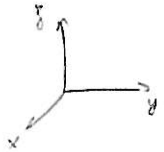
Stern- Gerlach experiment

{ www.youtube.com/watch?v=waK4eKNXB4A

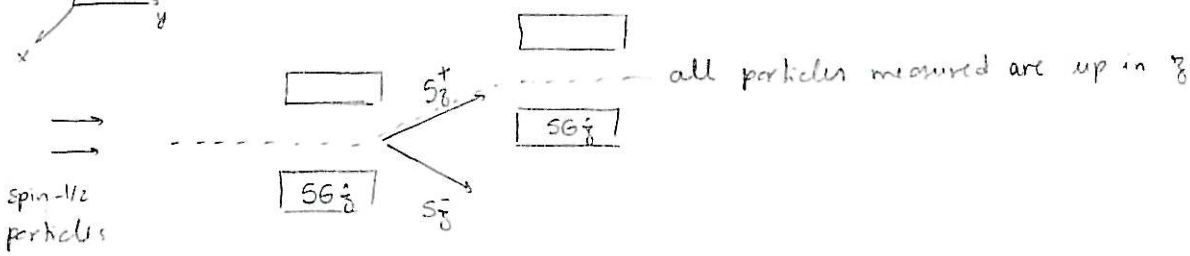
Modern Quantum Mechanics by J. J. Sakurai

S_x, S_y, S_z do not commute $\Rightarrow$ cannot know S_x and S_z precisely at the same time

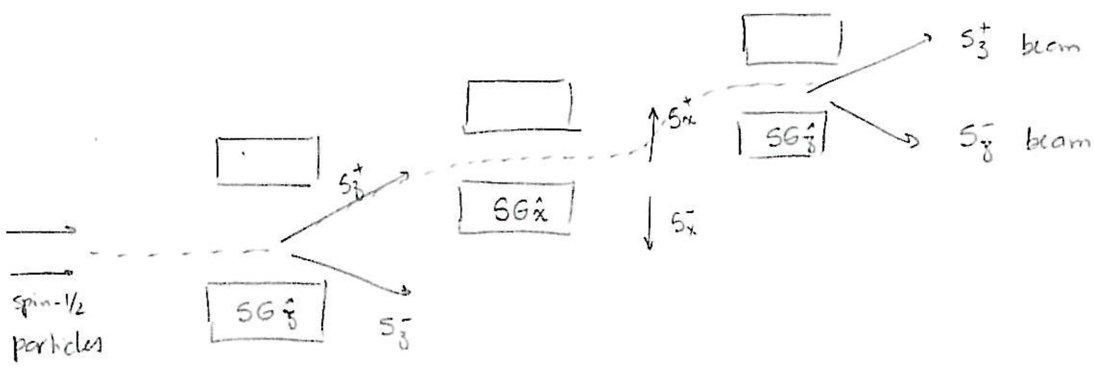
uncertainty principle



(1)



(2)



Measurement disturbs the system

HW 4.26, 4.27, 4.28, 4.29

4.34, 4.38, 4.49, ~~[illegible]~~

Addition of Angular Momenta

Suppose we have two spin-1/2 particles. Example: e + p in the ground state of H ($\ell=0$)

What is the TOTAL angular momentum of the atom?

$$\vec{S} = \underbrace{\vec{S}^{(1)}}_{\text{acts only on spin 1 } \chi_1} + \underbrace{\vec{S}^{(2)}}_{\text{acts only on spin 2 } \chi_2}$$

∘) There are four possibilities:

$$\begin{aligned} S_z \chi_1 \chi_2 &= (S_z^{(1)} + S_z^{(2)}) \chi_1 \chi_2 \\ &= (S_z^{(1)} \chi_1) \chi_2 + \chi_1 (S_z^{(1)} \chi_2) \\ &= \hbar m_1 \chi_1 \chi_2 + \hbar m_2 \chi_1 \chi_2 \\ &= \boxed{\hbar (m_1 + m_2) \chi_1 \chi_2} \end{aligned}$$

↑↑	$m=1$	
↑↓	$m=0$	? → eigenstates of S_z but not of S^2
↓↑	$m=0$	
↓↓	$m=-1$	

it appears that $\boxed{s=1}$, but there is an extra $m=0$

∘) Apply $S_- = S_-^{(1)} + S_-^{(2)}$ to state ↑↑

$$S_-(\uparrow\uparrow) = (S_-^{(1)} \uparrow)\uparrow + \uparrow(S_-^{(2)}\uparrow) = \hbar(\downarrow\uparrow + \uparrow\downarrow) \Longrightarrow \boxed{m=0}$$

so the three states with $\boxed{s=1}$ are

$$|s\,m\rangle = \left\{ \begin{array}{l} |1\,1\rangle = \uparrow\uparrow \\ |1\,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ |1\,{-1}\rangle = \downarrow\downarrow \end{array} \right\} s=1 \text{ (triplet)}$$

But we can also have a state orthogonal to the triplet where

$s=0$ (<u>singlet</u>)

$$|s\,m\rangle = |0\,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

Therefore, the combination of two spin-1/2 particles can carry a <u>total</u> spin of 1 or 0

