Linear Momentum

Conservation of
- energy
- ***linear momentum***
- angular momentum
- electric charge

To study collisions: Conservation of energy + Conservation of ***linear momentum***

Linear momentum: (VECTOR)

$$\vec{p} = m\vec{v}$$

SI unit: kg.m/s

p(faster car)>p(slower car)
p(heavy truck)>p(light car) – both at the same speed

Newton's 2nd Law

A force is needed to change the momentum
- increase it
- decrease it
- change its direction

Newton's 2nd law – originally in terms of momentum:

$$\sum \vec{F} = m\vec{a} \Leftrightarrow \boxed{\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}}$$

(applies even when the mass changes)

$$\frac{\Delta \vec{p}}{\Delta t} = \frac{m(\vec{v}_2 - \vec{v}_1)}{\Delta t} = ma$$

Ex.7-1 A 0.060 kg ball leaves the racket with v=55m/s. It is in contact with the racket for 4ms. What is the average force on the ball? Compare it with the weight of a 60-kg person.

F~800N

Newton's 2nd Law

Newton's 2nd law – originally in terms of momentum:

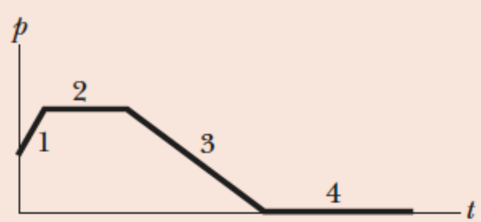
The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \quad (9\text{-}23)$$

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}.$$

CHECKPOINT 3

The figure gives the magnitude p of the linear momentum versus time t for a particle moving along an axis. A force directed along the axis acts on the particle. (a) Rank the four regions indicated according to the magnitude of the force, greatest first. (b) In which region is the particle slowing?

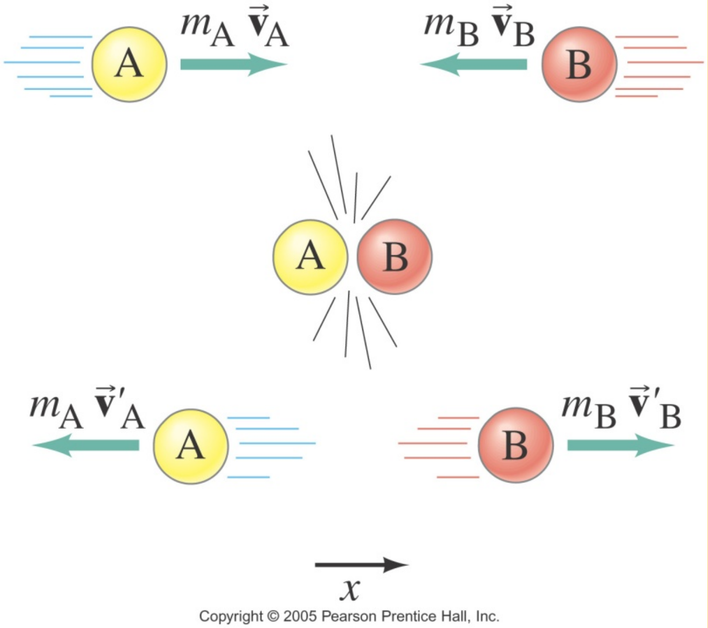


Conservation of Momentum

If the system is ***ISOLATED*** (the net external force on the system is zero) then the sum of momenta before and after a collision is the same.

$$m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B = m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B$$

Head-on collision:



$$\Delta\vec{p} = \vec{F}\Delta t$$

$$\Delta\vec{p}_B = m_B(\vec{v}'_B - \vec{v}_B) = \vec{F}_{BA}\Delta t$$

$$\Delta\vec{p}_A = m_A(\vec{v}'_A - \vec{v}_A) = \vec{F}_{AB}\Delta t = -\vec{F}_{BA}\Delta t$$

$$m_A(\vec{v}'_A - \vec{v}_A) = -m_B(\vec{v}'_B - \vec{v}_B)$$

Isolated System

Isolated system = net external force is zero,
only internal forces are significant

