Work and Energy

We have studied motion in terms of force, now we consider ***energy and momentum – CONSERVED*** quantities

This approach is helpful when dealing with many objects and considering all forces involved become very difficult.

In this chapter we study **WORK and ENERGY,** both are SCALARS

There are different kinds of energy.

Kinetic energy and potential energy are examples of mechanical energy.

An object in motion has ***kinetic energy.***

There are also thermal energy (heat), nuclear energy, etc.

The sum of all types of energy is ***CONSERVED.***

Energy is not destroyed, only transformed.

Work

To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

$$W = F_x d.$$

$$W = \vec{F} \cdot \vec{d} \qquad \text{(work done by a constant force)}$$

$$W = Fd \cos \phi \qquad \text{(work done by a constant force)}$$

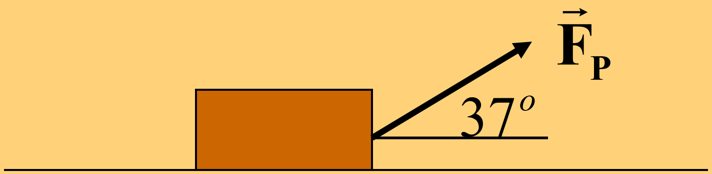
The work done on an object by a force can be either positive work or negative work, depending on the angle.

Units

1 N.m = 1 J (joule)

1 dyne.cm = 1 erg

Net Work



Ex. 01 A person pulls a 50 kg crate 40m along a horizontal floor by a constant force Fp=100N, which acts at a 37 degree angle. Ffr=50 N.

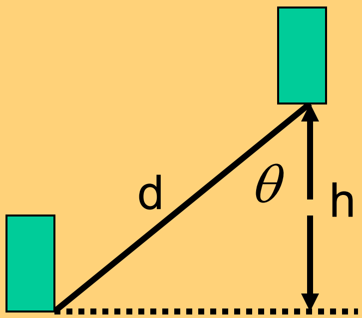
a) What is the work done by each force acting on the crate?

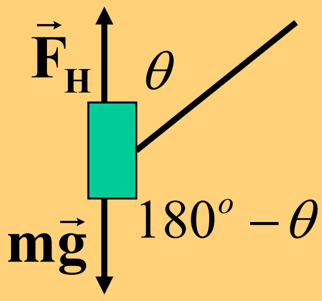
b) What is the net force done on the crate?

W_G=0, W_N = 0, W_P=3200 J, Wfr=-2000J
Wnet=1200J

Ex. 02 A box is dragged across a floor by a force as in the figure. If the angle is increased from 0 to 90 degree, what happens to the work done to the box?

Exercises





Ex. 03 (a) Determine the work a hiker must do on a 15.0kg backpack to Carry it up a hill of height h=10.0m with constant speed.
Determine also (b) the work done by gravity on the backpack,
(c) the net work done on the backpack

W_H=1470J W_G=-1470J W_{net}=0

Ex.04 Does Earth do work on the Moon?

NO, because the radial force is perpendicular to tangential motion of the Moon

Kinetic Energy

7-3 Kinetic Energy

Kinetic energy K is energy associated with the *state of motion* of an object. The faster the object moves, the greater is its kinetic energy. When the object is stationary, its kinetic energy is zero.

For an object of mass m whose speed v is well below the speed of light,

$$K = \frac{1}{2}mv^2 \quad \text{(kinetic energy).} \tag{7-1}$$

For example, a 3.0 kg duck flying past us at 2.0 m/s has a kinetic energy of $6.0\ \text{kg} \cdot \text{m}^2/\text{s}^2$; that is, we associate that number with the duck's motion.

The SI unit of kinetic energy (and every other type of energy) is the **joule** (J), named for James Prescott Joule, an English scientist of the 1800s. It is defined directly from Eq. 7-1 in terms of the units for mass and velocity:

$$1\ \text{joule} = 1\ \text{J} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^2. \tag{7-2}$$

Thus, the flying duck has a kinetic energy of 6.0 J.

Units

1 N.m = 1 J (joule)

1 dyne.cm = 1 erg

Example

Kinetic energy, train crash

In 1896 in Waco, Texas, William Crush parked two locomotives at opposite ends of a 6.4-km-long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed (Fig. 7-1) in front of 30,000 spectators. Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed 1.2×10^6 N and its acceleration was a constant 0.26 m/s^2, what was the total kinetic energy of the two locomotives just before the collision?

Fig. 7-1 The aftermath of an 1896 crash of two locomotives. *(Courtesy Library of Congress)*

KEY IDEAS

(1) We need to find the kinetic energy of each locomotive with Eq. 7-1, but that means we need each locomotive's speed just before the collision and its mass. (2) Because we can assume each locomotive had constant acceleration, we can use the equations in Table 2-1 to find its speed v just before the collision.

Calculations: We choose Eq. 2-16 because we know values for all the variables except v:

$$v^2 = v_0^2 + 2a(x - x_0).$$

With $v_0 = 0$ and $x - x_0 = 3.2 \times 10^3$ m (half the initial separation), this yields

$$v^2 = 0 + 2(0.26 \text{ m/s}^2)(3.2 \times 10^3 \text{ m}),$$

or

$$v = 40.8 \text{ m/s}$$

(about 150 km/h).

