## Lesson 01

## Mathematica

Mathematica is a program that allows you to:
*) Make numeric and symbolic calculations.
*) Simplify complicated mathematical expressions.
*) Compute derivatives and integrals.
*) Solve equations.
*) Plot graphs in 2D and 3D.
*) Create animations.
*) Write programs and manipulate data.

## Advantages:

*) It is easy to use.
*) It allows for symbolic calculations.
Disadvantages:
*) You need to pay to have it (students have good discounts).
*) It is not as efficient as other languages as Fortran or $C$ when dealing with very involved computations. But it is a good way to learn the basic of programming and then move to other languages.

Good idea!
If you have, bring a flash drive to save your work of every class.

## Before starting Mathematica

Create a folder where you will save all the exercises done in class, the assignments, and the exams.

## Starting Mathematica

In "All Programs" look for "Wolfram Mathematica". There select "Wolfram Mathematica 7" (or a newer version if you have)

## Front End

You are now in front of a Mathematica notebook, where you will document all your work.

There are three important parts to Mathematica:
*) The kernel is the computational engine. You input instructions and the kernel gives you the desired output.
${ }^{*}$ ) The front end is an interface between you and the kernel.
*) The medium of the front end is the Mathematica notebook, which allows you to communicate with the kernel.

## Format, Save and Reopen

You can use Mathematica simply as a word processor, as I have done so far.

1) Write:

Lecture 1 (date)
2) Notice that a bracket appeared at the far right. This is called a cell.
3) Click on the cell so that it is highlighted.
4) At the Tool Bar, go to Format -> Style -> Title
(play with the various other options for styles, fonts, text color, etc)
5) Follow items 1, 2, 3 above and now select at the keyboard "Alt+1". You get the same effect.
(in Format-> Style, you can find the equivalent "Alt+." for each style)
6) Delete one of the two cells by clicking on one of the right brackets and pressing "delete".
7) Save this file in the folder you created:

File -> Save As
Some suggestions for its name:
Lesson01
Lesson01_2011_08_31
-) Notice that the default name of the file was "Untitled-1" and you changed it to your desired name, which now appears at the title bar of the window.
8) Close Mathematica.
9) Go to the folder you created and click twice on the file you saved to open Mathematica and the file together with it.

## The Basics

## To execute an instruction, press SHIFT + ENTER

1) Write

The Basics
"Alt+4"
2) Write

To execute an instruction,
type the instruction and press SHIFT+ENTER
"Alt+5"
3) Type $2+3$ and then press SHIFT+ENTER (i.e. hold down the Shift key and then the Enter key)
4) Click of the input cell,

Ctrl+C to save
$\mathrm{Ctrl}+\mathrm{V}$ to paste it in a new cell
Add +8

## Comments

A comment is a useful way to remind us of important details. It is written in between (* ... *)

1) For example :

On the same cell, type
(* I can perform different calculations on the same cell*)
2 + 3
2-3
$2^{\wedge} 3$
$9^{\wedge}(1 / 2)$
3!

## 2) Another example :

On the same cell, type
2*3
(* To multiply I can use the symbol * or just give a space between the numbers *)
23

## Exact vs Approximate

On the same cell, type
2/3
2./3
2./3

2/3.
What is the difference?
Mathematica sees 2 and 3 as exact numbers and for $2 / 3$, it simply gives the exact fraction. To get an approximate answer to the division of 2 by 3 , we need to use the decimal representation to at least one of the numbers.

## Functions

Mathematica has thousands of built-in functions.
CAREFUL!

1) They always start with a *CAPITAL* letter. All Mathematica-defined symbols, commands and functions begin with a capital letter.
2) SQUARE BRACKETS need to be used for the function arguments.

Type

Sqri[9]
Sqrt[10]
Sqrt[10.]

Sqrt[-16]
(* imaginary number I *)
$\operatorname{Re}[2+4 \mathrm{I}]$
Im[2+4 I]
Abs[2+4 I]
(* $\mathrm{Pi}^{*}$ )

```
Pi
1. Pi
N[Pi, 20]
(* natural logarithm *)
E
Exp[1]
Exp[1.]
N[E,20]
Exp[-3.]
6!
0!
Cos[Pi/3]
Cos[60.]
Cos[60. Degree]
ArcCos[0.5]
Pi/3.
```


## Brackets

1) SQUARE BRACKETS are used for the function arguments: Sqrt[9]
2) ROUND BRACKETS are used for grouping: $(2+3) * 4$ and NOT $[2+3]^{*} 4$
3) CURLY BRACKETS are used for lists: $\{1,2,3,4\}$

To open a new window
File -> New -> Notebook
or
CTRL+N

## Variables

We can introduce variables and give them values.
-) Example:
$a=2$
b=3
a+b
From now on, whenever you type "a", Mathematica will equal it to 2 . This will only disappear from its memory if you assign a new value to it, close Mathematica, or if you use the Clear function.
-) Type:
Clear[a]
a+b
*) You cannot start a variable name with a number.
$2 x$ is not accepted, but $x 2$ is.
*) You cannot use words or letters that already have a meaning in Mathematica, such as E, N, Pi
-) Example:
Type:
$\mathrm{E}=4$

Mathematica will complain.
Click on ">>" to find out why it did not like your choice.

