Potential Energy

Potential energy – energy that is stored on a object. When it is released, it leads to the increase of kinetic energy and therefore to work.

Potential energy – energy associated with forces that depend on position or configuration of an object with respect to the surroundings.

One job of physics is to identify the different types of energy in the world, especially those that are of common importance. One general type of energy is **potential energy** U. Technically, potential energy is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another.

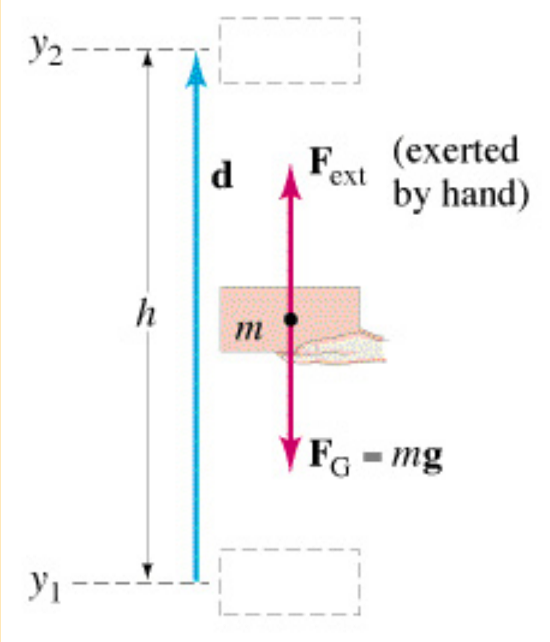
An example might help better than the definition: A bungee-cord jumper plunges from a staging platform (Fig. 8-1). The system of objects consists of Earth and the jumper. The force between the objects is the gravitational force. The configuration of the system changes (the separation between the jumper and Earth decreases — that is, of course, the thrill of the jump). We can account for the jumper's motion and increase in kinetic energy by defining a **gravitational potential energy** U. This

Gravitational potential energy, elastic potential energy, chemical potential energy.

Potential Energy

Potential energy – energy associated with forces that depend on position or configuration of an object with respect to the surroundings

Examples: Gravitational potential energy (object at a certain height)
Elastic potential energy (spring)



Gravitational potential energy:

1) Hand does work on the brick
exerted force: F_{ext}

$$W_{\mathbf{ext}} = F_{\mathbf{ext}} d \cos 0 = mg(y_2 - y_1) = mgh$$

2) Gravity does work on the brick
(against the motion), F_G

$$W_{\mathbf{G}} = F_{\mathbf{G}} d \cos 180^o = -mg(y_2 - y_1) = -mgh$$

$$U_{grav} = mgy$$

gravitational potential energy

Gravitational Potential Energy

$$U_{grav} = mgy$$

The higher an object is above the ground the more gravitational potential energy it has.

Work done by **gravity** as the object moves from point 1 to 2.

$$\Delta U = -W_{grav}$$

$$W_{grav} = -\ mg(y_2-y_1) = -(U_2-U_1)$$

Work done by an external force to move the object from point 1 to 2 (a=0)

$$W_{ext} = mg(y_2-y_1) = (U_2-U_1)$$

At the top, if the object is released,
the potential energy is transformed into kinetic energy

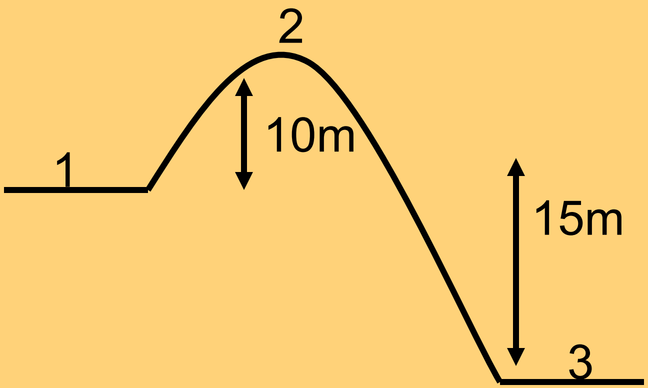
$$v^2 = 2gh$$

$$\frac{1}{2}mv^2 = mgh$$

Exercise

Reference point for zero gravitational potential energy is arbitrary

Ex. A 1000-kg roller-coaster car moves from point 1 to point 2 and then to point 3. (a) What is the gravitational potential energy at 2 and 3 RELATIVE to point 1? That is, take y=0 at point 1. (b) What is the change in potential energy when the car goes from point 2 to point 3? (c) Repeat parts (a) and (b) but take the reference point (y=0) to be at point 3.



$U_2 = 9.8 \times 10^4$ J $U_3 = -1.5 \times 10^5$ J

$U_3 - U_2 = -2.5 \times 10^5$ J

$U_2 = -2.5 \times 10^5$ J $U_3 = 0$

$U_3 - U_2 = -2.5 \times 10^5$ J

What is physically important is the **CHANGE** in potential energy, because this is what is related to work and this is what can be measured.

Potential Energy

For either rise or fall, the change ΔU in gravitational potential energy is defined as being equal to the negative of the work done on the tomato by the gravitational force. Using the general symbol W for work, we write this as

$$\Delta U = -W. \quad (8\text{-}1)$$

$$W = \int_{x_i}^{x_f} F(x)\,dx. \quad (8\text{-}5)$$

$$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx. \quad (8\text{-}6)$$

Gravitational Potential Energy

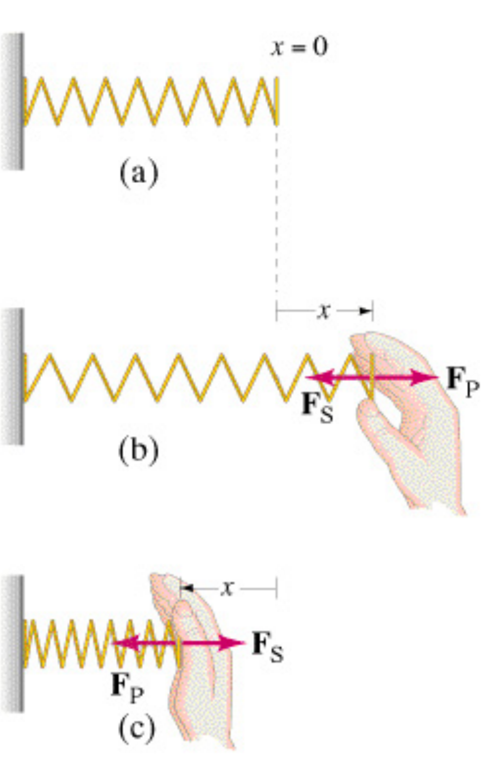
$$\Delta U = -\int_{y_i}^{y_f} (-mg)\,dy = mg\int_{y_i}^{y_f} dy = mg\Big[y\Big]_{y_i}^{y_f}, \quad (8\text{-}7)$$

$$\Delta U = mg(y_f - y_i) = mg\,\Delta y.$$

Elastic Potential Energy

Each form of potential energy is associated with a particular force.

The change in potential energy is the work required of an external force to move the object without acceleration between two points.



