Assignment 05

(1) Find the limit of $\frac{\sin x^2}{x}$ as x-> 0

(2) Find the derivative of

(i) y(x) = $\sin(e^{x^2})$

(ii) y(x) = $\frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$ [Use Simplify[...] to get the answer in a nice shape]

(iii) y(x) = x arcsinh(x/3) - $\sqrt{9+x^2}$

(iv) The second derivative of y(x)=e^xsinx

(v) The partial derivative with respect to x of f = $\frac{\sin(xy)}{\cos(x+y)}$

(vi) Find $\frac{\partial^3 u}{\partial y^2 \partial x}$ given u=ln(x^2+y)

(3) Maxima and minima

The function f(x)=x+sin(5x) has three relative maxima and two relative minima in the interval [0,Pi].

(i) Plot the function to get an idea of their locations.

(ii) Find them.

(4) Suppose f(x) = -x^3 + x + 1.

(i) Find f'(x) using a single command from *Mathematica.*

(ii) Find f'(x) using the command "Limit" from *Mathematica.*

(iii) Find f'(1) using item (i) and compare it with results from the definition of derivative obtained with a do-loop. In this case, use increments h to x, which vary as 0.1, 0.01, ...0.000001

(iv) Determine the equation of the line tangent to the graph of f(x) at the point (1,f(1)).

[Hint: the equation of the tangent line to a curve at xo is
f(xo) + f'(xo) (x-xo)]

(v) Plot both f(x) and the tangent line in the interval [-1.5,2].

(5) Repeat the last exercise item above, but embed it all within "Manipulate" so that the point of tangency between the line and the cubic function goes continuosuly from x=-1.5 to x=2.0

(6) Repeat the exercise above, but now with Animate and this time for the function: f(x)=x*sin(x^2) +1
[This is the same function used in wikipedia to illustrate the notion of derivate: http://en.wikipedia.org/wiki/Derivative].
The plot range should be {-2,4} for x and also for y, whereas the function and the tangent should appear just for x from -1 to 3.

(7)
(i) Find the Taylor expansion of $e^{\sin(u)}$ about u=0 to order u^7
(ii) Convert the power series above to an ordinary expression (that is use Normal[...])
(iii) Find the Taylor expansion of $(1+x^4)^{1/3}$ about x=0 up to order x^4,x^8, and x^{24} and integrate each result for {x, 0, 1}
(iv) Find the integral
$\int_0^1 (1+x^4)^{1/3}\, dx$
and compare with the results from item (iii)

(8)
(i) Evaluate $\int_0^1 \frac{4}{1+x^2}\, dx$
(ii) Evaluate the integral and then differentiate the result to recover the integrand
[you may need to use Simplify[...]]
$\int \frac{x}{a^3+x^3}\, dx$
(iii) $\int \frac{2x^2-x+4}{x^3+4x}\, dx$
(iv) Assuming that the constant a is both real and positive, evaluate the integral
$\int_{-\infty}^{\infty} x^2 e^{-2 a x^2}\, dx$
(v) Assuming that a>0 and m≥0, evaluate
$\int_0^{\infty} \frac{\cos^2(mx)}{a^2+x^2}\, dx$

(9) Integrate the two functions below:
f1= x^5 Sin[2 x] + x./3   in the interval [2.5,7]

f2= x^5 Exp[Sin[2 x]] + x./3   in the interval [2.5,7]

(i) Using a single command from *Mathematica*
(ii) Using a sum of small areas, as we did in class.
(iii) Using a sum of small areas, as we did in class, but avoid the commad “Sum” and instead use a do-loop.
[In (ii) and (iii), use a bin size that gives results very close to the integral done with *Mathematica*]