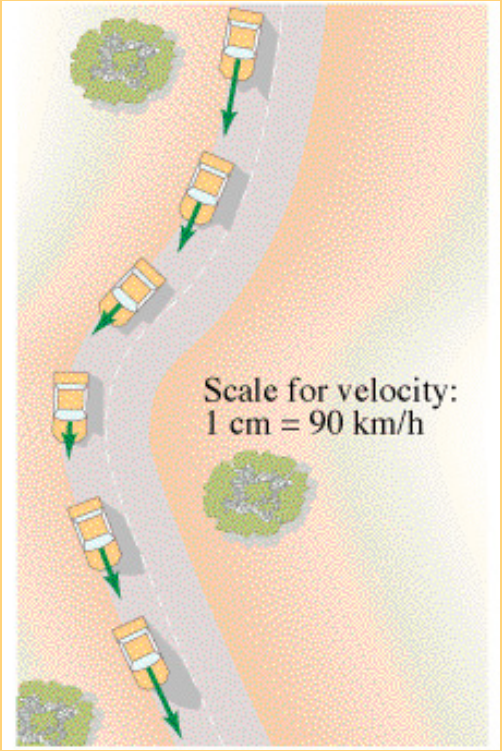
Chapter 3

Kinematics in Two Dimensions (motion of an object on a plane);



VECTOR
is a quantity that has magnitude AND direction
Examples: displacement, velocity, acceleration

SCALAR
is a quantity that has only magnitude
and NO direction
Examples: mass, time, temperature

Diagram:
vector is represented by an **arrow**
where it points indicates the **direction**
its size gives the **magnitude**

Addition of Vectors

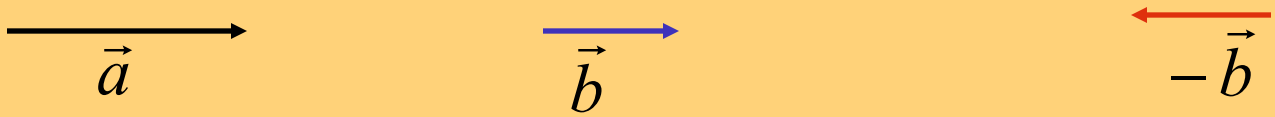
The symbol for vector: use boldface type and an arrow over the symbol

Example: velocity should be indicated as $\vec{v}$

Rule for adding vectors *(tail to tip method)*

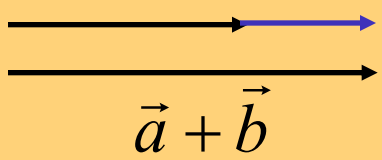
1) Place the Tail of the second vector at the Tip of the first one
2) The arrow drawn from the tail of the first vector to the tip of the second vector is the sum (**resultant**) of the two vectors

Vectors in a straight line



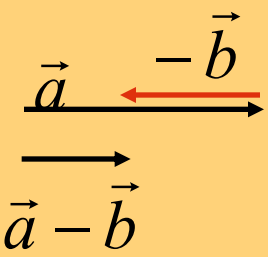
Addition:

$\vec{a}+\vec{b}$



Subtraction:

$\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$



It is not important in which order the vectors are added

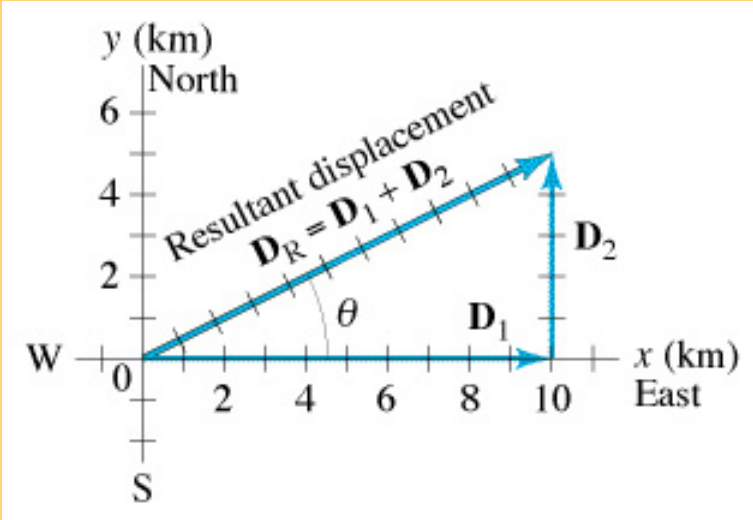
Addition of Vectors

Vectors NOT in a straight line

$\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2$ This is a vector equation

In class, solve

$\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_1 - \vec{\mathbf{D}}_2$



Vectors not in the same line:

The magnitude of the resultant vector is NOT equal to the sum of magnitudes of the two separate vectors