To confirm: prove that the <u>triplet</u> states are eigenvectors of S^2 with eigenvalue $2\hbar^2$ $\longleftarrow$ $S^2|\text{triplet}\rangle \underset{(s=1)}{=} \hbar^2 1(1+1)|\text{triplet}\rangle$

and the <u>singlet</u> is an eigenvector of S^2 with eigenvalue 0 $\longleftarrow$ $S^2|\text{singlet}\rangle \underset{(s=0)}{=} 0$

$$S^2 = (\vec{S}^{(1)} + \vec{S}^{(2)}).(\vec{S}^{(1)} + \vec{S}^{(2)}) = (S^{(1)})^2 + 2\,\vec{S}^{(1)}.\vec{S}^{(2)} + (S^{(2)})^2$$

$$\left.\begin{aligned} \circ)\ (S^{(1)})^2 \uparrow &= \hbar^2 \tfrac{1}{2}\left(\tfrac{3}{2}\right)\uparrow = \tfrac{3}{4}\hbar^2\uparrow \\ \circ)\ (S^{(1)})^2 \downarrow &= \hbar^2 \tfrac{1}{2}\left(\tfrac{3}{2}\right)\downarrow = \tfrac{3}{4}\hbar^2\uparrow \end{aligned}\right\} \text{ same for } (S^{(2)})^2$$

$$\circ)\ \vec{S}^{(1)}.\vec{S}^{(2)} = S_x^{(1)} S_x^{(2)} + S_y^{(1)} S_y^{(2)} + S_z^{(1)} S_z^{(1)}$$

$$\left\{\begin{aligned} S_x &= \tfrac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix} \Rightarrow S_x\begin{pmatrix}1\\0\end{pmatrix} = \tfrac{\hbar}{2}\begin{pmatrix}0\\1\end{pmatrix},\ S_x\begin{pmatrix}0\\1\end{pmatrix} = \tfrac{\hbar}{2}\begin{pmatrix}1\\0\end{pmatrix} \\ S_y &= \tfrac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix} \Rightarrow S_y\begin{pmatrix}1\\0\end{pmatrix} = \tfrac{\hbar i}{2}\begin{pmatrix}0\\1\end{pmatrix},\ S_y\begin{pmatrix}0\\1\end{pmatrix} = -\tfrac{\hbar i}{2}\begin{pmatrix}1\\0\end{pmatrix} \\ S_z &= \tfrac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix} \Rightarrow S_z\begin{pmatrix}1\\0\end{pmatrix} = \tfrac{\hbar}{2}\begin{pmatrix}1\\0\end{pmatrix},\ S_z\begin{pmatrix}0\\1\end{pmatrix} = -\tfrac{\hbar}{2}\begin{pmatrix}0\\1\end{pmatrix} \end{aligned}\right.$$

$$\circ)\ \vec{S}^{(1)}.\vec{S}^{(2)}(\uparrow\downarrow) = \hbar^2/4\,(\downarrow\uparrow) + \hbar^2/4\,(\downarrow\uparrow) - \hbar^2/4\,(\uparrow\downarrow) = \hbar^2/4\,(2\downarrow\uparrow - \uparrow\downarrow)$$

$$\circ)\ \vec{S}^{(1)}.\vec{S}^{(2)}(\downarrow\uparrow) = \hbar^2/4\,(2\uparrow\downarrow - \downarrow\uparrow)$$

∘) $\vec{S}^{(1)} \cdot \vec{S}^{(2)} |10\rangle = \vec{S}^{(1)} \cdot \vec{S}^{(2)} \frac{1}{\sqrt{2}} (\uparrow\downarrow + \downarrow\uparrow) =$

$$= \frac{\hbar^2}{4} \frac{1}{\sqrt{2}} \left(2 \downarrow\uparrow - \uparrow\downarrow + 2 \uparrow\downarrow - \downarrow\uparrow \right) = \left(\frac{\hbar^2}{4}\right) |10\rangle$$

∘) $\vec{S}^{(1)} \cdot \vec{S}^{(2)} |00\rangle = \vec{S}^{(1)} \cdot \vec{S}^{(2)} \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) =$

$$= \frac{\hbar^2}{4} \frac{1}{\sqrt{2}} \left(2 \downarrow\uparrow - \uparrow\downarrow - 2 \uparrow\downarrow + \downarrow\uparrow \right) = \left(-\frac{3\hbar^2}{4}\right) |00\rangle$$

$$\Rightarrow \quad S^2 |10\rangle = \Big(\underbrace{\frac{3}{4}\hbar^2}_{(S^{(1)})^2} + \underbrace{\frac{3}{4}\hbar^2}_{(S^{(2)})^2} + 2 \underbrace{\frac{\hbar^2}{4}}_{\vec{S}^{(1)} \cdot \vec{S}^{(2)}} \Big) |10\rangle$$

$$\boxed{S^2 |10\rangle = 2\hbar^2 |10\rangle}$$

$$\Rightarrow \quad S^2 |00\rangle = \left(\frac{3}{4}\hbar^2 + \frac{3}{4}\hbar^2 + 2 \left(-\frac{3}{4}\hbar^2 \right) \right)$$

$$\boxed{S^2 |00\rangle = 0}$$