Chapter 7 – Linear Momentum

Conservation of
- energy
- ***linear momentum***
- angular momentum
- electric charge

To study collisions
- Conservation of energy
- +
- Conservation of ***linear momentum***

Linear momentum:
(VECTOR)

$$\boxed{\vec{p} = m\vec{v}}$$

SI unit: kg.m/s

p(faster car)>p(slower car)
p(heavy truck)>p(light car) – both at the same speed

Newton's 2nd Law

A force is needed to change the momentum $\begin{cases} \text{increase it} \\ \text{decrease it} \\ \text{change its direction} \end{cases}$

Newton's 2nd law – originally in terms of momentum:

$$\sum \vec{F} = m\vec{a} \Leftrightarrow \boxed{\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}}$$

(applies even when the mass changes)

$$\frac{\Delta \vec{p}}{\Delta t} = \frac{m(\vec{v}_2 - \vec{v}_1)}{\Delta t} = ma$$

Ex.7-1 A 0.060 kg ball leaves the racket with v=55m/s. It is in contact with the racket for 4ms. What is the average force on the ball? Compare it with the weight of a 60-kg person.

F~800N

Conservation of Momentum

If the system is ***ISOLATED*** ,
the sum of momenta before and after a collision is the same

$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}'_A + m_B \vec{v}'_B$$

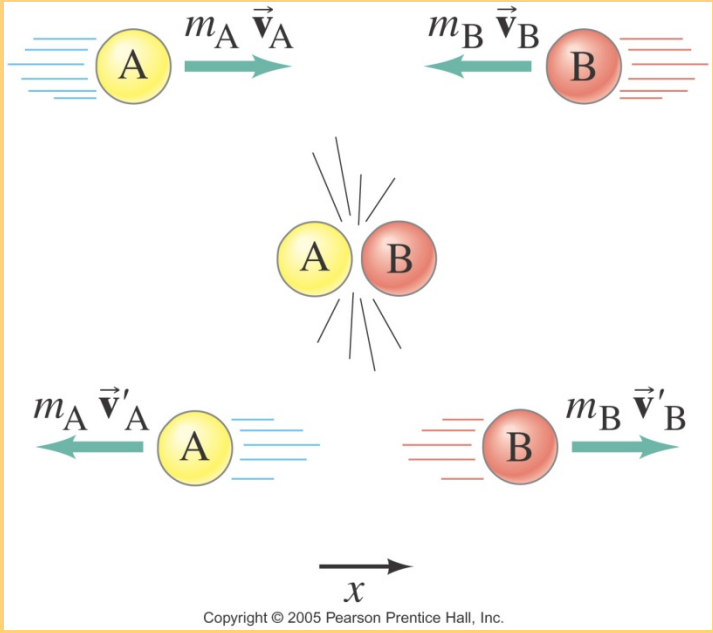
Head-on collision:

$$\Delta\vec{p} = \vec{F}\Delta t$$

$$\Delta\vec{p}_B = m_B(\vec{v}'_B - \vec{v}_B) = \vec{F}_{BA}\Delta t$$

$$\Delta\vec{p}_A = m_A(\vec{v}'_A - \vec{v}_A) = \vec{F}_{AB}\Delta t = -\vec{F}_{BA}\Delta t$$

$$m_A(\vec{v}'_A - \vec{v}_A) = -m_B(\vec{v}'_B - \vec{v}_B)$$



Isolated System

Isolated system = net external force is zero,
only internal forces are significant