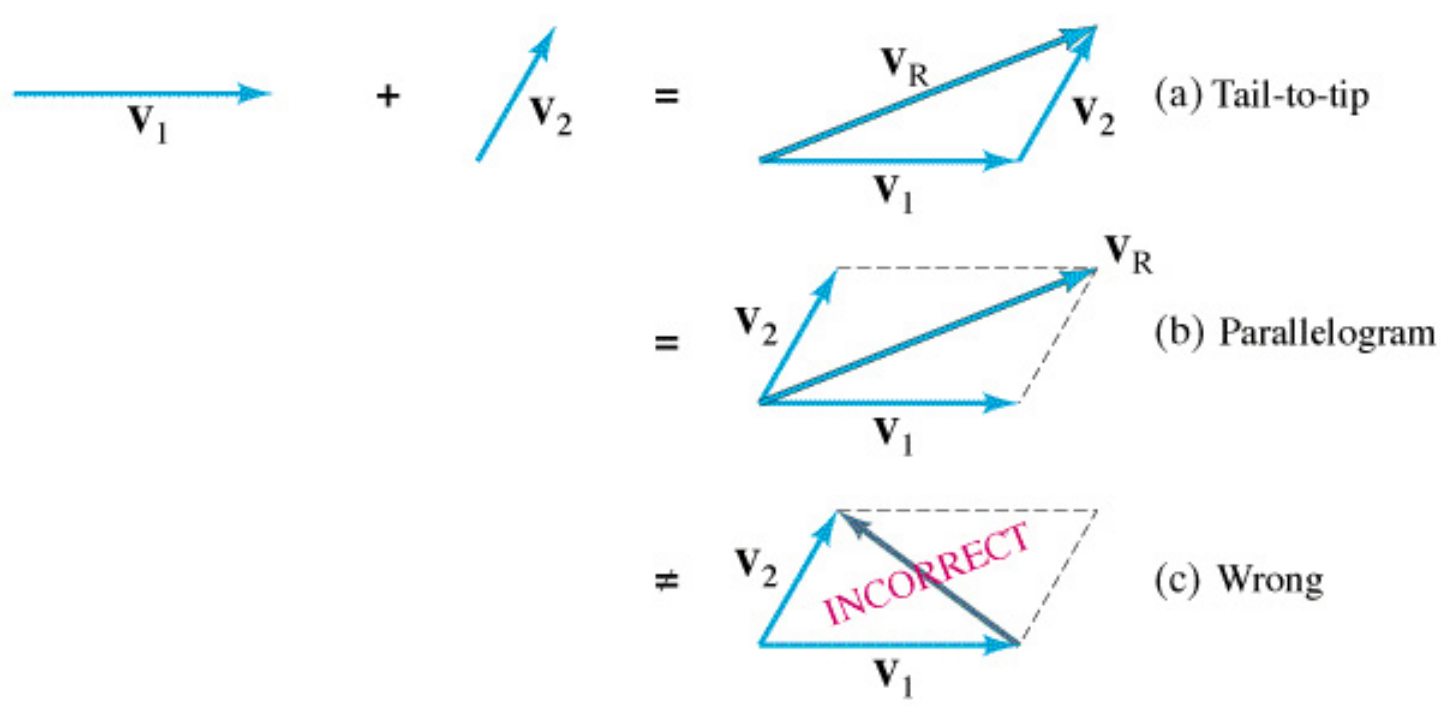
Magnitude of the resultant
*(theorem of Pythagoras – ONLY if vectors are **perpendicular** to each other)*

$$D_R = \sqrt{D_1^2 + D_2^2} = \sqrt{(10.0km)^2 + (5.0km)^2} = 11.2km$$

$$D_R < D_1 + D_2$$

$D_R = D_1 + D_2$ only when both vectors point in the same direction

Addition of Vectors



Exercise: Suppose two vectors each have length 3.0 m.

a) What is the range of possible lengths for the sum of the two?

b) If they are perpendicular to each other, what is the sum?

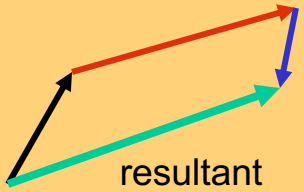
a) From 0 (antiparallel) to 6.0 m (parallel)

b) $3\sqrt{2}m$

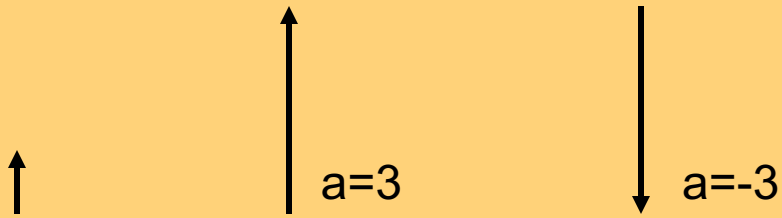
Multiplication by scalar

It is not important in which order the vectors are added

The tail to tip method of adding vectors can be extended to three or more vectors



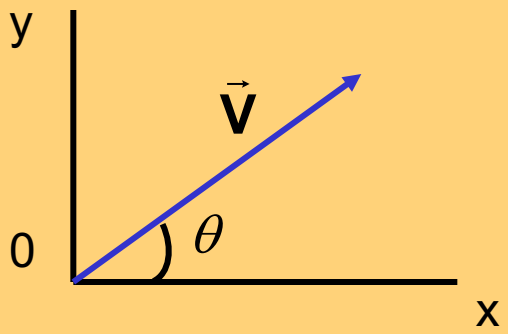
A vector can be **multiplied by a scalar**.
Multiplication by a positive scalar (a) changes the magnitude but not the direction
Multiplication by a negative scalar changes the magnitude and the direction becomes the opposite one



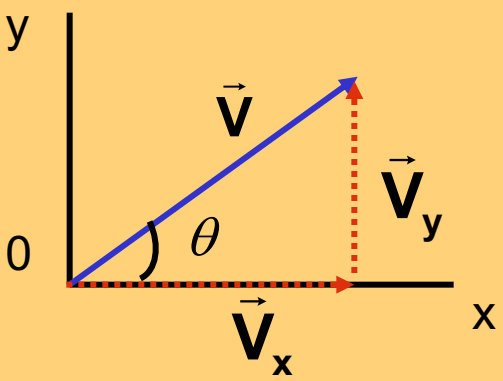
Components of a vector

There are two ways to specify a vector:

1. We give its magnitude V and the angle θ it makes with the positive x axis.

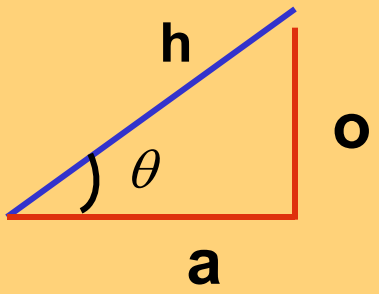


2. We give its components, V_x and V_y



Trigonometric Functions

Right triangle:



$$\sin\theta = \frac{opposite\ side}{hypotenuse} = \frac{o}{h}$$

$$\cos\theta = \frac{adjacent\ side}{hypotenuse} = \frac{a}{h}$$

$$\tan\theta = \frac{opposite\ side}{adjacent\ side} = \frac{o}{a}$$

$$h^2 = o^2 + a^2 \quad (Pythagorean\ theorem)$$

$$h^2 = o^2 + a^2$$

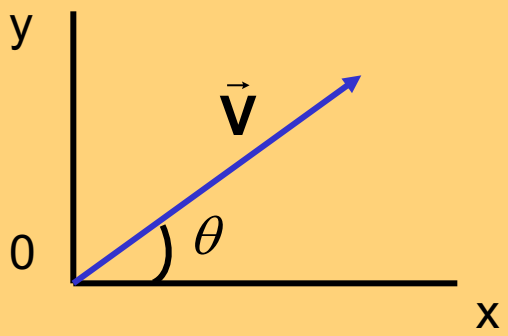
$$\Rightarrow \frac{h^2}{h^2} = \frac{o^2}{h^2} + \frac{a^2}{h^2}$$

$$\Rightarrow \sin^2\theta + \cos^2\theta = 1$$

Components of a vector

We can shift from one description to the other

1. If we have its magnitude V and the angle θ it makes with the positive x axis.

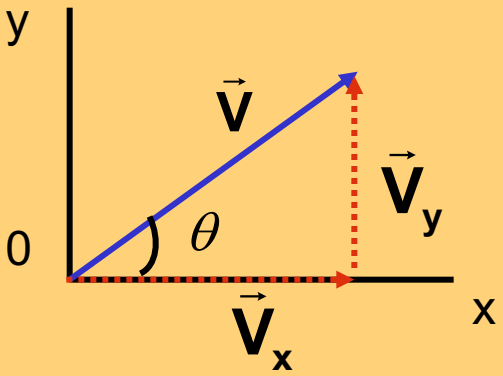


Then we can find the components

$$V_x = V\cos\theta$$

$$V_y = V\sin\theta$$

2. We give its components, V_x and V_y

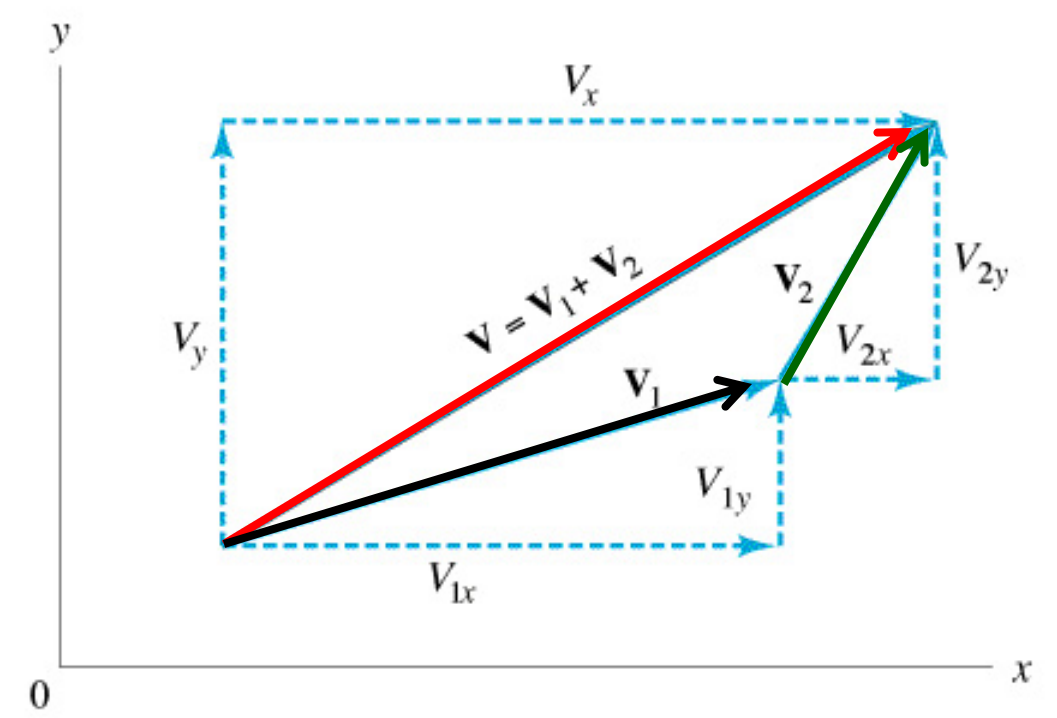


Then we can find V and the angle

$$V = \sqrt{V_x^2 + V_y^2}$$

$$\tan\theta = \frac{V_y}{V_x}$$

Sum of the components



$$\vec{V} = \vec{V}_1 + \vec{V}_2$$

$$V_x = V_{1x} + V_{2x}$$

$$V_y = V_{1y} + V_{2y}$$

Given the components of two vectors:
The sum of the x components equals the x component of the resultant (similarly for y)
In hands of V_x and V_y , we can find the magnitude and direction of the resultant

Example: Sum of the components

The choice of coordinate axes is **arbitrary**.
A good choice reduces the work of adding vectors.
Example: choose one of the axes to be in the same direction as one of the vectors

Example 3-2 of Giancoli's book:
A rural mail carrier leaves the post office and drives 22.0km north. She then drives in a direction 60.0 degrees south of east for 47.0km.
What is her displacement from the post office?

$$D_{2x} = 47.0\cos 60^o = 23.5km \qquad D_x = +23.5km$$

$$D_{2y} = -47.0\sin 60^o = -40.7km \qquad D_y = D_{1y} + D_{2y} = -18.7km$$

$$D = \sqrt{D_x^2 + D_y^2} = 30.0km$$

$$\tan\theta = |D_y| / |D_x| = 0.796 \Rightarrow \theta = 38.5^o$$

$$D_x +,\ D_y -\ \ so\ \theta = -38.5 = 321.5^o$$

Example 3-3 of Giancoli's book:
A plane flies 620 km east, then 440 km southeast (45 degrees) and then 550 km at 53 degrees south of west. What is the plane's total displacement?