In the real world:
external forces are
friction, gravity, etc

Examples:

system: rock falling – momentum **is not conserverd**
(external force = gravity)

system: rock + Earth – momentum **is conserverd**

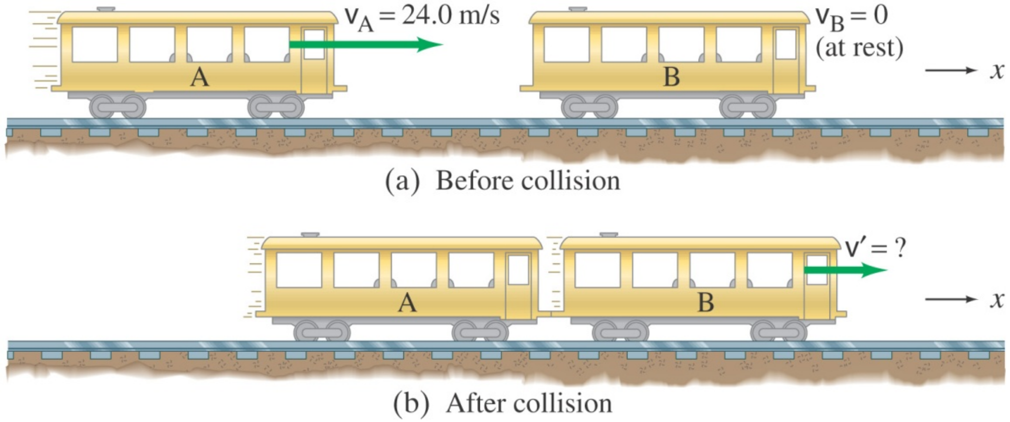
CHECKPOINT 6

An initially stationary device lying on a frictionless floor explodes into two pieces, which then slide across the floor. One piece slides in the positive direction of an x axis. (a) What is the sum of the momenta of the two pieces after the explosion? (b) Can the second piece move at an angle to the x axis? (c) What is the direction of the momentum of the second piece?

Isolated System

Collision in one dimension

Ex.: A 10,000-kg railroad car A traveling at 24.0 m/s strikes an identical car B at rest. If they lock together, what is their common speed afterwards?



$$v' = \frac{m_A}{m_A + m_B} v_A$$

$$m_A >> m_B \Rightarrow v' = v_A$$

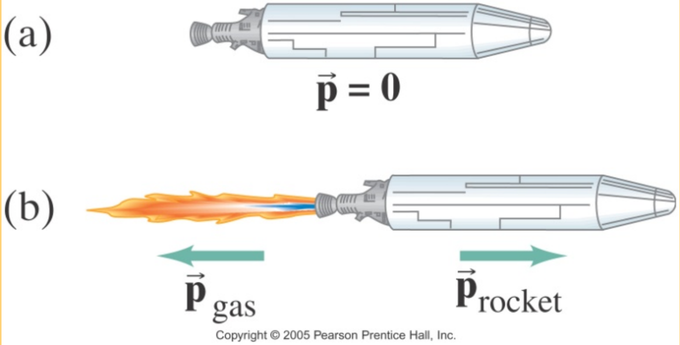
$$m_A << m_B \Rightarrow v' = 0$$

Examples

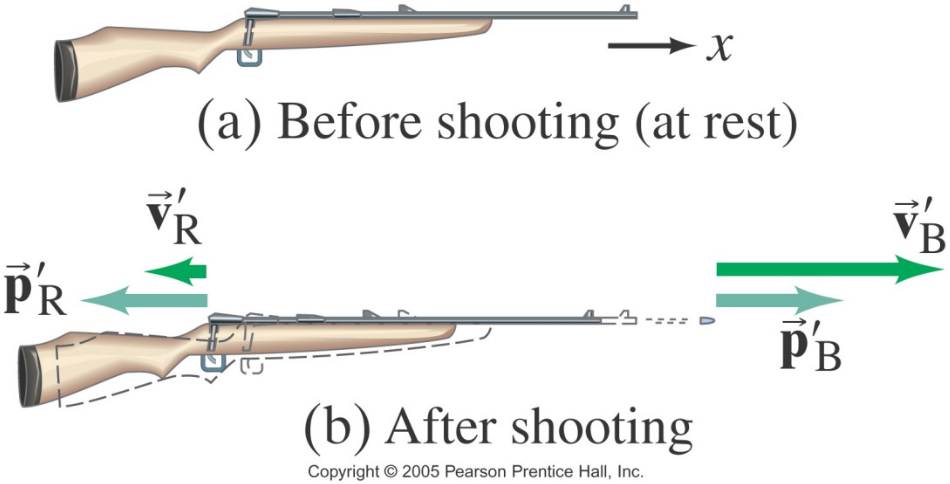
Momentum conservation works for a rocket as long as we consider the rocket and its fuel to be one system, and account for the mass loss of the rocket.