Elastic materials:

Force by the spring on the hand (Hooke's law)

$$F_S = -kx$$

Work done BY the spring

$$W_{elastic} = -(U_2 - U_1)$$

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\,dx = k\int_{x_i}^{x_f} x\,dx = \tfrac{1}{2}k\Big[x^2\Big]_{x_i}^{x_f},$$

$$\Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2.$$

Elastic potential energy

$$U_{el} = \frac{1}{2}kx^2$$

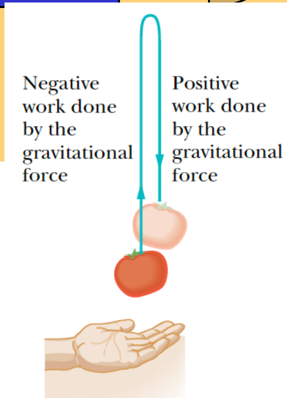
Reference point for zero potential energy is the spring's natural position

Comments

In the examples of potential energy: object has the *POTENTIAL* to do work even though it is not actually doing it.
Energy can be *STORED* in the form of potential energy.

There is a single formula for kinetic energy, but the mathematical form for the potential energy depends on the force involved.

Conservative vs Nonconservative F



In a situation in which $W_1 = -W_2$ is always true, the other type of energy is a potential energy and the force is said to be a **conservative force.** As you might suspect, the gravitational force and the spring force are both conservative (since otherwise we could not have spoken of gravitational potential energy and elastic potential energy, as we did previously).

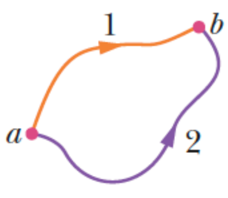
A force that is not conservative is called a **nonconservative force.** The kinetic frictional force and drag force are nonconservative. For an example, let us send a block sliding across a floor that is not frictionless. During the sliding, a kinetic frictional force from the floor slows the block by transferring energy from its kinetic energy to a type of energy called *thermal energy* (which has to do with the random motions of atoms and molecules). We know from experiment that this energy transfer cannot be reversed (thermal energy cannot be transferred back to kinetic energy of the block by the kinetic frictional force). Thus, although we have a system (made up of the block and the floor), a force that acts between parts of the system, and a transfer of energy by the force, the force is not conservative. Therefore, thermal energy is not a potential energy.

Path Independence

Conservative Forces:

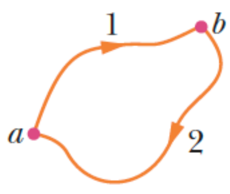
forces for which the work done does *NOT* depend on the *PATH* taken, but only on the final and initial position (Ex.: gravity, elastic force).

An object that starts at a point and returns to the same point under the action of a conservative force has no net work done on it.



(a)

The force is conservative. Any choice of path between the points gives the same amount of work.



(b)

And a round trip gives a total work of zero.

The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

$$W_{ab,1} + W_{ba,2} = 0,$$

$$W_{ab,1} = -W_{ba,2}.$$

The net work done by a conservative force on a particle moving around any closed path is zero.

Nonconservative Forces:

forces for which the work done *DEPENDS* on the *PATH* taken (Ex.: friction, force exerted by a person, tension in a rope).

Example

Equivalent paths for calculating work, slippery cheese

Figure 8-5*a* shows a 2.0 kg block of slippery cheese that slides along a frictionless track from point *a* to point *b*. The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.80 m. How much work is done on the cheese by the gravitational force during the slide?

KEY IDEAS

(1) We *cannot* calculate the work by using Eq. 7-12 ($W_g = mgd\cos\phi$). The reason is that the angle ϕ between the directions of the gravitational force $\vec{F}_g$ and the displacement $\vec{d}$ varies along the track in an unknown way. (Even if we did know the shape of the track and could calculate ϕ along it, the calculation could be very difficult.) (2) Because $\vec{F}_g$ is a conservative force, we can find the work by choosing some other path between *a* and *b*—one that makes the calculation easy.

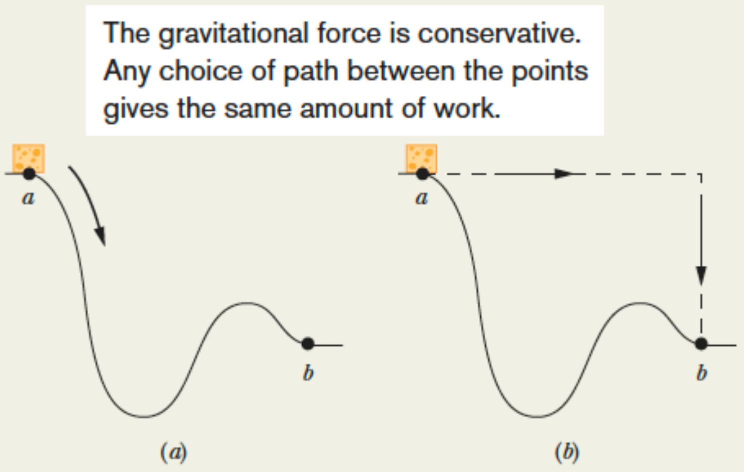


Fig. 8-5 (*a*) A block of cheese slides along a frictionless track from point *a* to point *b*. (*b*) Finding the work done on the cheese by the gravitational force is easier along the dashed path than along the actual path taken by the cheese; the result is the same for both paths.

Calculations: Let us choose the dashed path in Fig. 8-5*b*; it consists of two straight segments. Along the horizontal segment, the angle ϕ is a constant 90°. Even though we do not know the displacement along that horizontal segment, Eq. 7-12 tells us that the work W_h done there is

$$W_h = mgd\cos 90° = 0.$$

Along the vertical segment, the displacement d is 0.80 m and, with $\vec{F}_g$ and $\vec{d}$ both downward, the angle ϕ is a constant 0°. Thus, Eq. 7-12 gives us, for the work W_v done along the vertical part of the dashed path,

$$W_v = mgd\cos 0°$$
$$= (2.0\text{ kg})(9.8\text{ m/s}^2)(0.80\text{ m})(1) = 15.7\text{ J}.$$

The total work done on the cheese by $\vec{F}_g$ as the cheese moves from point *a* to point *b* along the dashed path is then

$$W = W_h + W_v = 0 + 15.7\text{ J} \approx 16\text{ J}. \quad \text{(Answer)}$$

This is also the work done as the cheese slides along the track from *a* to *b*.

Work-Energy Principle

Suppose several forces, conservative and nonconservative, act on an object
W_C – work done by conservative forces
W_{NC} – work done by nonconservative forces

$$W_{\mathbf{net}} = W_{\mathbf{C}} + W_{\mathbf{NC}}$$

$$W_{\mathbf{net}} = \Delta K$$

$$W_{\mathbf{C}} + W_{\mathbf{NC}} = \Delta K$$

$$W_{\mathbf{NC}} = \Delta K - W_{\mathbf{C}}$$

Remember that the work done BY a *conservative force* (gravitational, elastic) is

$$W_C = -\Delta U$$

$$W_{NC} = \Delta K + \Delta U$$

Work done by nonconservative forces acting on an object is equal to the total change in kinetic and potential energies.

Conservation of Mechanical Energy

If all forces acting on an object are conservative: $W_{NC} = \Delta K + \Delta U = 0$

$$\Delta K + \Delta U = 0$$

$$(K_2 - K_1) + (U_2 - U_1) = 0$$

Define a quantity E called **total MECHANICAL energy:** $E_{mech} = K + U$

$$K_2 + U_2 = K_1 + U_1$$

$$E_2 = E_1 = Const$$

<u>Principle of conservation of mechanical energy:</u>

If only conservative forces are acting, the total mechanical energy is conserved

Problems: Cons. of Mechanical Energy

Gravitational Potential Energy

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{mv_1^2}{2} + mgy_1 = \frac{mv_2^2}{2} + mgy_2$$

Ex. A rock at 3.0 m from the ground is dropped. Calculate the rock's speed when it has fallen to 1.0 m above the ground.

v=6.3 m/s

Ex. Assume that a roller-coaster at 40m above the ground starts from rest. Calculate (a) the speed it has at the bottom of the hill; (b) at what height it will have half this speed. Take y=0 at the bottom of the hill.