2nd quadrant x-, y+	1st quadrant x+, y+
3rd quadrant x-, y-	4th quadrant x+, y-

$$D = \sqrt{D_x^2 + D_y^2} = 960.0km$$

$$\tan\theta = |D_y| / |D_x| = 1.25$$

$$D_x +,\ D_y - \Rightarrow \theta = -51^o$$

Galileo Galilei

http://en.wikipedia.org/wiki/Galileo_Galilei

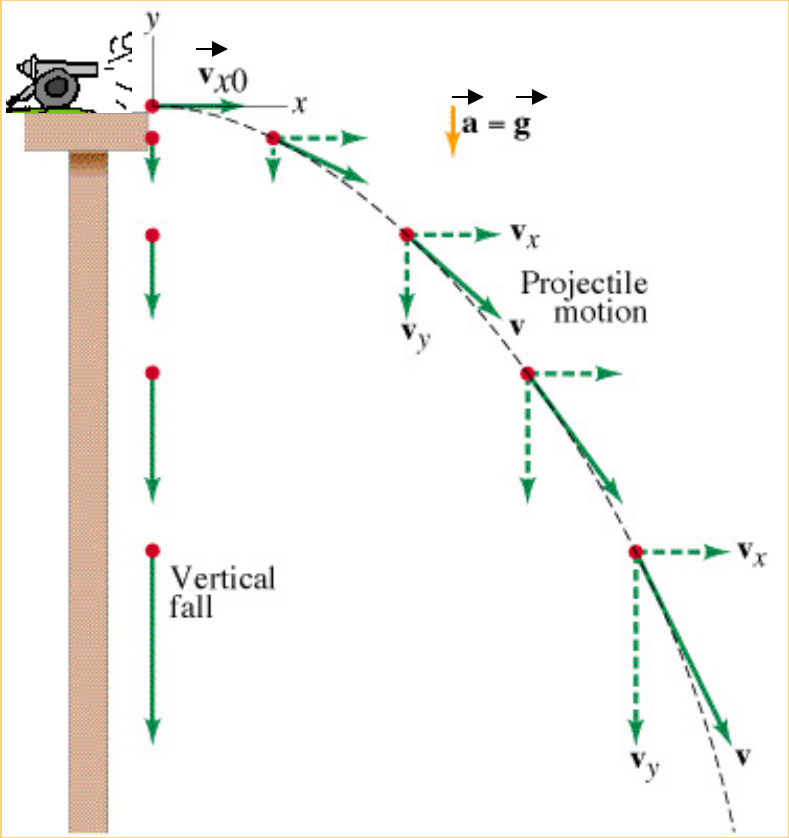
http://en.wikipedia.org/wiki/Dialogue_Concerning_the_Two_Chief_World_Systems
The ***Dialogue Concerning the Two Chief World Systems*** was a 1632 book by Galileo, comparing the Copernican system, and the traditional Ptolemaic system. In the Copernican system the Earth and other planets orbit the Sun, while in the Ptolemaic system everything in the Universe circles around the Earth. The *Dialogue* was published in Florence under a formal license from the Inquisition. In 1633, Galileo was convicted of "grave suspicion of heresy" based on the book, which was then placed on the Index of Forbidden Books, from which it was not removed until 1835.

http://en.wikipedia.org/wiki/Two_New_Sciences
The ***Discourses and Mathematical Demonstrations Relating to Two New Sciences*** (*Discorsi e dimostrazioni matematiche, intorno a due nuove scienze*, 1638) was Galileo's final book and a sort of scientific testament covering much of his work in physics over the preceding thirty years. Unlike the ***Dialogue Concerning the Two Chief World Systems***, it was not published with a license from the Inquisition; after the heresy trial based on the earlier book, the Roman Inquisition had banned publication of any work by Galileo, including any he might write in the future. After the failure of attempts to publish the work in France, Germany, or Poland, it was picked up by Lowys Elsevier in Leiden, The Netherlands, where the writ of the Inquisition was of little account.
The same three men as in the *Dialogue* carry on the discussion, but they have changed. **Simplicio**, in particular, is no longer the stubborn and rather dense Aristotelian; to some extent he represents the thinking of Galileo's early years, as **Sagredo** represents his middle period. **Salviati** remains the spokesman for Galileo.

The whole book ***Discourses and Mathematical Demonstrations Relating to Two New Sciences*** can be found at http://oll.libertyfund.org//files/753/0416_Bk.pdf

Projectile Motion

Galileo: projectile motion can be understood by analyzing the **horizontal and vertical** components of the motion **separately**.



$\vec{v}$ is tangent to the path

For convenience: $t_0 = 0, \;\; x_0 = 0, \;\; y_0 = 0,$

According to the reference frame of the figure:

vertical motion

constant acceleration

$$v_{y0} = 0 \qquad a_y = -g$$

$$v_y = -gt \qquad y = -\frac{g}{2}t^2$$

horizontal motion (no air resistance)

constant velocity

$$v_x = v_{x0} \qquad a_x = 0$$

$$x = v_{x0}t$$

An object projected horizontally will reach the ground in the same time as an object dropped vertically.

