Chapter 2

Mechanics – study the motion of objects (classical mechanics)

Galileo Galilei (1564-1642)
Uniformly accelerated motions
Telescope, astronomical observations
Copernican theory

Isaac Newton (1642-1727)
Universal gravitation
Laws of motion, calculus (Liebniz)
Optics

“Eppur se muove”
Bertold Brecht – Galileo

Dava Sobel
Galileo’s Daughter
Longitude
Chapter 2

Galileo Galilei (1564-1642)

Émilie du Châtelet (1706-1749)

Isaac Newton (1642-1727)
Mechanics – study the motion of objects
(classical mechanics)

Kinematics – how objects move
Dynamics – deals with forces and why objects move the way they do

Kinematics

**Translational** motion [as in (a)]
no rotation [as in (b)]

**One-dimensional** translational motion (straight line)

Idealized **particle**
(mathematical point with no spatial extent)

Idealizations, simplifications are common in modern science
Any measurement of position, distance, or speed is made with respect to a **reference frame** or **frame of reference**.

One-dimensional motion: **position** given by $x$ coordinate (horizontal motion) or $y$ coordinate (vertical motion).

To specify the **translational** motion of an object one needs:
- set of **coordinate axes**
- direction
- speed
Distance vs. Displacement

**Distance:**
how much the object traveled
It is a SCALAR (=number) with units
Its value is always positive

**Displacement:**
how far the object is from its starting point
has *magnitude* and *direction* - VECTOR
In one dimension, its direction is defined by a sign (+ or -)

\[ \Delta x = x_2 - x_1 \]

- $x_2$ - final position
- $x_1$ - initial position
**Speed and Velocity**

**Average speed:** total distance divided by time elapsed

\[
\text{average speed} = \frac{\text{distance}}{\Delta t}
\]

Units: m/s

Example: 70 m east and 30 m west, time elapsed = 10 s

Average speed = 10 m/s;

magnitude of the average velocity = 4 m/s
direction: + (positive), East, to the right

**Average velocity:** total displacement divided by time elapsed

\[
\vec{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}
\]

Direction of the average velocity = direction of displacement

Average speed and average velocity have the same magnitude if the motion is all in one direction.
1) (a) During a 3.00s time interval, a runner’s position changes from 30.5m to 50.0m. What is the runner’s average velocity?

(b) During a 3.00s time interval, a runner’s position changes from 50.0m to 30.5m. What is the runner’s average velocity?

2) How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18km/h?

3) A boat can move at 30 km/h in still water. How long will it take to move 12 km upstream in a river flowing 6.0 km/h?

1) (a) $+6.50 \text{ m/s}$ (magnitude and direction)

   [Motion/displacement in the POSITIVE direction, average velocity is POSITIVE]

(b) $-6.50 \text{ m/s}$ (magnitude and direction)

   [Motion/displacement in the NEGATIVE direction, average velocity is NEGATIVE]

2) 45 km

3) 30 min
Uniform velocity

In one dimension: direction is + or -

West = - \( x \)

East = + \( x \)

Vector \( \rightarrow \) has positive direction

Vector \( \leftarrow \) has negative direction

\[
\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\text{displacement}}{\text{elapsed time}}
\]

Direction of the average velocity = direction of displacement

The magnitude is the absolute value

Uniform velocity:
constant \( v = \bar{v} \)

Linear equation in time

\[
\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} \Rightarrow v = \frac{x - x_0}{t}
\]

\[
x - x_0 = vt \Rightarrow x = x_0 + vt
\]
This is a graph of position $x$ versus time $t$ for a stationary object.

Same position for any time.

Average velocity is zero.
Red curve: the average velocity is the slope of the line. The instantaneous velocity is constant.