(a) V_2=28 m/s

(b) y_2=30 m

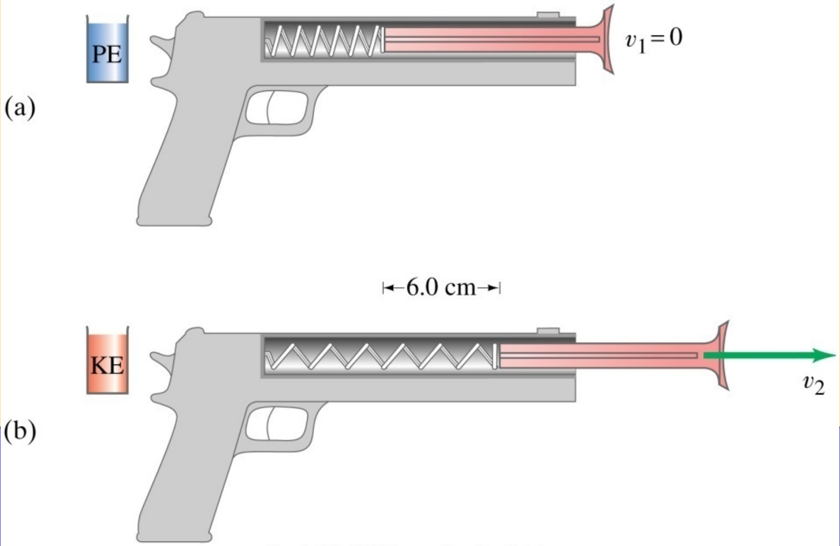
Problems: Cons. of Mechanical Energy

Elastic Potential Energy

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

Ex. A dart of mass 0.100 kg is pressed against the spring of a toy dart gun. The spring (with spring stiffness constant k=250N/m) is compressed 6.0 cm and released. If the dart detaches from the spring when the spring reaches its natural length (x=0), what speed does the dart acquire?

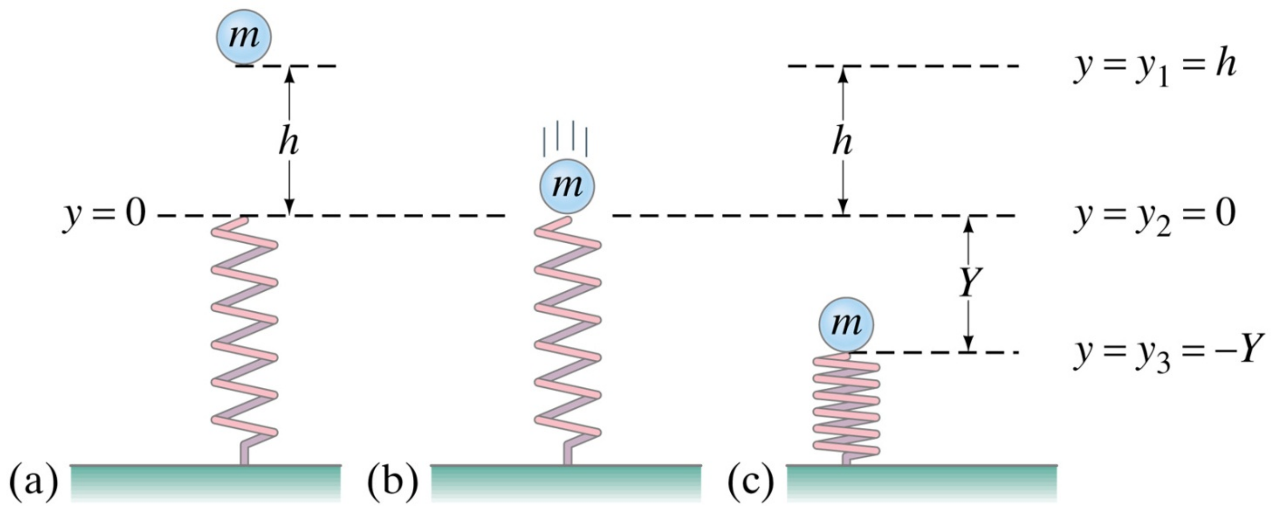


v=3.0m/s

Problems: Cons. of Mechanical Energy

Ex. A ball of mass m=2.60 kg, starting from rest, falls a vertical distance h=55.0 cm before striking a vertical coiled spring, which it compresses an amount Y=15.0 cm. Determine the spring stiffness constant of the spring. Assume the spring has negligible mass and ignore air resistance.

$$k = \frac{2mg(h+Y)}{Y^2} = 1590 N/m$$



Problems:

Conservation of mechanical energy, water slide

In Fig. 8-8, a child of mass m is released from rest at the top of a water slide, at height $h = 8.5$ m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.

KEY IDEAS

(1) We cannot find her speed at the bottom by using her acceleration along the slide as we might have in earlier chapters because we do not know the slope (angle) of the slide. However, because that speed is related to her kinetic energy, perhaps we can use the principle of conservation of mechanical energy to get the speed. Then we would not need to know the slope. (2) Mechanical energy is conserved in a system *if* the system is isolated and *if* only conservative forces cause energy transfers within it. Let's check.

Forces: Two forces act on the child. The *gravitational force,* a conservative force, does work on her. The *normal force* on her from the slide does no work because its direction at any point during the descent is always perpendicular to the direction in which the child moves.

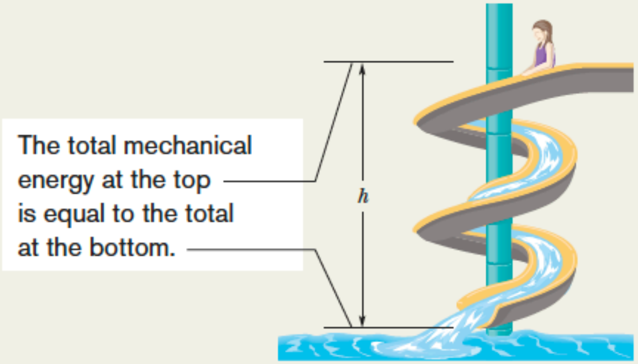


Fig. 8-8 A child slides down a water slide as she descends a height h.

System: Because the only force doing work on the child is the gravitational force, we choose the child–Earth system as our system, which we can take to be isolated.

Thus, we have only a conservative force doing work in an isolated system, so we *can* use the principle of conservation of mechanical energy.

Calculations: Let the mechanical energy be $E_{mec,t}$ when the child is at the top of the slide and $E_{mec,b}$ when she is at the bottom. Then the conservation principle tells us

$$E_{mec,b} = E_{mec,t}. \qquad (8\text{-}19)$$

To show both kinds of mechanical energy, we have

$$K_b + U_b = K_t + U_t, \qquad (8\text{-}20)$$

or

$$\tfrac{1}{2}mv_b^2 + mgy_b = \tfrac{1}{2}mv_t^2 + mgy_t.$$

Dividing by m and rearranging yield

$$v_b^2 = v_t^2 + 2g(y_t - y_b).$$

Putting $v_t = 0$ and $y_t - y_b = h$ leads to

$$v_b = \sqrt{2gh} = \sqrt{(2)(9.8\ \text{m/s}^2)(8.5\ \text{m})}$$
$$= 13\ \text{m/s}. \qquad \text{(Answer)}$$

This is the same speed that the child would reach if she fell 8.5 m vertically. On an actual slide, some frictional forces would act and the child would not be moving quite so fast.