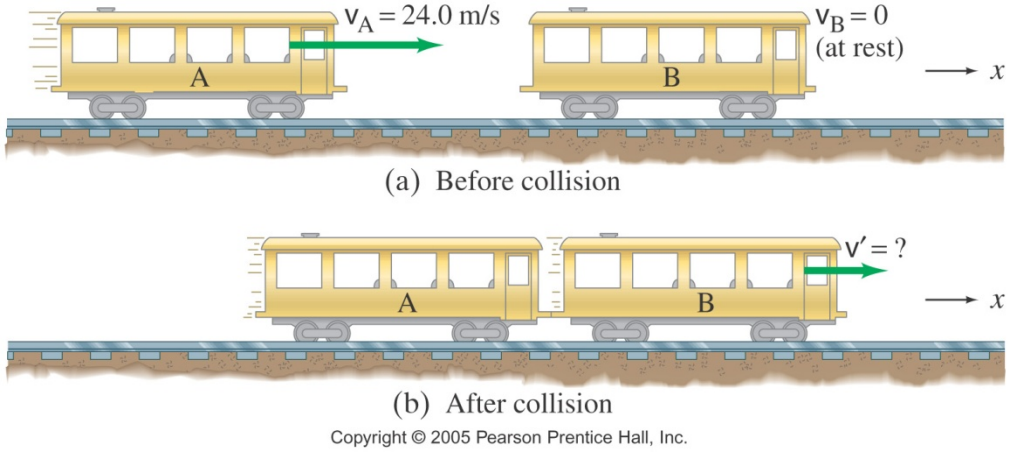
In the real world:
external forces are
friction, gravity, etc

Examples:

system: rock falling – momentum **is not conserverd**
(external force = gravity)

system: rock + Earth – momentum **is conserverd**

Ex. 7-3: A 10,000-kg railroad car A traveling at 24.0 m/s strikes an identical car B at rest. If they lock together, what is their common speed afterwards?



$$\boxed{v' = \frac{m_A}{m_A + m_B} v_A}$$

$$m_A >> m_B \Rightarrow v' = v_A$$

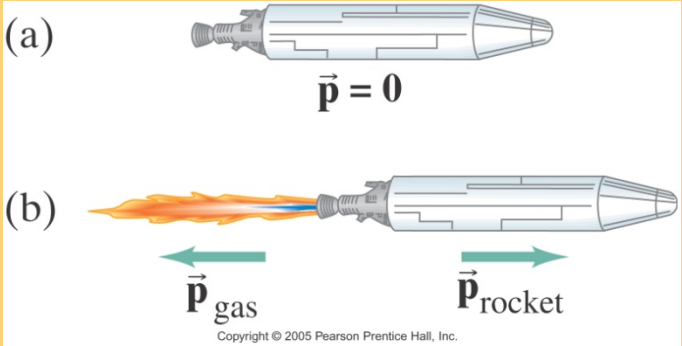
$$m_A << m_B \Rightarrow v' = 0$$

Examples

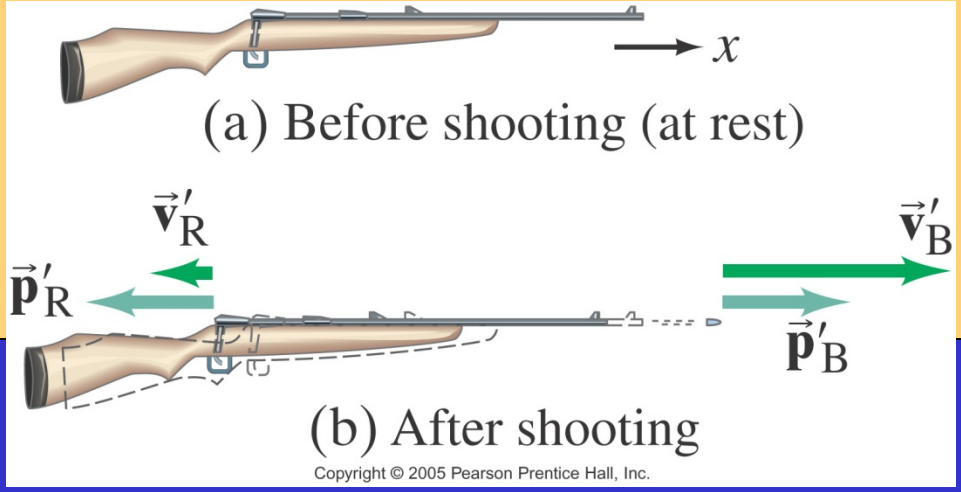
Momentum conservation works for a rocket as long as we consider the rocket and its fuel to be one system, and account for the mass loss of the rocket.

