Solution to Assignment 01

Compute the square root of π approximately.

```
Sqrt[1. Pi]
```

1.77245

Which one is bigger e^{π} or π^{e}?

```
Exp[1. Pi]
Pi^(Exp[1.])
```

23.1407

22.4592

e^{π} is bigger than π^{e}

Compute the cube root of 26 approximately.

```
26.^(1/3)
```

2.9625

What is the volume and the surface area of Earth (the radius of Earth is 6378.1 km)?

```
R = 6378.1;
V = (4/3) Pi R^3
A = 4 Pi R^2
```

1.08683×10^{12}

5.11202×10^{8}

Volume in km^3 and area in km^2

Compute the sine of 45^o.

```
Sin[45 Degree]
Sin[45. Degree]
```

$$\frac{1}{\sqrt{2}}$$

```
0.707107
```

What is a Fibonacci number (check online)? What is the 9th Fibonacci number? [hint: use help to find how to obtain a Fibonacci number in *Mathematica*]

The first two Fibonacci numbers, F_0 and F_1, are 0 and 1. Each subsequent number is the sum of the previous two. They are 0, 1, 1, 2, 3, 5, 8, 13, 21,... obtained from $F_n=F_{n-1}+F_{n-2}$
In *Mathematica*, F_1 is Fibonacci[1].

```
Fibonacci[9]
```

```
34
```

Compute the sum of the reciprocals of the odd numbers from 1 to 31. Give the exact and the approximate answer.

```
Sum[1 / k, {k, 1, 31, 2}]
Sum[1. / k, {k, 1, 31, 2}]
```

$$\frac{10\,686\,452\,707\,072}{4\,512\,611\,027\,925}$$

```
2.36813
```

Compute the infinite sum:
$\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+...$

```
Sum[1 / k^2, {k, 1, Infinity}]
```

$$\frac{\pi^2}{6}$$

Compute approximately (S1+S2)/P1, where

S1 is the sum of the first 10 primes numbers;
S2 is the sum of the first 20 numbers which are multiples of 10;
P1 is the product of the square root of the first 10 even numbers;

```
S1 = Sum[Prime[k], {k, 1, 10}];
S2 = Sum[10. x, {x, 1, 20}];
P1 = Product[Sqrt[x], {x, 2, 20, 2}];
(S1 + S2) / P1
```

```
0.0365661
```

Where have you encountered the binomial coefficient
C(n,k) = $\frac{n!}{k!\,(n-k)!}$?

(i) Use its definition to compute C(10,4).

(ii) The binomial coefficient can also be expressed as
$\left(\frac{n}{k}\right)\left(\frac{n-1}{k-1}\right)\left(\frac{n-2}{k-2}\right)\ldots\left(\frac{n-k+1}{1}\right)$
Use this representation to compute C(10,4) [hint: use "Product"]

It is the coefficient of the x^k term in the polynomial expansion of the binomial power (1 + x)^n.

```
(* Item (i) *)
(* one option *)
10! / (4! × 6!)
(* another option *)
Nk! / (k! (Nk - k)!) /. {Nk → 10, k → 4}
(* yet another option *)
Clear[f1, Nk, k]
f1 = Nk! / (k! (Nk - k)!);
f1 /. {Nk → 10, k → 4}
```

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210
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210
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210
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```
(* Item (ii) *)
(* one option *)
Product[(10 - x) / (4 - x), {x, 0, 3}]
(* another option *)
Nj = 10;
kj = 4;
Product[(Nj - x) / (kj - x), {x, 0, kj - 1}]
```

```
210
```

```
210
```