Two Sciences – page 244

FOURTH DAY

SALVIATI. Once more, Simplicio is here on time; so let us without delay take up the question of motion. The text of our Author is as follows:

THE MOTION OF PROJECTILES

In the preceding pages we have discussed the properties of uniform motion and of motion naturally accelerated along planes of all inclinations. I now propose to set forth those properties which belong to a body whose motion is compounded of two other motions, namely, one uniform and one naturally accelerated; these properties, well worth knowing, I propose to demonstrate in a rigid manner. This is the kind of motion seen in a moving projectile; its origin I conceive to be as follows:

Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits. But if the plane is limited and elevated, then the moving particle, which we imagine to be a heavy one, will on passing over the edge of the plane acquire, in addition to its previous uniform and perpetual motion, a downward propensity due to its own weight; so that the resulting motion which I call projection [*projectio*], is compounded of one which is uniform and horizontal and of another which is vertical and naturally accelerated. We now proceed to demonstrate

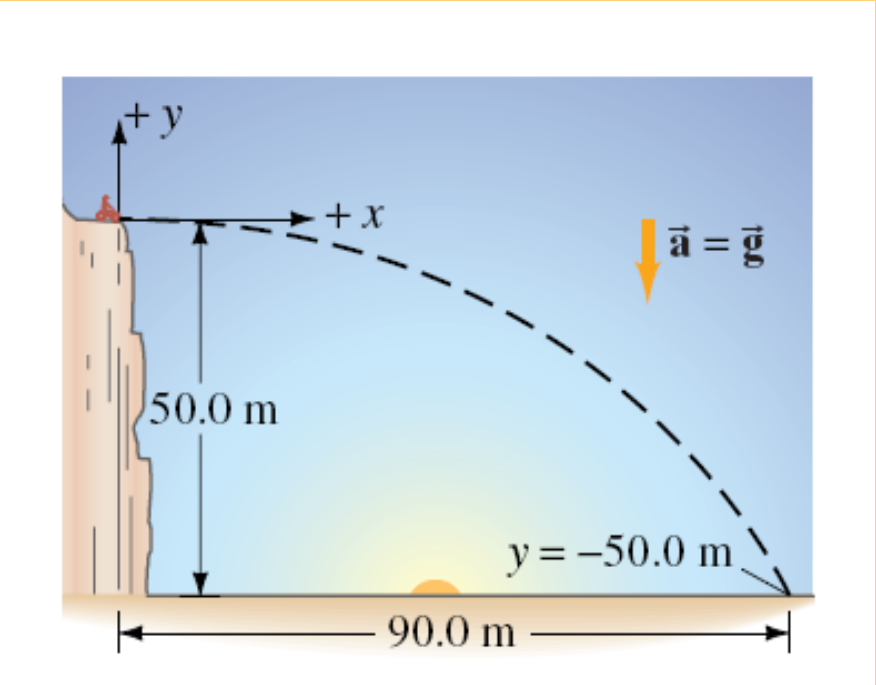
Example

Exercise C Two balls having different speeds roll off the edge of a horizontal table at the same time. Which hits the floor sooner, the faster or the slower ball?

They hit at the same time

Example

Ex. 3-4 A motorcycle speeds horizontally off a 50 m high cliff. How fast must it leave the cliff top to land on level ground 90 m from the base of the cliff?



+y

$x_0 = 0, \quad y_0 = 0,$

+x

$$y = -\frac{g}{2}t^2 \Rightarrow -50 = -\frac{9.8}{2}t^2$$

$$x = v_{x0}t \Rightarrow 90 = v_{x0}t$$

-y

$x_0 = 0, \quad y_0 = 0,$

+x

$$y = \frac{g}{2}t^2 \Rightarrow 50 = \frac{9.8}{2}t^2$$

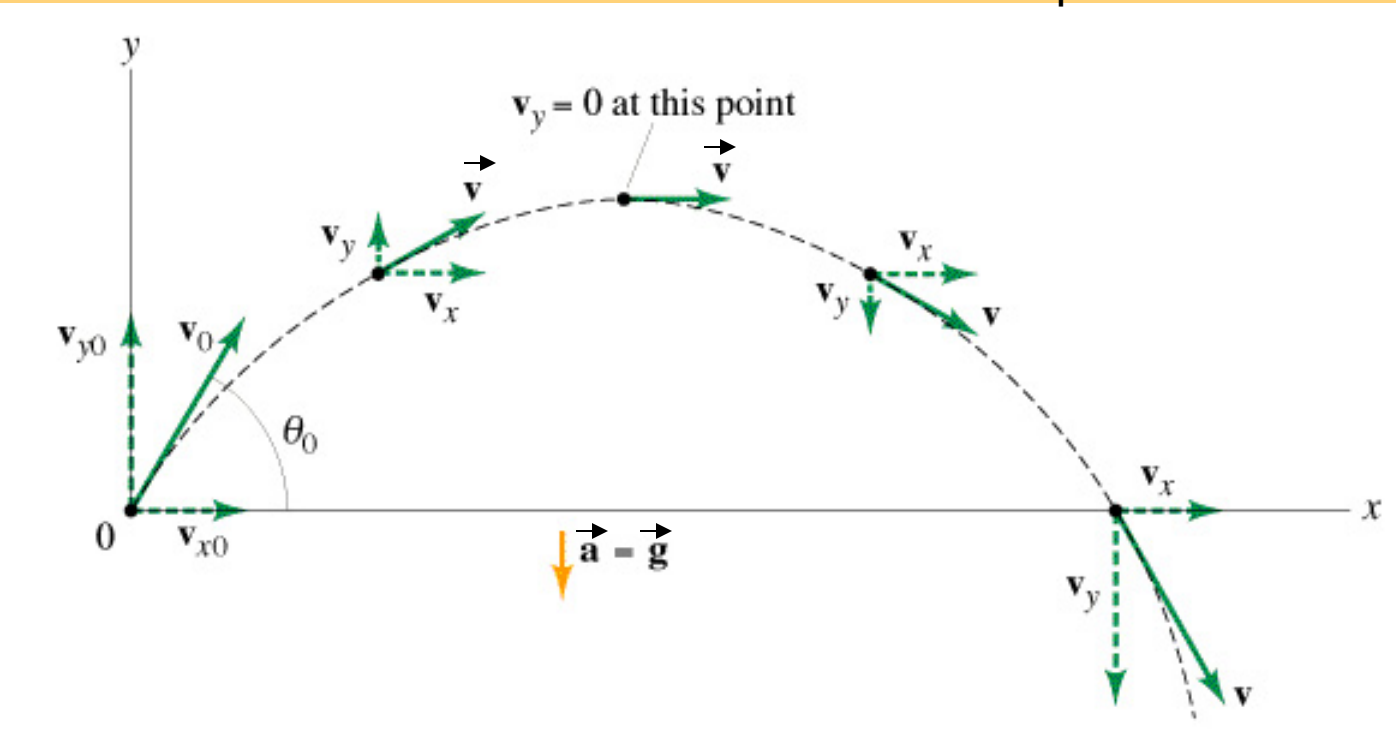
$$x = v_{x0}t \Rightarrow 90 = v_{x0}t$$