Comments: Although this problem is hard to solve directly with Newton's laws, using conservation of mechanical energy makes the solution much easier. However, if we were asked to find the time taken for the child to reach the bottom of the slide, energy methods would be of no use; we would need to know the shape of the slide, and we would have a difficult problem.

Conservation of Energy

Electric energy, nuclear energy, thermal energy, chemical energy.

In atomic physics, they are seen as kinetic or potential energy at the atomic level.

Thermal energy – kinetic energy of moving molecules

Energy stored in food and fuel – energy stored in the chemical bounds.

Work is done when energy is transferred from one object to another.

(spring to ball, water at the top of a damn to turbine blades, person to cart, etc)

Accounting for all forms of energy, we find that the total energy neither increases nor decreases.

Energy as a whole is conserved.

Dissipative Forces

Frictional forces reduce the total **mechanical** energy,
but NOT the total energy
They are called ***dissipative forces.***

Where do kinetic and potential energies go? **– *they become heat***

$$W_{NC} = \Delta K + \Delta U = -F_{fr} d$$

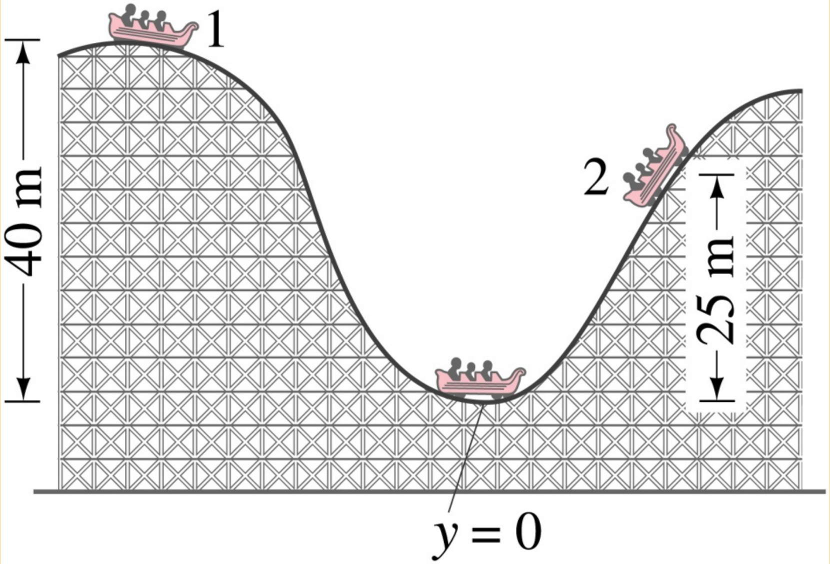
$$-F_{fr} d = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgy_2 - mgy_1$$

$$\frac{1}{2} m v_1^2 + mgy_1 = \frac{1}{2} m v_2^2 + mgy_2 + F_{fr} d$$

Example: in the roller-coaster the initial total energy will be equal to K+P at any subsequent point along the path PLUS the thermal energy produced.

Example

Ex. Assume that a roller-coaster of 1000 kg at 40m above the ground starts from rest. It reaches only 25m at the second hill before coming to a momentary stop. It traveled a total distance of 400 m. Estimate the average friction force.



$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 + F_{fr}d$$

$$F_{fr} = 370N$$

Energy, friction, spring, and tamales

In Fig. 8-17, a 2.0 kg package of tamales slides along a floor with speed $v_1 = 4.0$ m/s. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic frictional force from the floor, of magnitude 15 N, acts on the package. If $k = 10\,000$ N/m, by what distance d is the spring compressed when the package stops?

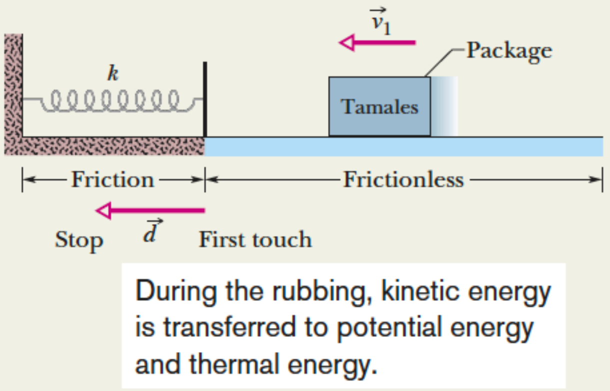


Fig. 8-17 A package slides across a frictionless floor with velocity $\vec{v}_1$ toward a spring of spring constant k. When the package reaches the spring, a frictional force from the floor acts on the package.

KEY IDEAS

We need to examine all the forces and then to determine whether we have an isolated system or a system on which an external force is doing work.

Forces: The normal force on the package from the floor does no work on the package because the direction of this force is always perpendicular to the direction of the package's displacement. For the same reason, the gravitational force on the package does no work. As the spring is compressed, however, a spring force does work on the package, transferring energy to elastic potential energy of the spring. The spring force also pushes against a rigid wall. Because there is friction between the package and the floor, the sliding of the package across the floor increases their thermal energies.

System: The package–spring–floor–wall system includes all these forces and energy transfers in one isolated system. Therefore, because the system is isolated, its total energy cannot change. We can then apply the law of conservation of energy in the form of Eq. 8-37 to the system:

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}}. \tag{8-42}$$

Calculations: In Eq. 8-42, let subscript 1 correspond to the initial state of the sliding package and subscript 2 correspond to the state in which the package is momentarily stopped and the spring is compressed by distance d. For both states the mechanical energy of the system is the sum of the package's kinetic energy ($K = \frac{1}{2}mv^2$) and the spring's potential energy ($U = \frac{1}{2}kx^2$). For state 1, $U = 0$ (because the spring is not compressed), and the package's speed is v_1. Thus, we have

$$E_{\text{mec},1} = K_1 + U_1 = \tfrac{1}{2}mv_1^2 + 0.$$

For state 2, $K = 0$ (because the package is stopped), and the compression distance is d. Therefore, we have

$$E_{\text{mec},2} = K_2 + U_2 = 0 + \tfrac{1}{2}kd^2.$$

Finally, by Eq. 8-31, we can substitute $f_k d$ for the change ΔE_{th} in the thermal energy of the package and the floor. We can now rewrite Eq. 8-42 as

$$\tfrac{1}{2}kd^2 = \tfrac{1}{2}mv_1^2 - f_k d.$$

Rearranging and substituting known data give us

$$5000d^2 + 15d - 16 = 0.$$

Solving this quadratic equation yields

$$d = 0.055 \text{ m} = 5.5 \text{ cm}. \qquad \text{(Answer)}$$

Dissipative and External Forces

Frictional forces reduce the total **mechanical** energy.
External forces can increase the mechanical energy.

$$W_{NC} = \Delta K + \Delta U = Fd - F_{fr}d$$

$$Fd = \Delta E_{\text{mec}} + \Delta E_{\text{th}}. \quad (8\text{-}32)$$

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}} \quad \text{(work done on system, friction involved).}$$

(8-33)

Dissipative and External Forces

Work, friction, change in thermal energy, cabbage heads

A food shipper pushes a wood crate of cabbage heads (total mass $m = 14$ kg) across a concrete floor with a constant horizontal force $\vec{F}$ of magnitude 40 N. In a straight-line displacement of magnitude $d = 0.50$ m, the speed of the crate decreases from $v_0 = 0.60$ m/s to $v = 0.20$ m/s.