In the reference frame of the rocket:

When the fuel burns and gases are expelled:
the rocket gains momentum,
it can accelerate in empty space.



Ex. Calculate the recoil **velocity** of a 5.0-kg rifle that shoots a 0.020-kg bullet at a speed of 620 m/s



$$v'_R = -2.5 m/s$$

Examples

One-dimensional explosion, relative velocity, space hauler

One-dimensional explosion: Figure 9-12*a* shows a space hauler and cargo module, of total mass M, traveling along an x axis in deep space. They have an initial velocity $\vec{v}_i$ of magnitude 2100 km/h relative to the Sun. With a small explosion, the hauler ejects the cargo module, of mass $0.20M$ (Fig. 9-12*b*). The hauler then travels 500 km/h faster than the module along the x axis; that is, the relative speed v_{rel} between the hauler and the module is 500 km/h. What then is the velocity $\vec{v}_{HS}$ of the hauler relative to the Sun?

KEY IDEA

Because the hauler–module system is closed and isolated, its total linear momentum is conserved; that is,

$$\vec{P}_i = \vec{P}_f, \qquad (9\text{-}44)$$

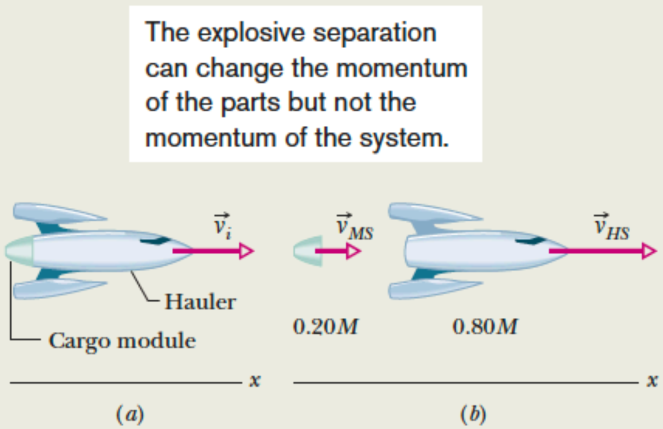


Fig. 9-12 (*a*) A space hauler, with a cargo module, moving at initial velocity $\vec{v}_i$. (*b*) The hauler has ejected the cargo module. Now the velocities relative to the Sun are $\vec{v}_{MS}$ for the module and $\vec{v}_{HS}$ for the hauler.

where the subscripts i and f refer to values before and after the ejection, respectively.

Calculations: Because the motion is along a single axis, we can write momenta and velocities in terms of their x components, using a sign to indicate direction. Before the ejection, we have

$$P_i = Mv_i. \qquad (9\text{-}45)$$

Let v_{MS} be the velocity of the ejected module relative to the Sun. The total linear momentum of the system after the ejection is then

$$P_f = (0.20M)v_{MS} + (0.80M)v_{HS}, \qquad (9\text{-}46)$$

where the first term on the right is the linear momentum of the module and the second term is that of the hauler.

We do not know the velocity v_{MS} of the module relative to the Sun, but we can relate it to the known velocities with

$$\begin{pmatrix}\text{velocity of}\\ \text{hauler relative}\\ \text{to Sun}\end{pmatrix} = \begin{pmatrix}\text{velocity of}\\ \text{hauler relative}\\ \text{to module}\end{pmatrix} + \begin{pmatrix}\text{velocity of}\\ \text{module relative}\\ \text{to Sun}\end{pmatrix}.$$

In symbols, this gives us

$$v_{HS} = v_{rel} + v_{MS} \qquad (9\text{-}47)$$

or

$$v_{MS} = v_{HS} - v_{rel}.$$

Substituting this expression for v_{MS} into Eq. 9-46, and then substituting Eqs. 9-45 and 9-46 into Eq. 9-44, we find

$$Mv_i = 0.20M(v_{HS} - v_{rel}) + 0.80Mv_{HS},$$

which gives us

$$v_{HS} = v_i + 0.20v_{rel},$$

or

$$v_{HS} = 2100 \text{ km/h} + (0.20)(500 \text{ km/h})$$
$$= 2200 \text{ km/h}. \qquad \text{(Answer)}$$

Examples

Two-dimensional explosion, momentum, coconut

Two-dimensional explosion: A firecracker placed inside a coconut of mass M, initially at rest on a frictionless floor, blows the coconut into three pieces that slide across the floor. An overhead view is shown in Fig. 9-13*a*. Piece C, with mass $0.30M$, has final speed $v_{fC} = 5.0$ m/s.