+y

When we choose $x_0 = 0, \quad y_0 = 0,$ then x and y give the displacements in these directions

Object projected upward

Now there are initial vertical and horizontal components of velocity



$$v_{x0} = v_0 \cos\theta$$

$$v_{y0} = v_0 \sin\theta$$

vertical motion

constant acceleration

$$v_{y0} \neq 0 \qquad a_y = -g$$

$$v_y = v_{y0} - gt \qquad y = v_{y0}t - \frac{g}{2}t^2$$

horizontal motion (no air resistance)

constant velocity

$$v_x = v_{x0} \qquad a_x = 0$$

$$x = v_{x0}t$$

Important Notes

The projectile is launched with an initial velocity that has components

$$v_{0x} = v_0 \cos \theta_0 \quad \text{and} \quad v_{0y} = v_0 \sin \theta_0.$$

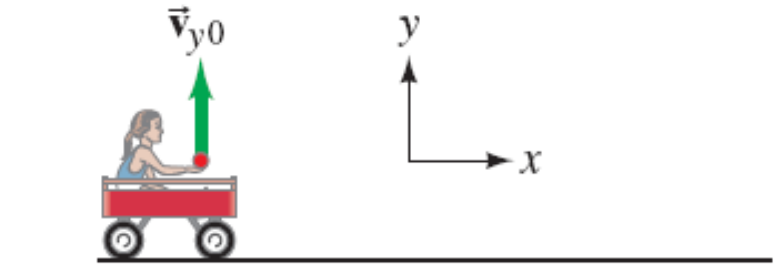
During its two-dimensional motion, the projectile's position vector $\vec{r}$ and velocity vector $\vec{v}$ change continuously, but its acceleration vector $\vec{a}$ is constant and *always* directed vertically downward. The projectile has *no* horizontal acceleration.

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

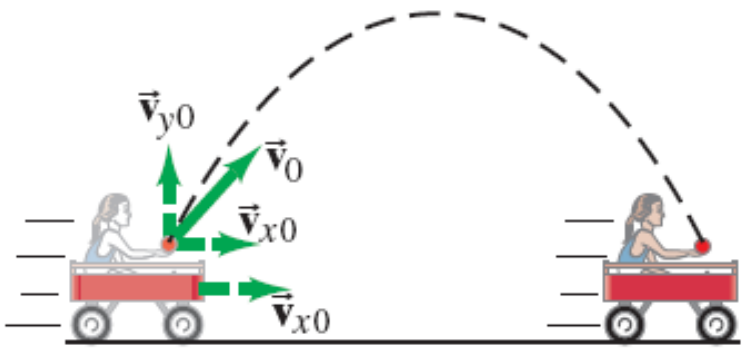
The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