(a) How much work is done by force $\vec{F}$, and on what system does it do the work?

KEY IDEA

Because the applied force $\vec{F}$ is constant, we can calculate the work it does by using Eq. 7-7 ($W = Fd\cos\phi$).

Calculation: Substituting given data, including the fact that force $\vec{F}$ and displacement $\vec{d}$ are in the same direction, we find

$$W = Fd\cos\phi = (40\text{ N})(0.50\text{ m})\cos 0^\circ$$
$$= 20\text{ J.} \qquad \text{(Answer)}$$

Reasoning: We can determine the system on which the work is done to see which energies change. Because the crate's speed changes, there is certainly a change ΔK in the crate's kinetic energy. Is there friction between the floor and the crate, and thus a change in thermal energy? Note that $\vec{F}$ and the crate's velocity have the same direction. Thus, if there is no friction, then $\vec{F}$ should be accelerating the crate to a *greater* speed. However, the crate is *slowing*, so there must be friction and a change ΔE_{th} in thermal energy of the crate and the floor. Therefore, the system on which the work is done is the crate–floor system, because both energy changes occur in that system.

(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

KEY IDEA

We can relate ΔE_{th} to the work W done by $\vec{F}$ with the energy statement of Eq. 8-33 for a system that involves friction:

$$W = \Delta E_{mec} + \Delta E_{th}. \qquad (8\text{-}34)$$

Calculations: We know the value of W from (a). The change ΔE_{mec} in the crate's mechanical energy is just the change in its kinetic energy because no potential energy changes occur, so we have

$$\Delta E_{mec} = \Delta K = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2.$$

Substituting this into Eq. 8-34 and solving for ΔE_{th}, we find

$$\Delta E_{th} = W - (\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2) = W - \tfrac{1}{2}m(v^2 - v_0^2)$$
$$= 20\text{ J} - \tfrac{1}{2}(14\text{ kg})[(0.20\text{ m/s})^2 - (0.60\text{ m/s})^2]$$
$$= 22.2\text{ J} \approx 22\text{ J.} \qquad \text{(Answer)}$$

Force: negative of the slope of U

$$\Delta U(x) = -W = -F(x)\,\Delta x. \qquad (8\text{-}21)$$

$$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx. \qquad (8\text{-}6)$$

$$F(x) = -\frac{dU(x)}{dx} \quad \text{(one-dimensional motion)} \qquad (8\text{-}22)$$

Elastic potential energy and Hooke's law

$$U(x) = \tfrac{1}{2}kx^2 \qquad F(x) = -kx$$

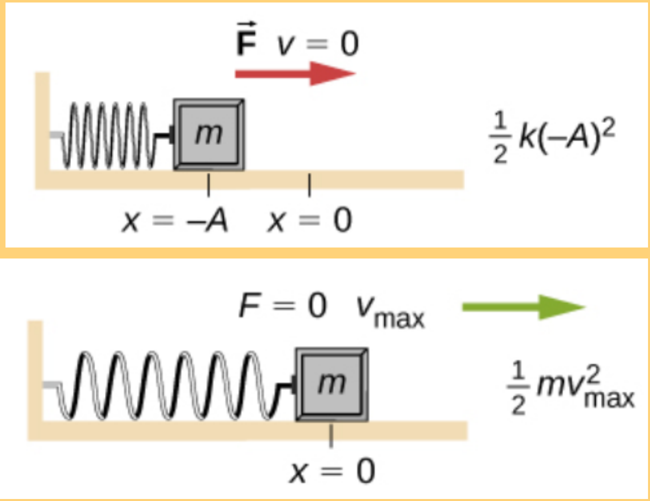
Gravitational potential energy and gravitational force

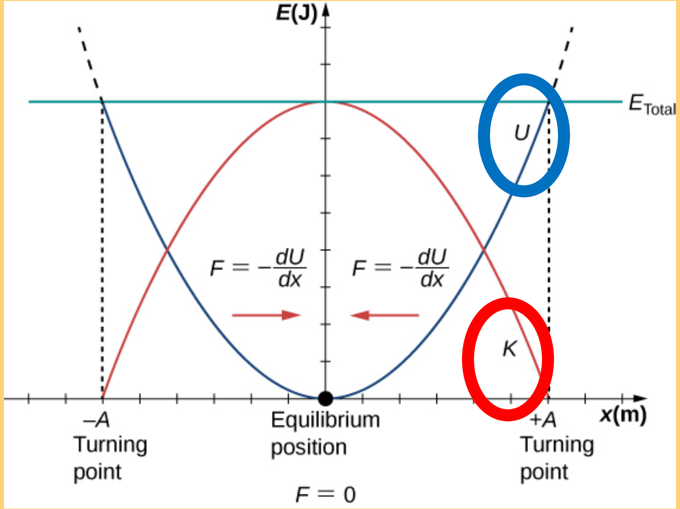
$$U(x) = mgx \qquad F = -mg$$

Reading a Potential Curve

$$F(x) = -\frac{dU(x)}{dx} \qquad \text{(one-dimensional motion)} \qquad (8\text{-}22)$$

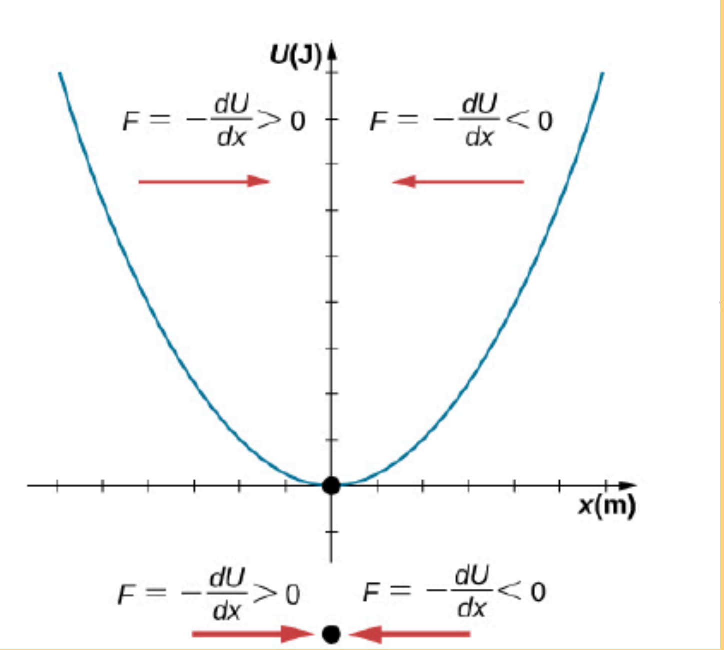
harmonic oscillator

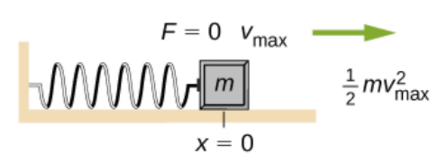


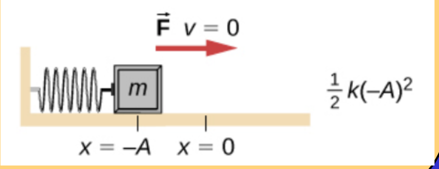


Reading a Potential Curve

$$F(x) = -\frac{dU(x)}{dx} \qquad \text{(one-dimensional motion)} \qquad (8\text{-}22)$$

