

The sum is $-\frac{8}{5}$ which coincides with $-\frac{8}{5}$

The product is $-\frac{63}{5}$ which coincides with $-\frac{63}{5}$

Sketch the graphs of f(x) = x^3 - 7 x^2 + 2 x + 20 and g(x)=x^2 on the same set of axes and find their points of intersection exactly and approximately

```
Clear[f, g]
f[x_] := x^3 - 7 x^2 + 2 x + 20;
g[x_] := x^2;
Plot[{f[x], g[x]}, {x, -10, 10}, PlotRange → {-100, 100}]
Solve[f[x] == g[x], x]
NSolve[f[x] == g[x], x]
```



$\{\{x \to 2\}, \{x \to 3 - \sqrt{19}\}, \{x \to 3 + \sqrt{19}\}\}$

{{x → -1.3589}, {x → 2.}, {x → 7.3589}}

(i) Find an equation of the line passing through (2,5) and (7,9)
[line: y=ax+b]
(ii) Find an equation of the circle passing through (1,4), (2,7) and (4,11)
[circle: x^2 + y^2 + a x + b y + c =0]

```
Print["(i)"];
Clear[line1, line2, a, b, sol, y, x];
(* Substituting the points in the equation *)
line1 = 2 a + b - 5;
line2 = 7 a + b - 9;
sol = Solve[{line1 == 0, line2 == 0} , {a, b}];
Print["The equation is y = ", (a /. Flatten[sol]) x + (b /. Flatten[sol])]
```

The equation is $y = \frac{17}{5} + \frac{4x}{5}$

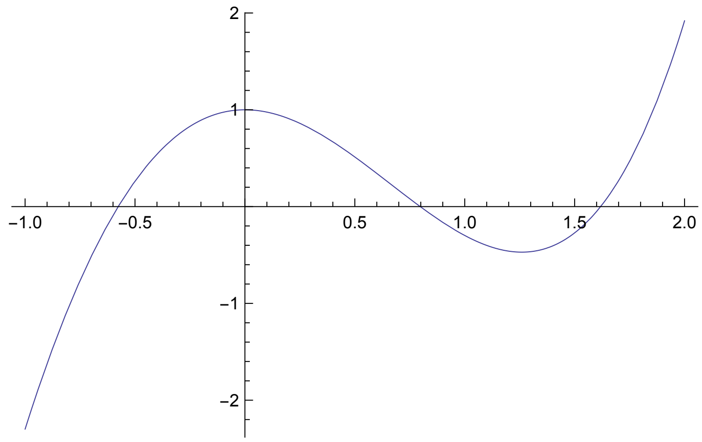
```
Print["(ii)"];
Clear[circ1, circ2, circ3, a, b, c, sol, y, x];
(* Substituting the points in the equation *)
circ1 = x^2 + y^2 + a x + b y + c /. {x → 1, y → 4};
circ2 = x^2 + y^2 + a x + b y + c /. {x → 2, y → 7};
circ3 = x^2 + y^2 + a x + b y + c /. {x → 4, y → 11};
sol = Solve[{circ1 == 0, circ2 == 0, circ3 == 0} , {a, b, c}];
Print["The equation is ", x^2 + y^2 + (a /. Flatten[sol]) x +
   (b /. Flatten[sol]) y + (c /. Flatten[sol]) , " = 0"];
```

(ii)

The equation is $13 - 54x + x^2 + 6y + y^2 = 0$

Solve the equation 5 cos(x) = 4 -x^3. Make sure you find all solutions.

```
Clear[f];
f[x_] := 5 Cos[x] + x^3 - 4;
Plot[f[x], {x, -1, 2}]
```




```
(* Now that I know more or less where the roots are, I can find them *)
FindRoot[f[x], {x, -0.5}]
FindRoot[f[x], {x, 0.7}]
FindRoot[f[x], {x, 1.5}]
```

{x → −0.576574}

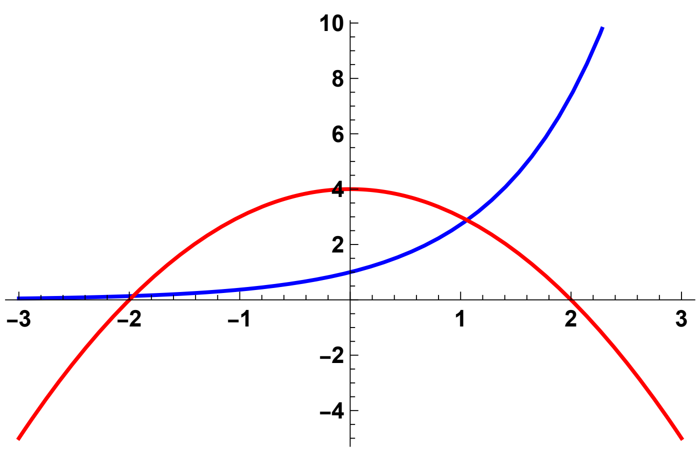
{x → 0.797323}

{x → 1.61805}

Find the area of the bounded region trapped between the graphs of the function exp(x) and the function 4-x^2.
Follow these steps:
(i) Plot together the two functions to picture where this area appears and which of the functions is larger.
(ii) Find the points where the two functions meet.
(iii) Compute the area.

```
Print["(i) Plot of the two functions. "];
Plot[{Exp[x], 4 - x^2}, {x, -3, 3}, PlotStyle → {{Blue, Thick}, {Red, Thick}},
 LabelStyle → Directive[Black, Bold, Medium]]
Print["From the figure above, we can see that the polynomial
    is larger than the exponential in the bounded area. We
    also see that the bounded area is somewhere between - 2
    and 1. I need to find exactly where these two points are."];
```

```
(i) Plot of the two functions.
```




```
From the figure above, we can see that the polynomial is larger than
  the exponential in the bounded area. We also see that the bounded area is
  somewhere between - 2 and 1. I need to find exactly where these two points are.
```

```
Clear[a, b, xin, xf];
a = FindRoot[Exp[x] == 4 - x^2, {x, -2}];
b = FindRoot[Exp[x] == 4 - x^2, {x, 1}];
Print["(ii) The curves meet at"]
xin = x /. a
xf = x /. b
Print["The integral of the bounded area is"]
NIntegrate[4 - x^2 - Exp[x], {x, xin, xf}]
```

```
(ii) The curves meet at
-1.96464
1.05801
The integral of the bounded area is
6.42769
```