Example

Discuss Ex. 3-6



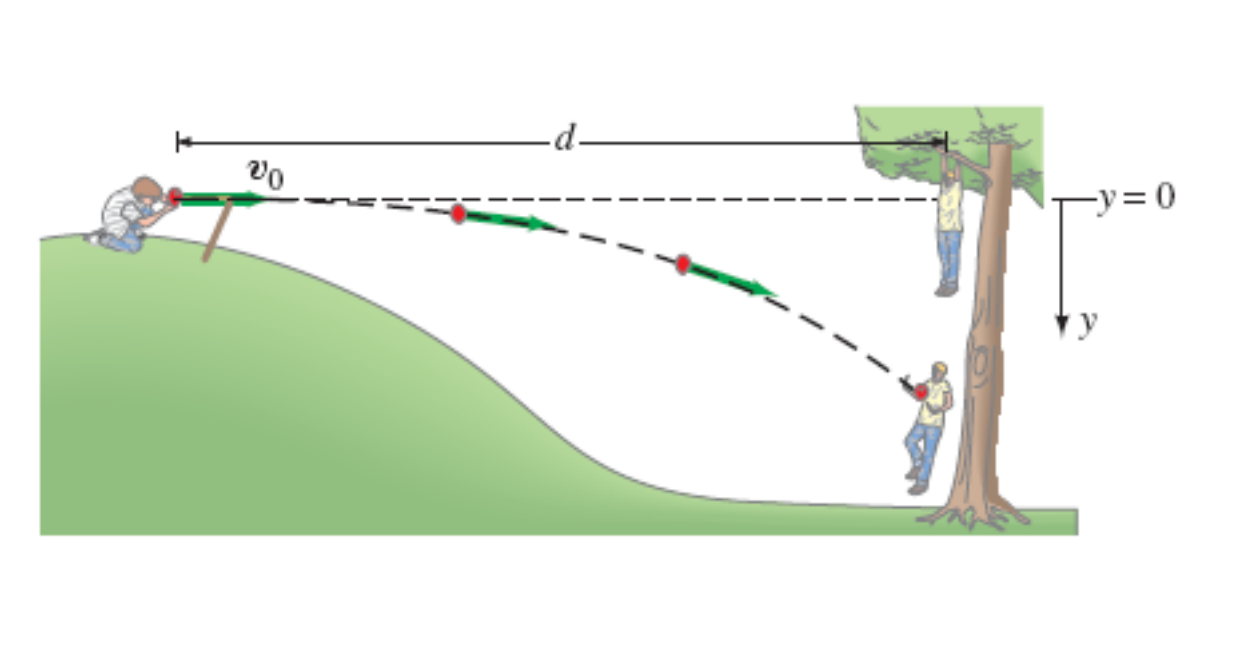
(a) Wagon reference frame



(b) Ground reference frame

Example

Discuss Ex. 3-7



Example

Where will the boy land?

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

Examples

Ex. 3-5 A football is kicked at an angle 37 degrees with velocity 20 m/s. Calculate: (a) the maximum height; (b) the time of travel before it hits the ground; (c) how far away it hits the ground; (d) the velocity vector at maximum height; (e) the acceleration vector at maximum height

$$a)\ v_y^2 = v_{y0}^2 - 2gy \Rightarrow y = \frac{v_{y0}^2}{2g} \qquad v_{x0} = v_0 \cos 37^o;\ v_{y0} = v_0 \sin 37^o$$

$$b)\ v_y = v_{y0} - gt \Rightarrow t = \frac{v_{y0}}{g} \Rightarrow 2t \qquad c)\ x = v_{x0} t$$

(d) there is no vertical component, only horizontal: 16m/s
(e) Acceleration is the same throughout, pointing downward

Read Ex. 3-8 and 3-9

Range (Ex. 3-8)

Derive a formula for the horizontal range R of a projectile in terms of its initial velocity v_0 and angle $theta_0$. The horizontal range is defined as the horizontal distance the projectile travels before returning to ***its original height***.

$$x_0 = 0, y_0 = 0, R = x = x_0 + v_{x0}t$$

$$\boxed{R \Rightarrow v_{x0}t = v_0 \cos\theta_0 t}$$

$$y = y_0 + v_0 \sin\theta_0 t - \frac{g}{2}t^2$$

$$0 = v_0 \sin\theta_0 t - \frac{g}{2}t^2$$

$$v_0 \sin\theta_0 t = \frac{g}{2}t^2$$

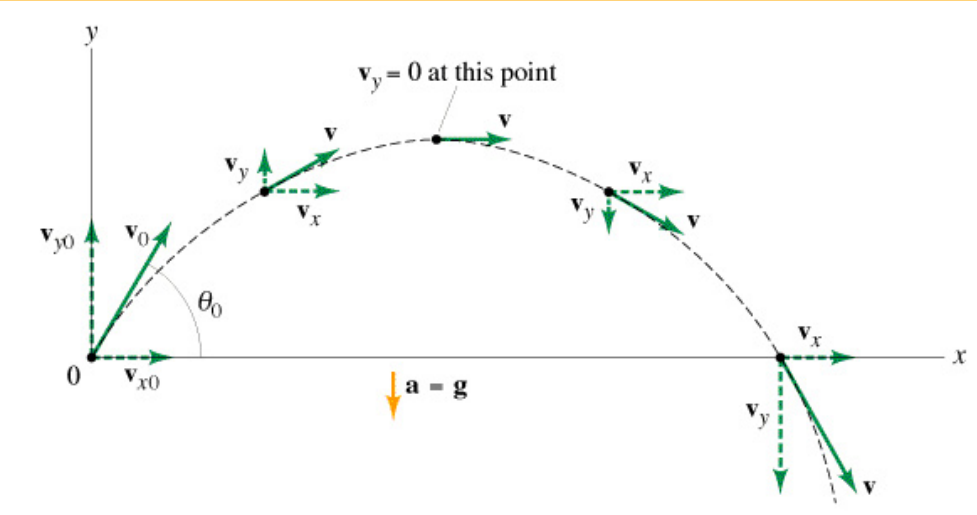
$$t = \frac{2v_0 \sin\theta_0}{g}$$

$$R = v_0 \cos\theta_0 \frac{2v_0 \sin\theta_0}{g} = \frac{v_0^2 2\cos\theta_0 \sin\theta_0}{g}$$

$$\boxed{R = \frac{v_0^2 \sin(2\theta_0)}{g}}$$

The horizontal range R is maximum for a launch angle of 45°.