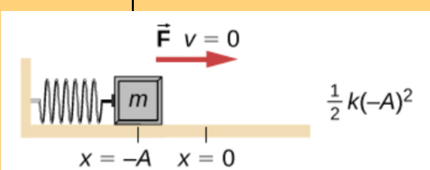


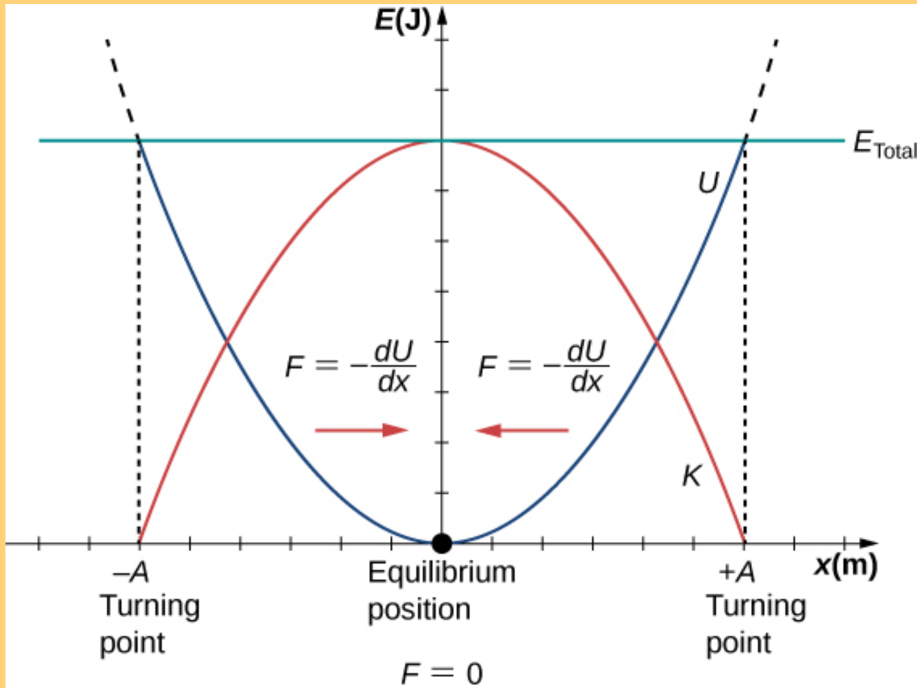


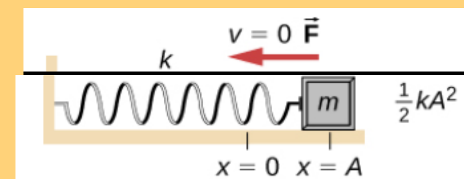


Turning Points

harmonic oscillator



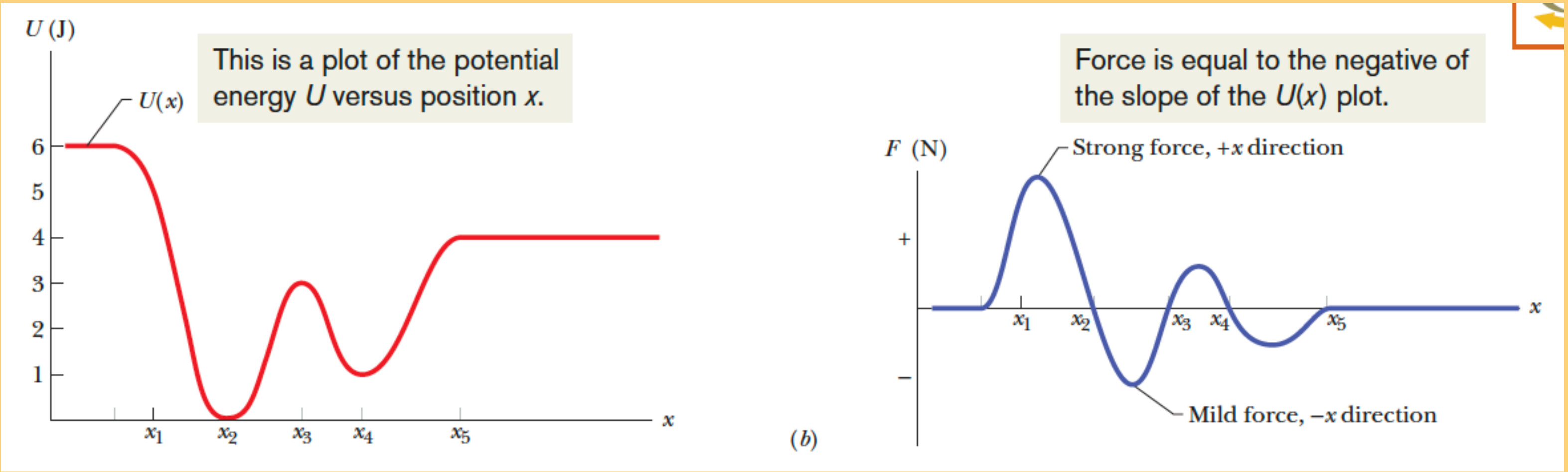




Turning point: place where K=0, U=E_{mech},
and the motion of the particle changes direction.