In the reference frame of the rocket:

When the fuel burns and gases are expelled:
the rocket gains momentum,
it can accelerate in empty space.



Ex. 7-4 Calculate the recoil **velocity** of a 5.0-kg rifle that shoots a 0.020-kg bullet at a speed of 620 m/s



$$v'_R = -2.5 m/s$$

Cons. of Energy and Momentum

No net external force: conservation of momentum

No loss of energy: (like heat or sound) **conservation of kinetic energy** --- ***ELASTIC*** collision

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B$$

Collisions of atoms and molecules are often elastic
Collisions of billiard balls are close to it

INELASTIC collision: kinetic energy is not conserved, part of it is lost into other forms

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B + \textbf{thermal and others}$$

NOTE: TOTAL energy is always conserved !

Elastic Collision – 1D

Conservation of momentum: $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$

Conservation of kinetic energy: $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B$

Simplification valid only for **head-on elastic collision**

Magnitude of relative speed of the two objects after the collision = before the collision, but opposite sign ---- no matter what the masses are

$$\frac{1}{2}m_A v_A^2 - \frac{1}{2}m_A v'^2_A = -\frac{1}{2}m_B v_B^2 + \frac{1}{2}m_B v'^2_B \qquad \frac{1}{2}m_A(v_A^2 - v'^2_A) = -\frac{1}{2}m_B(v_B^2 - v'^2_B)$$

$$\left.\begin{aligned} &\frac{1}{2}m_A(v_A - v'_A)(v_A + v'_A) = -\frac{1}{2}m_B(v_B - v'_B)(v_B + v'_B) \\ &m_A(v_A - v'_A) = -m_B(v_B - v'_B) \end{aligned}\right\}$$

$$v_A + v'_A = v'_B + v_B$$

$$v_A - v_B = -(v'_A - v'_B)$$

Examples

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B$$

$$v_A - v_B = -(v'_A - v'_B)$$

Ex. 7-7 A billiard ball A of mass m with speed v collides head-on with ball B of equal mass at rest. What are the speeds of the two balls after the collision, assuming it is elastic?

$$v'_A = 0, \quad v'_B = v$$

Ex. 7-8 A proton of mass 1.01 u (unified atomic mass units) traveling with a speed of 3.60x10^4 m/s has an elastic head-on collision with a helium nucleus, (mass = 4.00 u) initially at rest. What are the velocities of the proton and helium nucleus after the collision?

$$v'_{He} = 1.45\times10^4\, m/s$$

$$v'_p = -2.15\times10^4\, m/s$$

Proton reverses its direction after collision

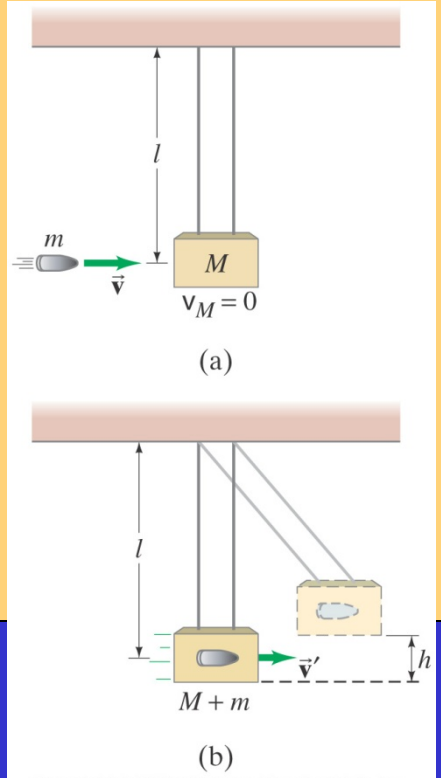
Inelastic Collision

Inelastic collision: kinetic energy is not conserved,
but <u>total energy</u> and total vector <u>momentum</u> are

COMPLETELY inelastic collision: two object stick together.