Blue curve: the instantaneous velocity is NOT constant.
Instantaneous velocity = the average velocity during an infinitesimal short time (VECTOR)

\[ v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \]

Instantaneous speed = magnitude of the instantaneous velocity

Object at **UNIFORM** (=constant) velocity then
instantaneous velocity = average velocity

**Velocity** = instantaneous velocity
vs.
Average velocity

(Instantaneous) **speed** = magnitude of the (instantaneous) velocity
Acceleration

Average acceleration = change in velocity divided by the elapsed time
it is a VECTOR

\[ a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \]

Instantaneous acceleration

\[ a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \]

ATTENTION!!!
direction of acceleration is not necessarily the same as the direction of displacement

Acceleration tells how quickly the velocity changes
Velocity tells how quickly the position changes

Units: \( \text{m/s}^2 \)

Deceleration DOES NOT NECESSARILY mean that the acceleration is negative
1) A car accelerates along a straight line from rest to 36km/h in 5.0s. What is magnitude of its average acceleration? (Careful with units)

2) A car accelerates along a straight line from rest to -36km/h in 5.0s. What is magnitude of its average acceleration? (Careful with units)

3) A car is moving in the positive direction. The driver puts on the brakes. If the initial velocity is 15m/s and it takes 5.0s to slow down to 5.0m/s, what is the car's average acceleration?

4) Same as (3), but the car is moving in the negative direction.

5) (a) If the acceleration is zero, does it mean that the velocity is zero? (b) If the velocity is zero, does it mean that the acceleration is zero?

1) + 2.0 m/s² (increase MAGNITUDE of velocity in the POSITIVE direction)
2) - 2.0 m/s² (increase MAGNITUDE of velocity in the NEGATIVE direction)
3) - 2.0 m/s² (decrease MAGNITUDE of velocity in the POSITIVE direction)
4) + 2.0 m/s² (decrease MAGNITUDE of velocity in the NEGATIVE direction)

5) (a) and (b) Not necessarily
Direction of the acceleration

In one dimension: direction is + or -

- West = - x 0
- East = + x

Vector has positive direction
Vector has negative direction

The magnitude is the absolute value

\[ \vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\text{change in velocity}}{\text{elapsed time}} \]

Magnitude of velocity is increasing:
Direction of the average acceleration = direction of the velocity (displacement)

Magnitude of velocity is decreasing:
Direction of the average acceleration is opposite to the direction of the velocity (displacement)

Uniform acceleration:
(constant) \[ a = \vec{a} \]

BUT velocity is NOT constant!! \[ \vec{v} \neq \vec{\bar{v}} \]
Constant Acceleration

Magnitude of acceleration is constant: instantaneous and average acceleration are equal

\[
\begin{align*}
\bar{a} &= a; \quad a = \frac{v - v_0}{t} \\
&\implies v = v_0 + at
\end{align*}
\]

(Linear in time) \hspace{1cm} (average velocity = midway)

\[
\bar{v} = \frac{x - x_0}{t}; \quad \bar{v} = \frac{v_0 + v}{2}; \quad v = v_0 + at
\]

\[
\implies x = x_0 + v_0 t + \frac{1}{2} at^2
\]

(Quadratic in time)

\[
\begin{align*}
x &= x_0 + \bar{v}t; \quad \bar{v} = \frac{v_0 + v}{2}; \quad v = v_0 + at \\
&\implies v^2 = v_0^2 + 2a(x - x_0)
\end{align*}
\]

(useful when time is unknown)

Classical mechanics: give us the initial conditions and we can predict the motion of any particle
Linear vs. Quadratic

\[ v = v_0 + at \]
\[ v_0 = 0; \quad a = 2m/s^2 \]

\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]
\[ x_0 = 0 \quad v_0 = 0 \quad a = 2m/s^2 \]
1. How long does it take a car to cross a 36.0 m wide intersection after the light turns green, if the car accelerates from rest at a constant acceleration of 2.00 \( m/s^2 \)?

2. Design an airport for small planes. One kind of airplane that uses this airfield must reach a speed before takeoff of at least 30.0 m/s and can accelerate at 4.50 \( m/s^2 \). (a) If the runway is 81 m long, can this plane reach the required speed for take off? (b) If not, what minimum length must the runway have?