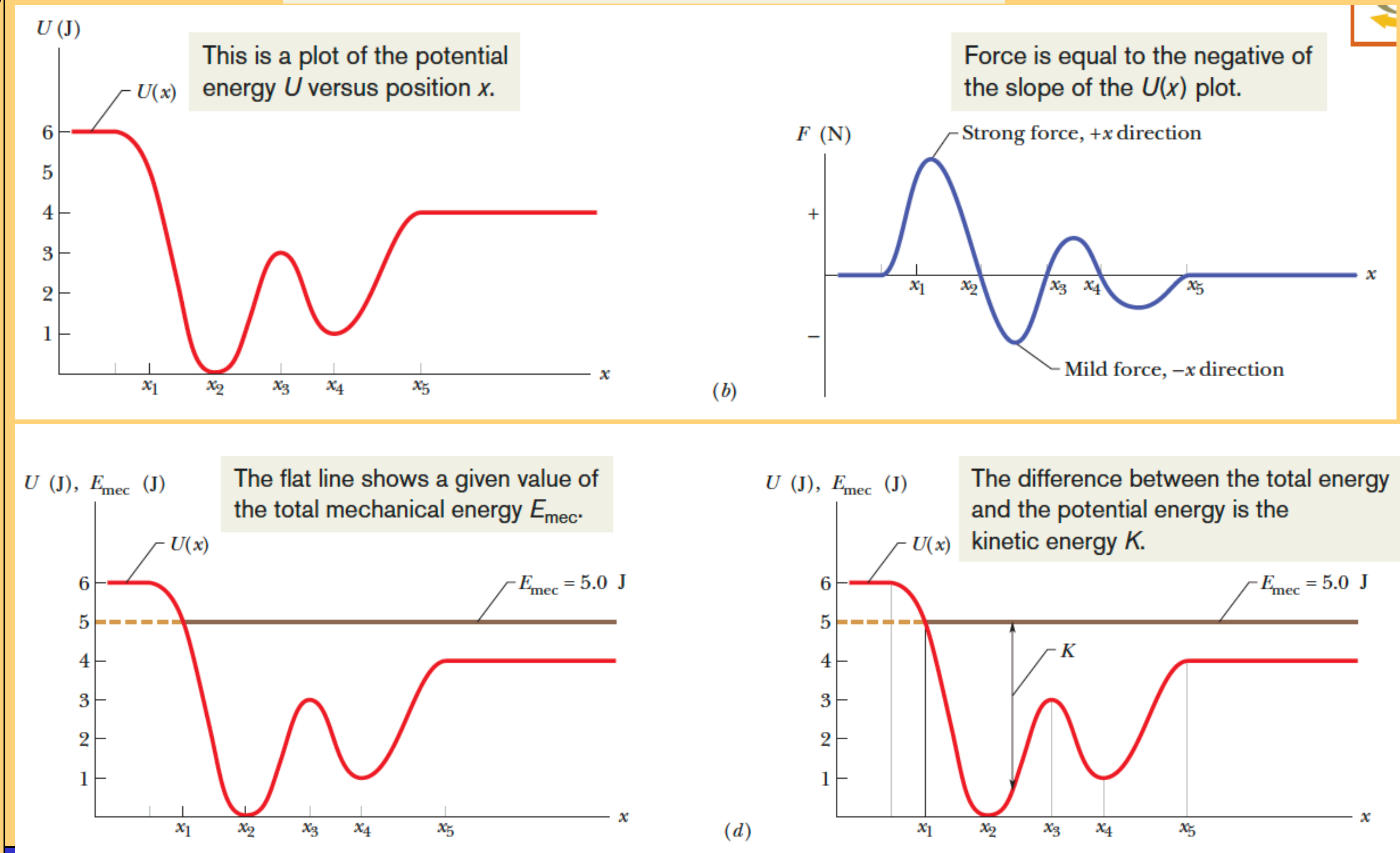
Reading a Potential Curve

$$F(x) = -\frac{dU(x)}{dx} \qquad \text{(one-dimensional motion)} \qquad (8\text{-}22)$$



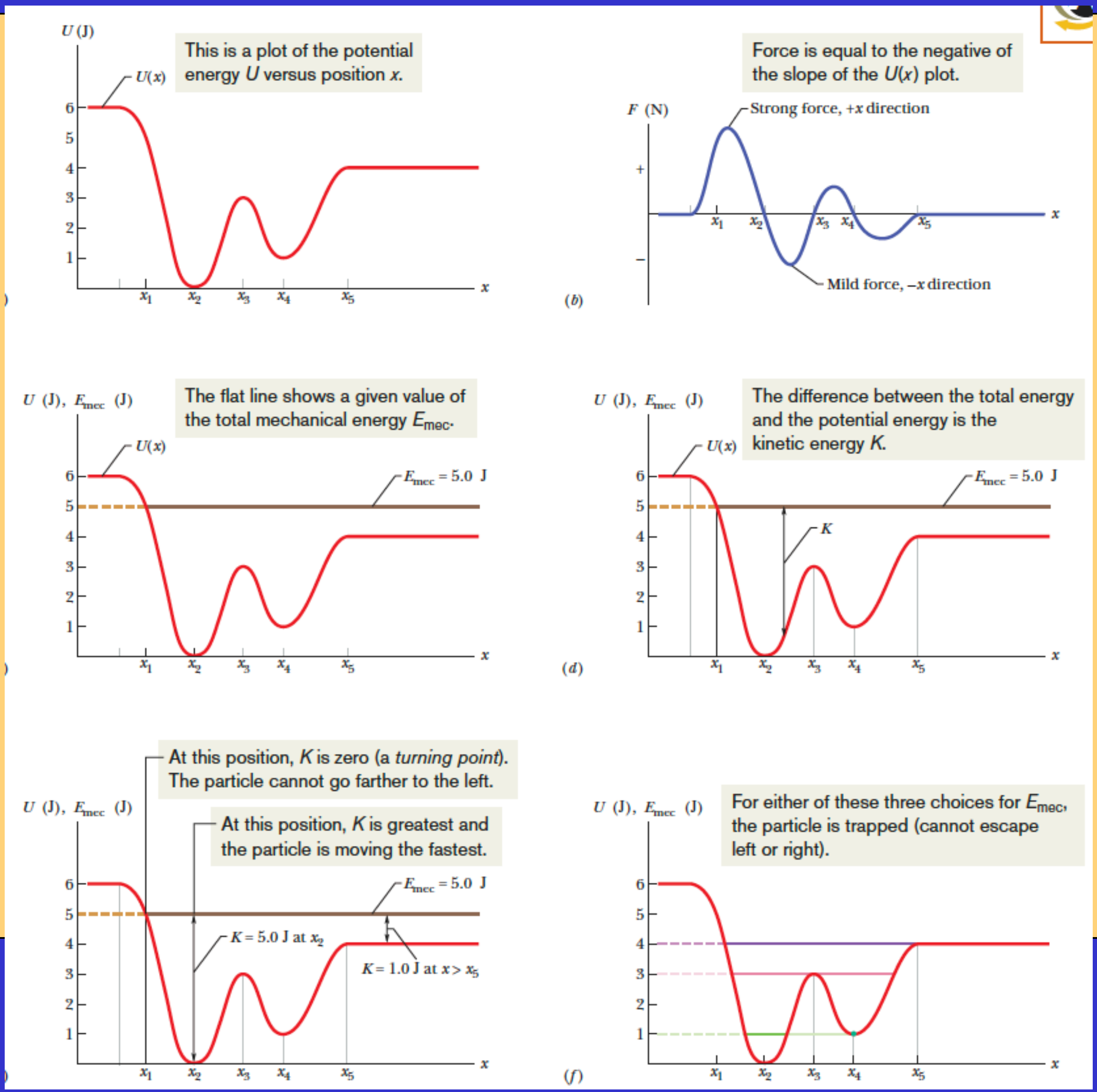
Reading a Potential Curve

$$F(x) = -\frac{dU(x)}{dx} \quad \text{(one-dimensional motion)} \qquad (8\text{-}22)$$



Since K can never be negative (because v^2 is always positive), the particle can never move to the left of x_1, where $E_{mec} - U$ is negative. Instead, as the particle moves toward x_1 from x_2, K decreases (the particle slows) until $K = 0$ at x_1 (the particle stops there).

Reading a Potential Curve



Equilibrium Points

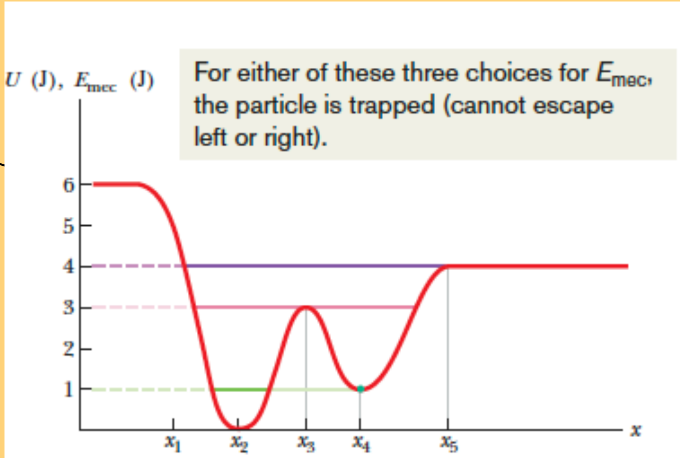


Figure 8-9*f* shows three different values for E_{mec} superposed on the plot of the potential energy function $U(x)$ of Fig. 8-9*a*. Let us see how they change the situation. If $E_{mec} = 4.0$ J (purple line), the turning point shifts from x_1 to a point between x_1 and x_2. Also, at any point to the right of x_5, the system's mechanical energy is equal to its potential energy; thus, the particle has no kinetic energy and (by Eq. 8-22) no force acts on it, and so it must be stationary. A particle at such a position is said to be in **neutral equilibrium.** (A marble placed on a horizontal tabletop is in that state.)