Ex. 7-9: A 10,000-kg railroad car A traveling at 24.0 m/s strikes an identical car B at rest and they lock together. Calculate how much of the initial kinetic energy is transformed to thermal or other forms of energy. (after the collision both cars are moving at 12 m/s – see Ex. 7-3)

1.44 x 10^6 J



Ex. 7-10 Ballistic pendulum. A projectile of mass m is fired into a large block of mass M, which is suspended like a pendulum. As a result of the collision, the pendulum swings up to a maximum height h. What is the relationship between the initial horizontal speed of the projectile, v, and the maximum height h?

(gravity neglected during the short collision time)

$$v = \frac{m+M}{m}\sqrt{2gh}$$

Collision in 2D

Conservation of momentum:
(we need the angles)

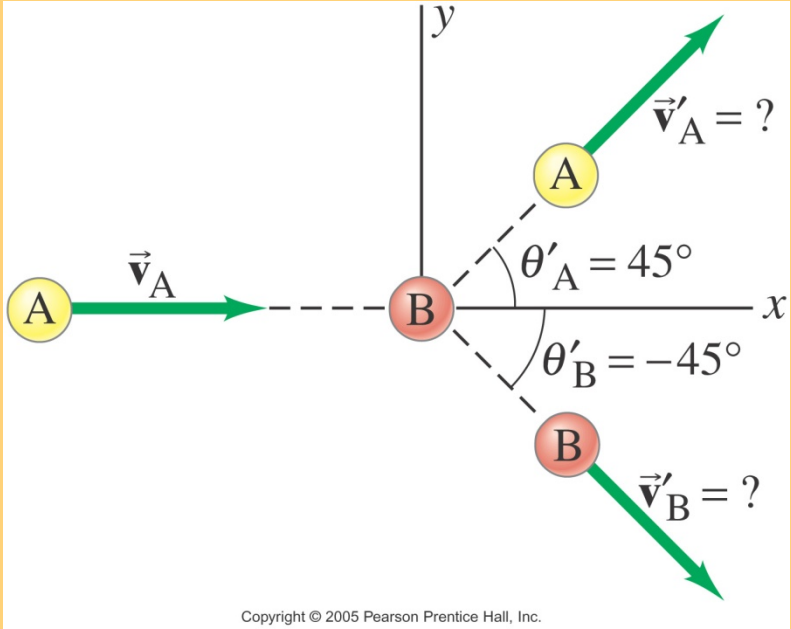
$$p_{Ax} + p_{Bx} = p'_{Ax} + p'_{Bx}$$

$$p_{Ay} + p_{By} = p'_{Ay} + p'_{By}$$

If the collision is elastic

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v'^2_A + \frac{1}{2} m_B v'^2_B$$

Three independent equations – we can find 3 unknowns



Ex.7-11 A more simple example, where there are only 2 unknowns:

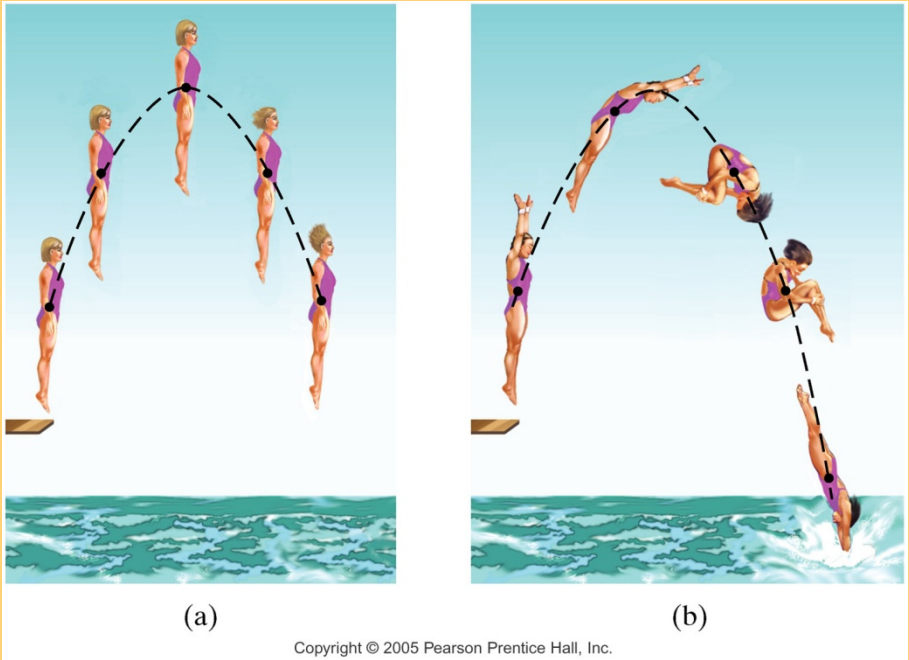
A billiard ball A at speed 3.0 m/s strikes an equal-mass ball B initially at rest. The angles are shown in the figure. What are the speeds of the two balls after the collision?