1) \( t = 6.00 \) s

2) (a) No, because it only reaches 27 m/s
(b) 100 m
Falling Objects

Free fall – uniformly accelerated motion (constant acceleration)

Galileo: Objects increase their speed as they fall

\[ d \propto t^2 \]

and experiments to support it

Air acts as a resistance to very light objects with a large surface, but in the absence of resistance all objects fall with the same constant acceleration
Same constant acceleration

**Free fall** – uniformly accelerated motion  \( \text{(constant acceleration)} \)

Heavy objects **DO NOT** fall faster than lighter objects
(mass does not appear in the equations)

*All* objects fall with the **same constant acceleration**
in the absence of air or any other resistance

Galileo is the father of **modern science** not only for the content of his science but also for his approach:

idealization, simplification, theory, experiments
Acceleration due to gravity is a **VECTOR** and its direction is toward the center of Earth

\[ g = 9.80 \text{ m/s}^2 \]

Here, we neglect the effects of air resistance

\[
\begin{align*}
 x &= x_0 + v_0 t + \frac{1}{2} a t^2 \\
y &= y_0 + v_0 t + \frac{1}{2} g t^2 \\
v &= v_0 + gt \\
v^2 &= v_0^2 + 2g(y - y_0) \\
\bar{v} &= \frac{v + v_0}{2}
\end{align*}
\]

*It is arbitrary to choose \( y \) positive in the upward or downward direction; but we must be consistent*
Free Fall - exercises

\[ g = 9.80 \ m / s^2 \]

1. A ball is dropped from a tower 70.0 m high. How far will the ball have fallen after 1.00s and after 2.00s? What will be the magnitude of its velocity at these points?

2. Now consider the same exercise, but suppose the ball is thrown downward with an initial velocity of 3.00m/s

3. A person throws a ball upward with initial velocity 15.0m/s. Calculate (a) how high it goes, (b) how long it takes to reach the maximum height, (c) how long the ball is in the air before it comes back to his hand again, (d) the velocity of the ball when it returns to the thrower’s hand, (e) the time the ball passes a point 8.00 m above the person’s hand.

1) 1s fell 4.9 m; 2s fell 19.6 m; \( v(1s) = 9.80 \ m/s; v(2s) = 19.6 \ m/s \)

2) 1s fell 7.90 m; 2s fell 25.6 m; \( v(1s) = 12.8 \ m/s; v(2s) = 22.6 \ m/s \)

3) (a) 11.5 m; (b) 1.53s; (c) 3.06s; (d) \( -15.0 \ m/s \); (e) 0.16s and 2.37s
A stone is thrown vertically upward with a speed of 12.0 m/s from the edge of a cliff 70.0 m high.

(a) How much later does it reach the bottom of the cliff? -2.7 s 5.2 s
(b) What is its speed just before hitting? 38.9 m/s (X)
(c) What total distance did it travel? 84.7 m

0 at the top of the cliff (more convenient to choose 0 where the motion starts)
70 m at the bottom of the cliff
initial velocity upward is negative
$g$ is positive

0 at the bottom of the cliff
70 m at the edge of the cliff
$g$ is negative
initial velocity upward is positive

In both frames: magnitudes are the same, but directions may change
Graphical Analysis of Linear Motion

The axes of any graph must have units

Slope of the graph of position vs. time gives the velocity

*uniform velocity*
Acceleration is zero throughout
Green: object moves with constant velocity in the positive direction
Blue: object moves with constant velocity in the positive direction
Here, the velocity is greatest, because the slope is the highest
Black: the object stopped from 4 s to 6 s
Red: the object moves with constant velocity in the negative direction
At 8 s: displacement=50m, total distance traveled=70m
The axes of any graph must have units.

Slope of the graph of velocity vs. time gives the acceleration.

**uniform acceleration**
Constant Acceleration

Green: + displacement, + velocity  
magnitude of velocity increases, + acceleration, constant acceleration
Blue:  + displacement, + velocity  
constant velocity, acceleration=0
Black: + displacement, + velocity  
magnitude of velocity decreases, - acceleration, constant acceleration
Yellow: the object stops
Red: - displacement, - velocity  
magnitude of velocity increases, - acceleration, constant acceleration

Black: largest magnitude of the acceleration – highest slope
Maximum magnitude of the velocity is 120m/s