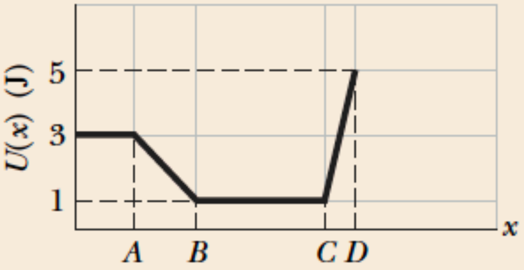
If $E_{mec} = 3.0$ J (pink line), there are two turning points: One is between x_1 and x_2, and the other is between x_4 and x_5. In addition, x_3 is a point at which $K = 0$. If the particle is located exactly there, the force on it is also zero, and the particle remains stationary. However, if it is displaced even slightly in either direction, a nonzero force pushes it farther in the same direction, and the particle continues to move. A particle at such a position is said to be in **unstable equilibrium.** (A marble balanced on top of a bowling ball is an example.)

Next consider the particle's behavior if $E_{mec} = 1.0$ J (green line). If we place it at x_4, it is stuck there. It cannot move left or right on its own because to do so would require a negative kinetic energy. If we push it slightly left or right, a restoring force appears that moves it back to x_4. A particle at such a position is said to be in **stable equilibrium.** (A marble placed at the bottom of a hemispherical bowl is an example.) If we place the particle in the cup-like *potential well* centered at x_2, it is between two turning points. It can still move somewhat, but only partway to x_1 or x_3.

Exercises

CHECKPOINT 4

The figure gives the potential energy function $U(x)$ for a system in which a particle is in one-dimensional motion. (a) Rank regions AB, BC, and CD according to the magnitude of the force on the particle, greatest first. (b) What is the direction of the force when the particle is in region AB?



Reading a potential energy graph

A 2.00 kg particle moves along an x axis in one-dimensional motion while a conservative force along that axis acts on it. The potential energy $U(x)$ associated with the force is plotted in Fig. 8-10*a*. That is, if the particle were placed at any position between $x = 0$ and $x = 7.00$ m, it would have the plotted value of U. At $x = 6.5$ m, the particle has velocity $\vec{v}_0 = (-4.00 \text{ m/s})\hat{\text{i}}$.

(a) From Fig. 8-10*a*, determine the particle's speed at $x_1 = 4.5$ m.

KEY IDEAS

(1) The particle's kinetic energy is given by Eq. 7-1 ($K = \frac{1}{2}mv^2$). (2) Because only a conservative force acts on the particle, the mechanical energy E_{mec} ($= K + U$) is conserved as the particle moves. (3) Therefore, on a plot of $U(x)$ such as Fig. 8-10*a*, the kinetic energy is equal to the difference between E_{mec} and U.

Calculations: At $x = 6.5$ m, the particle has kinetic energy

$$K_0 = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}(2.00 \text{ kg})(4.00 \text{ m/s})^2 = 16.0 \text{ J}.$$

Because the potential energy there is $U = 0$, the mechanical energy is

$$E_{mec} = K_0 + U_0 = 16.0 \text{ J} + 0 = 16.0 \text{ J}.$$

This value for E_{mec} is plotted as a horizontal line in Fig. 8-10*a*. From that figure we see that at $x = 4.5$ m, the potential energy is $U_1 = 7.0$ J. The kinetic energy K_1 is the difference between E_{mec} and U_1:

$$K_1 = E_{mec} - U_1 = 16.0 \text{ J} - 7.0 \text{ J} = 9.0 \text{ J}.$$

Because $K_1 = \frac{1}{2}mv_1^2$, we find

$$v_1 = 3.0 \text{ m/s}. \qquad \text{(Answer)}$$

(b) Where is the particle's turning point located?

KEY IDEA

The turning point is where the force momentarily stops and then reverses the particle's motion. That is, it is where the particle momentarily has $v = 0$ and thus $K = 0$.

Calculations: Because K is the difference between E_{mec} and U, we want the point in Fig. 8-10*a* where the plot of U rises to meet the horizontal line of E_{mec}, as shown in Fig. 8-10*b*. Because the plot of U is a straight line in Fig. 8-10*b*, we can draw nested right triangles as shown and then write

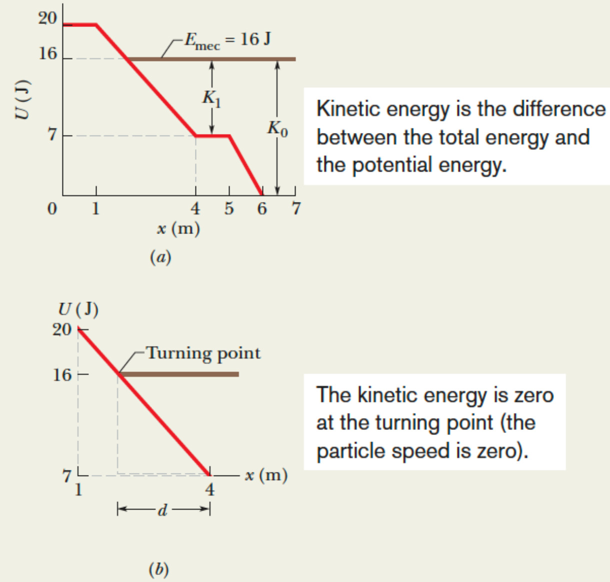


Fig. 8-10 (*a*) A plot of potential energy U versus position x. (*b*) A section of the plot used to find where the particle turns around.

the proportionality of distances

$$\frac{16 - 7.0}{d} = \frac{20 - 7.0}{4.0 - 1.0},$$

which gives us $d = 2.08$ m. Thus, the turning point is at

$$x = 4.0 \text{ m} - d = 1.9 \text{ m}. \qquad \text{(Answer)}$$

(c) Evaluate the force acting on the particle when it is in the region $1.9 \text{ m} < x < 4.0$ m.

KEY IDEA

The force is given by Eq. 8-22 ($F(x) = -dU(x)/dx$). The equation states that the force is equal to the negative of the slope on a graph of $U(x)$.

Calculations: For the graph of Fig. 8-10*b*, we see that for the range $1.0 \text{ m} < x < 4.0$ m the force is

$$F = -\frac{20 \text{ J} - 7.0 \text{ J}}{1.0 \text{ m} - 4.0 \text{ m}} = 4.3 \text{ N}. \qquad \text{(Answer)}$$

Thus, the force has magnitude 4.3 N and is in the positive direction of the x axis. This result is consistent with the fact that the initially leftward-moving particle is stopped by the force and then sent rightward.