(a) What is the speed of piece B, with mass $0.20M$?

KEY IDEA

First we need to see whether linear momentum is conserved. We note that (1) the coconut and its pieces form a closed system, (2) the explosion forces are internal to that system, and (3) no net external force acts on the system. Therefore, the linear momentum of the system is conserved.

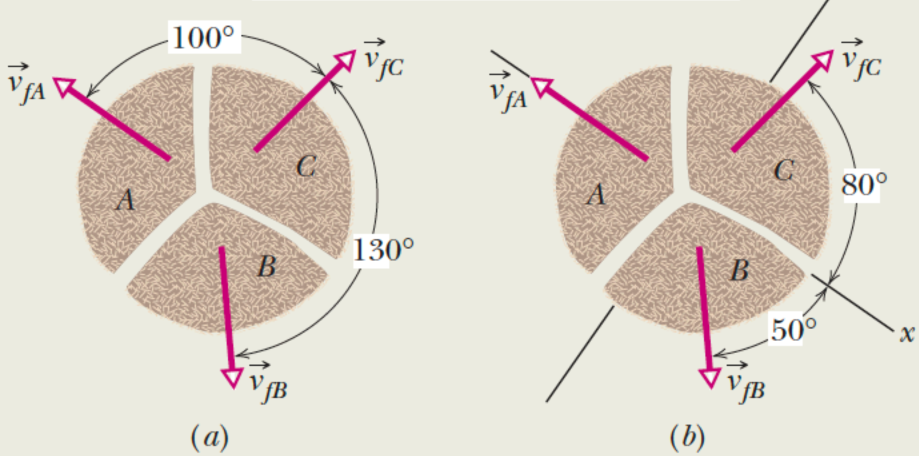
Calculations: To get started, we superimpose an xy coordinate system as shown in Fig. 9-13*b*, with the negative direction of the x axis coinciding with the direction of $\vec{v}_{fA}$. The x axis is at 80° with the direction of $\vec{v}_{fC}$ and 50° with the direction of $\vec{v}_{fB}$.

Linear momentum is conserved separately along each axis. Let's use the y axis and write

$$P_{iy} = P_{fy}, \qquad (9\text{-}48)$$

where subscript i refers to the initial value (before the explosion), and subscript y refers to the y component of $\vec{P}_i$ or $\vec{P}_f$.

The explosive separation can change the momentum of the parts but not the momentum of the system.



NOTE the choice of reference frame!!

Examples

The component P_{iy} of the initial linear momentum is zero, because the coconut is initially at rest. To get an expression for P_{fy}, we find the y component of the final linear momentum of each piece, using the y-component version of Eq. 9-22 ($p_y = mv_y$):

$$p_{fA,y} = 0,$$
$$p_{fB,y} = -0.20Mv_{fB,y} = -0.20Mv_{fB}\sin 50^\circ,$$
$$p_{fC,y} = 0.30Mv_{fC,y} = 0.30Mv_{fC}\sin 80^\circ.$$

(Note that $p_{fA,y} = 0$ because of our choice of axes.) Equation 9-48 can now be written as

$$P_{iy} = P_{fy} = p_{fA,y} + p_{fB,y} + p_{fC,y}.$$

The explosive separation can change the momentum of the parts but not the momentum of the system.

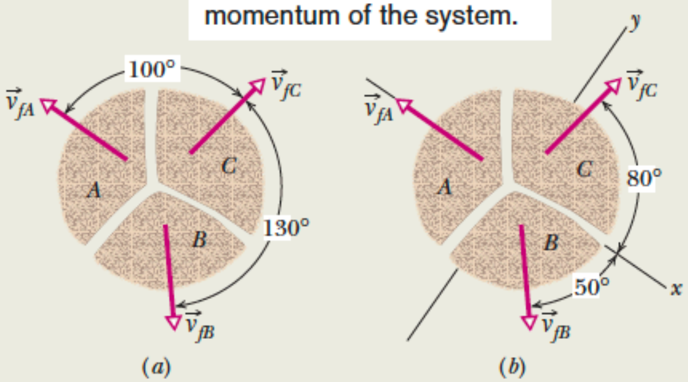


Fig. 9-13 Three pieces of an exploded coconut move off in three directions along a frictionless floor. (*a*) An overhead view of the event. (*b*) The same with a two-dimensional axis system imposed.