## Suppressing output

In the middle of long computations, you may want to hide some of the outputs.
-) Example: suppose you only want to see the final answer for $\mathrm{a}+\mathrm{b}$ in
$a=2$
b=3
$a+b^{\wedge} 2$
You can use semicolons for the first two lines, so their outputs will not appear.
Type:
$\mathrm{a}=2$;
$\mathrm{b}=3$;
$a+b^{\wedge} 2$

## Command "Print"

We may want to add text to the output, which we can do with "Print"
Type:
Clear[a,b,result];
a=Sqrt[2387];
b=Log[339.];
result=a*b;
Print["The result of my calculation was ", result];

## Palettes

To compute the square root of 12 , you can use any of the alternatives:
(i) $\mathrm{Sqrt}[12$.
(ii) $12 . \wedge^{\wedge}(1 / 2)$
(iii) From the menu bar: Palettes -> BasicMathAssistant -> select the square root symbol $\sqrt{\square}$ and you can now write the number 12. inside.

## Save your work and how to print

Before we continue, it may be a good idea to save your works so far:
From the menu bar: File -> Save

If you ever decide to print it, use
File -> Print

## Abort

Sometimes we might want to or need to interrupt a calculation, either because it will take days to give an answer or because we realized there was a mistake in the program we wrote.
From the menu bar: Evaluation -> Abort Evaluation
-) Example:
Abort the computation of $5^{\wedge} 10000000$

## Getting help

Mathematica has various files and tutorials that explain how to use the program. A lot can also be found online.
(i) Let us have a look at some of these files.

From the menu bar: Help -> Virtual Book
Open "Introduction"
Have a look at some examples at "Getting Started". Notice the "related tutorials" at the bottom.
(ii) From the menu bar: Help -> Documentation Center

You can either select one of the topics from the list or use the "SEARCH" field, if you know which command you want to have more information about.

## Sums and Products

Sum
-) Add the squares of the first 10 positive integers
Sum[ $x^{\wedge} 2,\{x, 1,10\}$ ]
-) Add the first 10 even numbers
Sum [ $k,\{k, 2,20,2\}]$

## Product

Multiply the first 10 primes
Product[ Prime[x], $\{x, 1,10\}$ ]

## Transforming Expressions

Expand, factor, simplify, etc
Cell 1:
poly1 $=$ Expand $\left[(1+x)^{\wedge} 3(x-2 y)\right]$
Factor[ poly1]
Simplify[ poly1]
Cell 2:
FullSimplify[ poly1]
Cell 3:
Collect[ poly1, y]
Cell 4:
poly1/(1+x)
Cancel[ poly $1 /(1+x)$ ]
Expand[ $\left.(1+x)^{\wedge} 2(x-2 y)\right]$
Cell 5:
Clear[a,b];
poly2 $=$ Sqrt[a^2] + Sqrt[ $\left.b^{\wedge} 3\right]$

PowerExpand[ poly2 ]

Cell 6:
$\operatorname{trig} 1=\operatorname{Sin}[x+y]+\operatorname{Cos}[x-y]$
Expand[ trig1]
trig2=TrigExpand[ trig1 ]
TrigReduce[ trig2 ]

Cell 7:
$\operatorname{trig} 3=\operatorname{Sin}[x+y]+\operatorname{Sin}[x-y]$
TrigExpand[ $\operatorname{Sin}[x+y]$ ]
trig4=TrigExpand[ trig3]
TrigReduce[ trig4 ]
Cell 8:
$f 1=\left(3-x+6 x^{\wedge} 2-x^{\wedge} 3+2 x^{\wedge} 5\right) /(-2+3 x)$
num=Numerator[ f 1 ]
den=Denominator[ f1 ]
Factor[ num]

## Replacement

poly1 /. x-> 2.
poly1 /. \{ x-> 2., y-> 3. \}
poly1 /. $\{x->p+1, y->q-2\}$

## There is much more!

Plot[Cos[x], $\{x, 0,8 \mathrm{Pi}\}]$
Plot3D[Cos[x] $\operatorname{Sin}[2 y],\{x, 0,24\},\{y, 2,7\}]$