Projectile Motion is Parabolic



$$y = v_{y0}t - \tfrac{1}{2}gt^2$$

$$x = v_{x0}t$$

$$t = \frac{x}{v_{x0}}$$

$$y = \frac{v_{y0}}{v_{x0}}x - \underbrace{\tfrac{1}{2}g\left(\frac{1}{v_{x0}}\right)^2 x^2 \qquad \frac{v_{y0}}{v_{x0}} = \tan\theta_0;\ v_{x0} = v_0\cos\theta_0}$$

$$y = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2 = Ax - Bx^2$$

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THIRD DAY

[190]

CHANGE OF POSITION. [*De Motu Locali*]

MY purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small; nevertheless I have discovered by experiment some properties of it which are worth knowing and which have not hitherto been either observed or demonstrated. Some superficial observations have been made, as, for instance, that the free motion [*naturalem motum*] of a heavy falling body is continuously accelerated;* but to just what extent this acceleration occurs has not yet been announced; for so far as I know, no one has yet pointed out that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity.†

It has been observed that missiles and projectiles describe a curved path of some sort; however no one has pointed out the fact that this path is a parabola. But this and other facts, not few in number or less worth knowing, I have succeeded in proving; and what I consider more important, there have been opened up to this vast and most excellent science, of which my

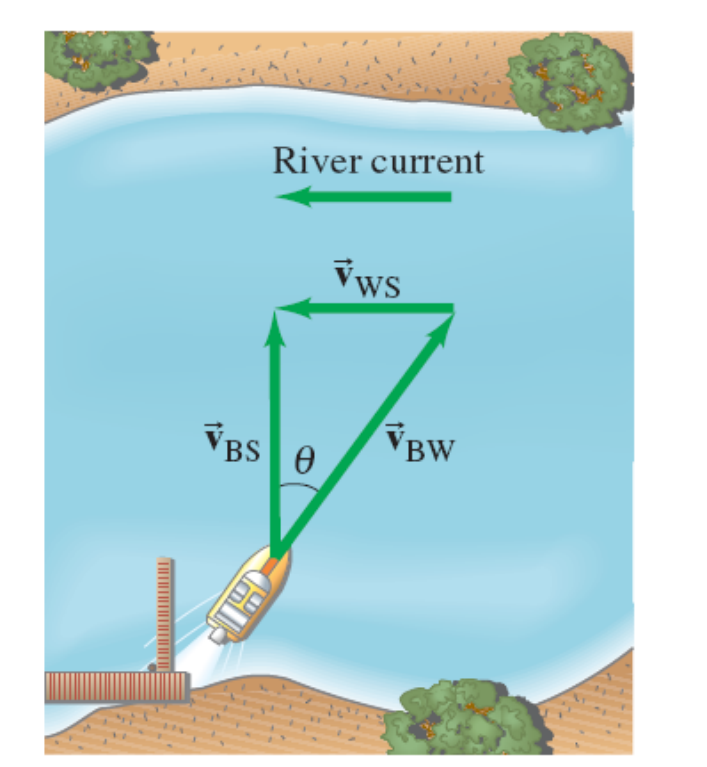
* "Natural motion" of the author has here been translated into "free motion"—since this is the term used to-day to distinguish the "natural" from the "violent" motions of the Renaissance. [*Trans.*]

† A theorem demonstrated on p. 175 below. [*Trans.*]

Relative Motion

Ex. 3-10

A man in a small boat is trying to cross a river that flows due west with a current. The man starts on the south bank and is trying to reach the north bank directly north from his starting point. In which direction should he head?

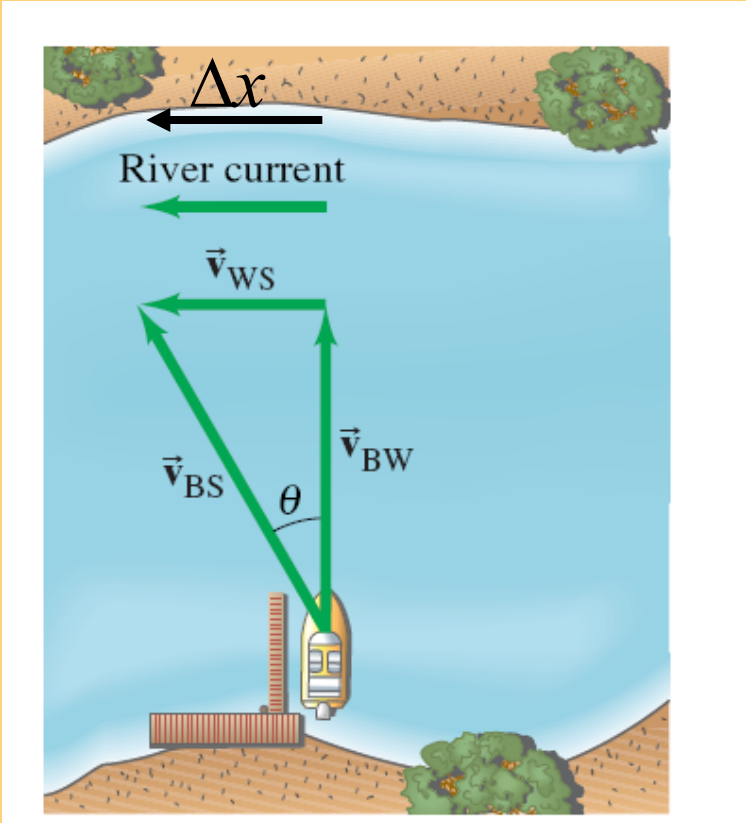


BW- boat with respect to the water
WS – water with respect to the shore

BS- boat with respect to the shore

Example

Ex. 3-12 A boat heads directly north from the south shore. The river has a current from east to west. Find the boat velocity relative to the shore. How far downstream will the boat arrive?



D

BW- boat with respect to the water
WS – water with respect to the shore

BS- boat with respect to the shore

$$\boldsymbol{v_{BW}} = 1.85\boldsymbol{m/s};\ \ \boldsymbol{v_{WS}} = 1.20\boldsymbol{m/s}$$

$$\boldsymbol{D} = 110\boldsymbol{m};$$

$$v_{BS} = ?;\ \ \theta = ?\ \ \boldsymbol{v_{BS}} = 2.21\boldsymbol{m/s};\ \ \theta = 33.0^{o}$$

$$x = ?\qquad \boldsymbol{x} = 72\boldsymbol{m}$$

Read Ex.3-11