Then, with $v_{fC} = 5.0$ m/s, we have

$$0 = 0 - 0.20Mv_{fB}\sin 50^\circ + (0.30M)(5.0 \text{ m/s})\sin 80^\circ,$$

from which we find

$$v_{fB} = 9.64 \text{ m/s} \approx 9.6 \text{ m/s}. \qquad \text{(Answer)}$$

(b) What is the speed of piece A?

Calculations: Because linear momentum is also conserved along the x axis, we have

$$P_{ix} = P_{fx}, \qquad (9\text{-}49)$$

where $P_{ix} = 0$ because the coconut is initially at rest. To get P_{fx}, we find the x components of the final momenta, using the fact that piece A must have a mass of $0.50M$ $(= M - 0.20M - 0.30M)$:

$$p_{fA,x} = -0.50Mv_{fA},$$
$$p_{fB,x} = 0.20Mv_{fB,x} = 0.20Mv_{fB}\cos 50^\circ,$$
$$p_{fC,x} = 0.30Mv_{fC,x} = 0.30Mv_{fC}\cos 80^\circ.$$

Equation 9-49 can now be written as

$$P_{ix} = P_{fx} = p_{fA,x} + p_{fB,x} + p_{fC,x}.$$

Then, with $v_{fC} = 5.0$ m/s and $v_{fB} = 9.64$ m/s, we have

$$0 = -0.50Mv_{fA} + 0.20M(9.64 \text{ m/s})\cos 50^\circ + 0.30M(5.0 \text{ m/s})\cos 80^\circ,$$

from which we find

$$v_{fA} = 3.0 \text{ m/s}. \qquad \text{(Answer)}$$

Cons. of Energy and Momentum

No net external force: conservation of momentum

No loss of energy: **conservation of kinetic energy** --- ***ELASTIC*** collision
(like heat or sound)

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B$$

Collisions of atoms and molecules are often elastic
Collisions of billiard balls are close to it

INELASTIC collision: kinetic energy is not conserved, part of it is lost into other forms

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B + \textbf{thermal and others}$$

NOTE: TOTAL energy is always conserved !

Elastic Collision – 1D

Conservation of momentum: $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$

Conservation of kinetic energy: $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B$

Simplification valid only for **head-on elastic collision**

Magnitude of relative speed of the two objects after the collision = before the collision, but opposite sign ---- no matter what the masses are

$$\frac{1}{2}m_A v_A^2 - \frac{1}{2}m_A v'^2_A = -\frac{1}{2}m_B v_B^2 + \frac{1}{2}m_B v'^2_B \qquad \frac{1}{2}m_A(v_A^2 - v'^2_A) = -\frac{1}{2}m_B(v_B^2 - v'^2_B)$$

$$\frac{1}{2}m_A(v_A - v'_A)(v_A + v'_A) = -\frac{1}{2}m_B(v_B - v'_B)(v_B + v'_B)$$

$$m_A(v_A - v'_A) = -m_B(v_B - v'_B)$$

$$v_A + v'_A = v'_B + v_B$$

$$v_A - v_B = -(v'_A - v'_B)$$

Examples

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B$$

$$v_A - v_B = -(v'_A - v'_B)$$

Ex. 7-7 A billiard ball A of mass m with speed v collides head-on with ball B of equal mass at rest. What are the speeds of the two balls after the collision, assuming it is elastic?

$$v'_A = 0, \quad v'_B = v$$

Ex. 7-8 A proton of mass 1.01 u (unified atomic mass units) traveling with a speed of 3.60x10^4 m/s has an elastic head-on collision with a helium nucleus, (mass = 4.00 u) initially at rest. What are the velocities of the proton and helium nucleus after the collision?

$$v'_{He} = 1.45\times 10^4 \, m/s$$

$$v'_p = -2.15\times 10^4 \, m/s$$

Proton reverses its direction after collision

Examples

Elastic collision, two pendulums

Two metal spheres, suspended by vertical cords, initially just touch, as shown in Fig. 9-20. Sphere 1, with mass $m_1 = 30$ g, is pulled to the left to height $h_1 = 8.0$ cm, and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2, whose mass $m_2 = 75$ g. What is the velocity v_{1f} of sphere 1 just after the collision?