$$v'_A = v'_B = 2.1 m/s$$

Center of Mass

The motion of extended objects or a system with many objects may be complicated,

but the **CENTER OF MASS (CM)** moves as a point particle.



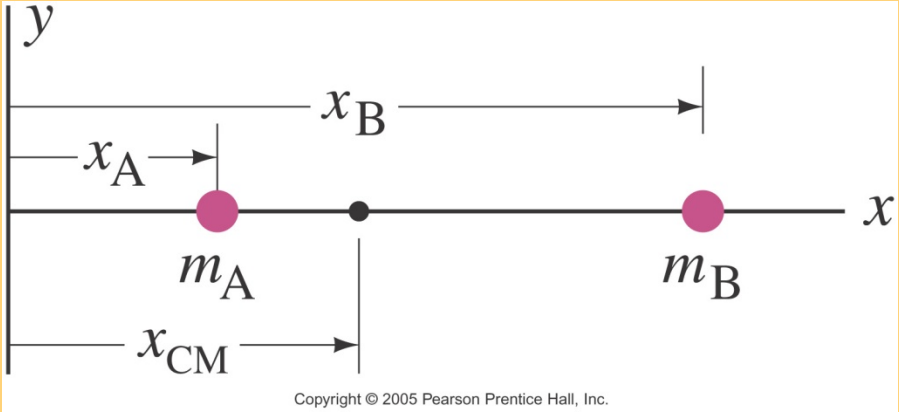
The general motion of an object can be considered as the sum of the translational motion of the CM, plus rotational, vibrational, or other forms of motion about the CM.

Center of Mass – 2 objects

For two particles, the center of mass lies closer to the one with the most mass:

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M}$$

where *M* is the total mass.



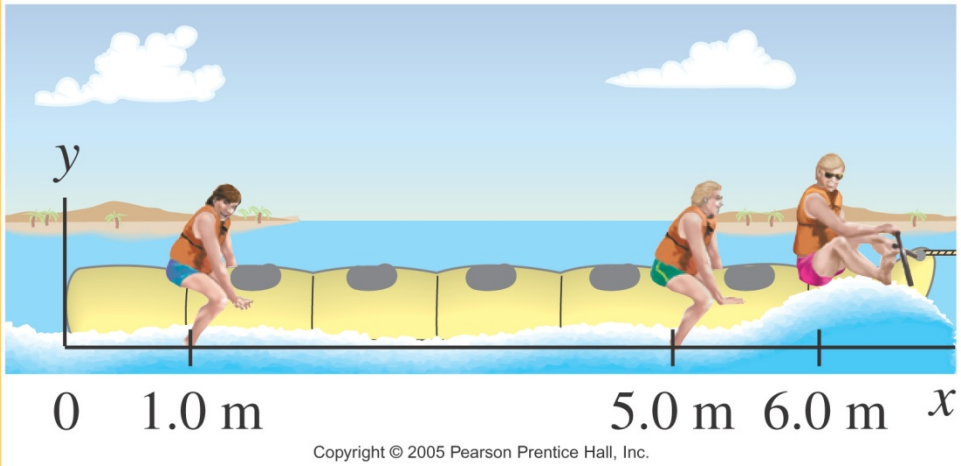
CM – more than 2 objects

If there are more than two particles

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C + \ldots}{M}$$

where M is the total mass of all the particles

Ex. 7-12 Three people of roughly equal masses m on a lightweight banana boat sit along the x axis at the positions x_A=1.0m, x_B=5.0m, and x_C=6.0m, as in the figure. Find the CM.