KEY IDEA

We can split this complicated motion into two steps that we can analyze separately: (1) the descent of sphere 1 (in which mechanical energy is conserved) and (2) the two-sphere collision (in which momentum is also conserved).

Step 1: As sphere 1 swings down, the mechanical energy of the sphere–Earth system is conserved. (The mechanical energy is not changed by the force of the cord on sphere 1 because that force is always directed perpendicular to the sphere's direction of travel.)

Calculation: Let's take the lowest level as our reference level of zero gravitational potential energy. Then the kinetic energy of sphere 1 at the lowest level must equal the gravitational potential energy of the system when sphere 1 is at height h_1. Thus,

$$\tfrac{1}{2}m_1 v_{1i}^2 = m_1 g h_1,$$

which we solve for the speed v_{1i} of sphere 1 just before the collision:

$$v_{1i} = \sqrt{2gh_1} = \sqrt{(2)(9.8\ \text{m/s}^2)(0.080\ \text{m})} = 1.252\ \text{m/s}.$$

Step 2: Here we can make two assumptions in addition to the assumption that the collision is elastic. First, we can assume that the collision is one-dimensional because the motions of the spheres are approximately horizontal from just before the collision to just after it. Second, because the collision is so brief, we can assume that the two-sphere system is closed and isolated. This means that the total linear momentum of the system is conserved.

Calculation: Thus, we can use Eq. 9-67 to find the velocity of sphere 1 just after the collision:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{0.030\ \text{kg} - 0.075\ \text{kg}}{0.030\ \text{kg} + 0.075\ \text{kg}}(1.252\ \text{m/s}) = -0.537\ \text{m/s} \approx -0.54\ \text{m/s}. \quad \text{(Answer)}$$

The minus sign tells us that sphere 1 moves to the left just after the collision.

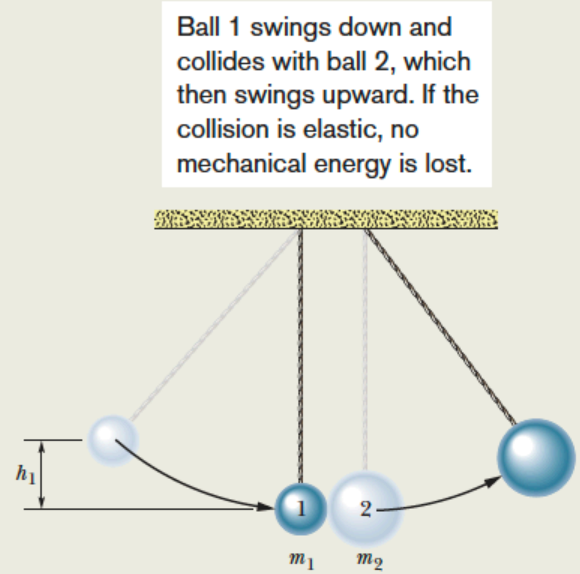


Fig. 9-20 Two metal spheres suspended by cords just touch when they are at rest. Sphere 1, with mass m_1, is pulled to the left to height h_1 and then released.

Inelastic Collision

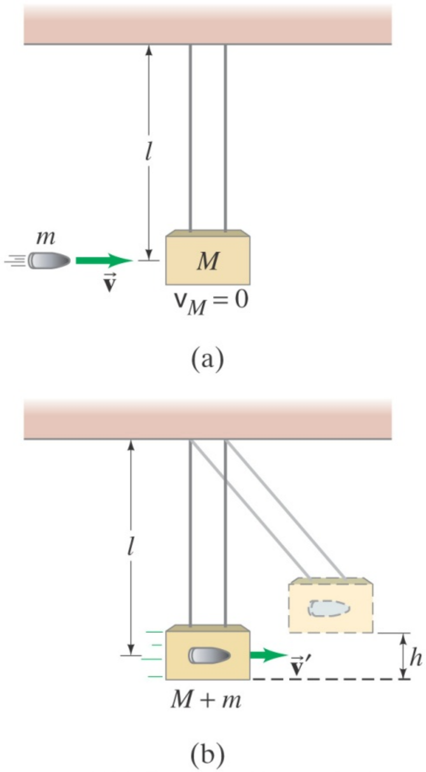
Inelastic collision: kinetic energy is not conserved, but <u>total energy</u> and total vector <u>momentum</u> are

COMPLETELY inelastic collision: ***two object stick together.***

Ex. 7-9: A 10,000-kg railroad car A traveling at 24.0 m/s strikes an identical car B at rest and they lock together. Calculate how much of the initial kinetic energy is transformed to thermal or other forms of energy. (after the collision both cars are moving at 12 m/s – see Ex. 7-3)

1.44 x 10^6 J

Ex. 7-10 Ballistic pendulum. A projectile of mass m is fired into a large block of mass M, which is suspended like a pendulum. As a result of the collision, the pendulum swings up to a maximum height h. What is the relationship between the initial horizontal speed of the projectile, v, and the maximum height h?



(gravity neglected during the short collision time)

$$v = \frac{m+M}{m}\sqrt{2gh}$$

Collision in 2D

Conservation of momentum:
(we need the angles)

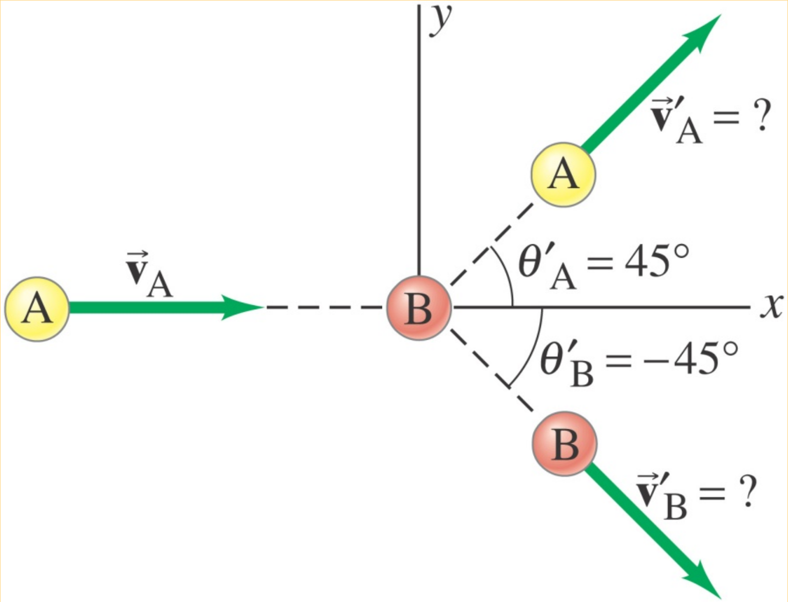
$$p_{Ax} + p_{Bx} = p'_{Ax} + p'_{Bx}$$

$$p_{Ay} + p_{By} = p'_{Ay} + p'_{By}$$

If the collision is elastic

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B$$

Three independent equations – we can find 3 unknowns



Ex.7-11 A more simple example, where there are only 2 unknowns:

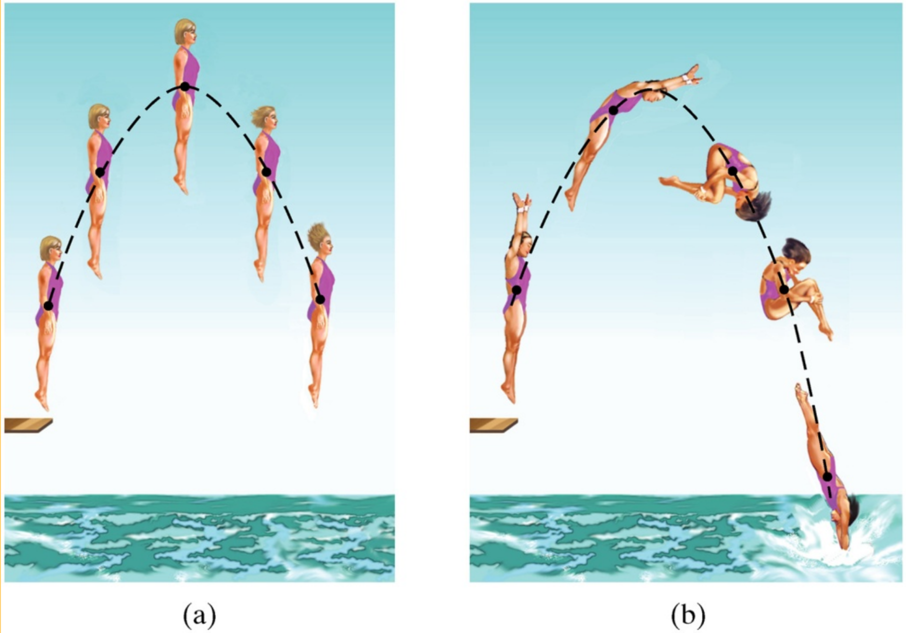
A billiard ball A at speed 3.0 m/s strikes an equal-mass ball B initially at rest. The angles are shown in the figure. What are the speeds of the two balls after the collision?

$$v'_A = v'_B = 2.1 m/s$$

Center of Mass

The motion of extended objects or a system with many objects may be complicated,

but the **CENTER OF MASS (CM)** moves as a point particle.