x_{CM}=4.0m

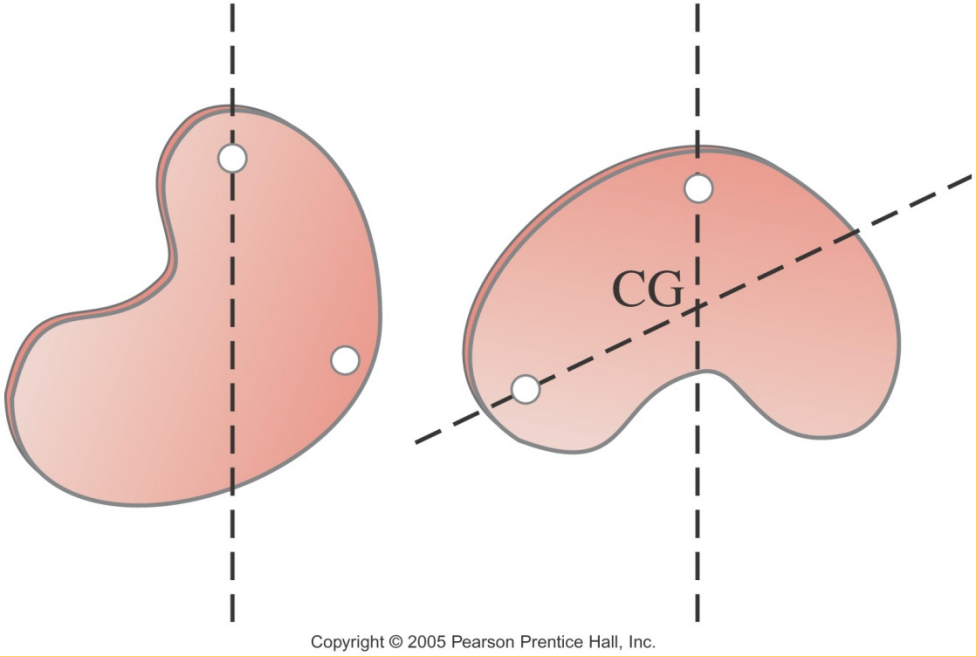
The coordinates of the CM depend on the reference frame, but the physical location of the CM is independent of that choice

The CM may lie outside the object
Ex: donuts, jumpers over bars

CM – 2D

The **center of gravity** is the point where the gravitational force can be considered to act. It is the same as the **center of mass** as long as the gravitational force does not vary among different parts of the object.

The center of gravity can be found **experimentally** by suspending an object from different points.



$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C + \ldots}{M}$$

$$y_{CM} = \frac{m_A y_A + m_B y_B + m_C y_C + \ldots}{M}$$

CM and Translational Motion

The **total momentum** of a system of particles is equal to the product of the **total mass** and the **velocity of the center of mass**.

$$Mv_{CM} = m_A v_A + m_B v_B + m_C v_C$$

The sum of all the forces acting on a system is equal to the **total mass** of the system multiplied by the **acceleration of the center of mass**:

$$F_{net} = Ma_{CM}$$

CM and Translational Motion

Jesse Owens won four gold medals in the 1936 Berlin summer Olympics. A film shows that his CM rose 1.1m from launch point to the top of the arc. What minimum speed did he need at launch if he was traveling at 6.5 m/s at the top of the arc?

$$K1 + P1 = K2 + P2$$

$$v=8.0 \text{ m/s}$$