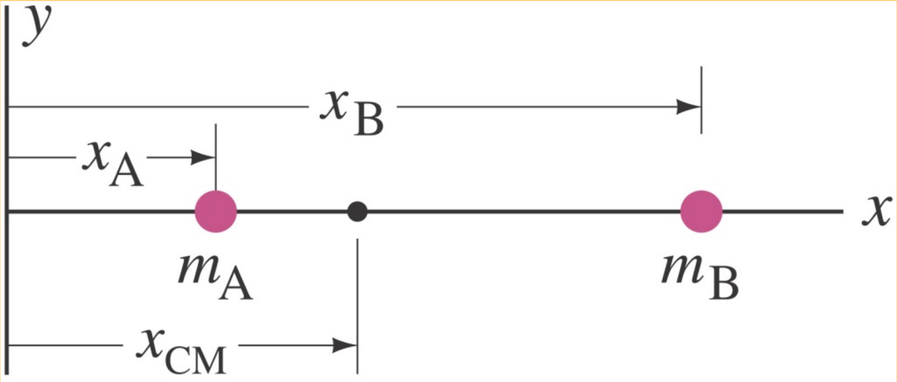
The general motion of an object can be considered as the sum of the translational motion of the CM, plus rotational, vibrational, or other forms of motion about the CM.

Center of Mass – 2 objects

For two particles, the center of mass lies closer to the one with the most mass:

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M}$$

where *M* is the total mass.



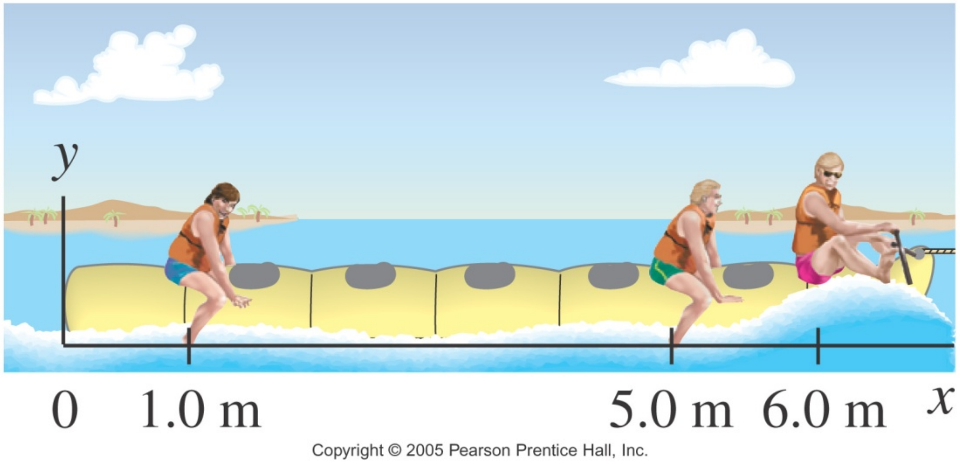
CM – more than 2 objects

If there are more than two particles

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C + ...}{M}$$

where M is the total mass of all the particles

Ex. 7-12 Three people of roughly equal masses m on a lightweight banana boat sit along the x axis at the positions x_A=1.0m, x_B=5.0m, and x_C=6.0m, as in the figure. Find the CM.



x_{CM}=4.0m

The coordinates of the CM depend on the reference frame, but the physical location of the CM is independent of that choice

The CM may lie outside the object

Ex: donuts, jumpers over bars

CM and Translational Motion

The **total momentum** of a system of particles is equal to

the product of the **total mass** and the **velocity of the center of mass**.

$$Mv_{CM} = m_A v_A + m_B v_B + m_C v_C$$

The sum of all the forces acting on a system is equal to

the **total mass** of the system multiplied by

the **acceleration of the center of mass**:

$$F_{net} = Ma_{CM}$$