We can find the mass of each locomotive by dividing its given weight by g:

$$m = \frac{1.2 \times 10^6 \text{ N}}{9.8 \text{ m/s}^2} = 1.22 \times 10^5 \text{ kg}.$$

Now, using Eq. 7-1, we find the total kinetic energy of the two locomotives just before the collision as

$$K = 2(\tfrac{1}{2}mv^2) = (1.22 \times 10^5 \text{ kg})(40.8 \text{ m/s})^2$$
$$= 2.0 \times 10^8 \text{ J}. \quad \text{(Answer)}$$

This collision was like an exploding bomb.

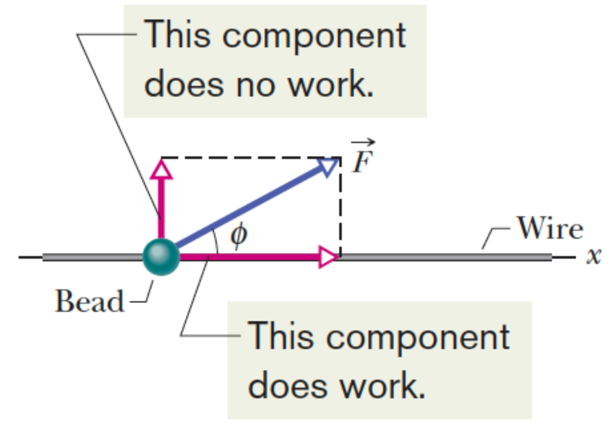
Work I

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

Finding an Expression for Work

Let us find an expression for work by considering a bead that can slide along a frictionless wire that is stretched along a horizontal x axis (Fig. 7-2). A constant force $\vec{F}$, directed at an angle ϕ to the wire, accelerates the bead along the wire. We can relate the force and the acceleration with Newton's second law, written for components along the x axis:

$$F_x = ma_x, \tag{7-3}$$



As the bead moves, the velocity changes

$$v^2 = v_0^2 + 2a_x d.$$

Using F_x in the equation above

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = F_x d.$$

The left side of the equation above tells us that the kinetic energy has been changed by the force,

Work II

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d.$$

The left side of the equation above tells us that the kinetic energy has been changed by the force. Therefore, the work W done on the bead by the force (the energy transfer due to the force) is

$$W = F_x d.$$

To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

$$W = \vec{F} \cdot \vec{d} \qquad \text{(work done by a constant force)}$$

$$W = Fd\cos\phi \qquad \text{(work done by a constant force)}$$

The work done on an object by a force can be either positive work or negative work, depending on the angle.

Work-Kinetic Energy Principle

Work – Kinetic Energy Theorem

Equation 7-5 relates the change in kinetic energy of the bead (from an initial $K_i = \frac{1}{2}mv_0^2$ to a later $K_f = \frac{1}{2}mv^2$) to the work W $(= F_x d)$ done on the bead. For such particle-like objects, we can generalize that equation. Let ΔK be the change in the kinetic energy of the object, and let W be the net work done on it. Then

$$\Delta K = K_f - K_i = W, \quad \text{(net work)} \qquad (7\text{-}10)$$

which says that

$$\begin{pmatrix}\text{change in the kinetic}\\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on}\\ \text{the particle}\end{pmatrix}.$$

We can also write

$$K_f = K_i + W, \qquad (7\text{-}11)$$

which says that

$$\begin{pmatrix}\text{kinetic energy after}\\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy}\\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net}\\ \text{work done}\end{pmatrix}.$$

These statements are known traditionally as the **work–kinetic energy theorem**

Work-kinetic energy principle:

The net work done on an object is equal to the change in the object's kinetic energy

Exercises

Ex. 05 A 145-g baseball is thrown so that it acquires a speed of 25m/s

(a) What is its kinetic energy?

(b) What was the net work done on the ball to make it reach this speed, if it started from rest?

$K=45J$ and $W_{net}=45J$

Ex. 06 How much work is required to accelerate a 1000-kg car from 20m/s to 30m/s

$$W_{net} = 2.5\mathbf{x}10^5 J$$

Exercise

Work done by two constant forces, industrial spies

Figure 7-4*a* shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m, straight toward their truck. The push $\vec{F}_1$ of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?

KEY IDEAS

(1) The net work W done on the safe by the two forces is the sum of the works they do individually. (2) Because we can treat the safe as a particle and the forces are constant in both magnitude and direction, we can use either Eq. 7-7 ($W = Fd\cos\phi$) or Eq. 7-8 ($W = \vec{F}\cdot\vec{d}$) to calculate those works. Since we know the magnitudes and directions of the forces, we choose Eq. 7-7.

Calculations: From Eq. 7-7 and the free-body diagram for the safe in Fig. 7-4*b*, the work done by $\vec{F}_1$ is

$$W_1 = F_1 d\cos\phi_1 = (12.0\text{ N})(8.50\text{ m})(\cos 30.0°) = 88.33\text{ J},$$

and the work done by $\vec{F}_2$ is

$$W_2 = F_2 d\cos\phi_2 = (10.0\text{ N})(8.50\text{ m})(\cos 40.0°) = 65.11\text{ J}.$$

Thus, the net work W is

$$W = W_1 + W_2 = 88.33\text{ J} + 65.11\text{ J} = 153.4\text{ J} \approx 153\text{ J}. \qquad \text{(Answer)}$$

During the 8.50 m displacement, therefore, the spies transfer 153 J of energy to the kinetic energy of the safe.

(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

KEY IDEA

Because these forces are constant in both magnitude and direction, we can find the work they do with Eq. 7-7.

Calculations: Thus, with mg as the magnitude of the gravitational force, we write

$$W_g = mgd\cos 90° = mgd(0) = 0 \qquad \text{(Answer)}$$

and

$$W_N = F_N d\cos 90° = F_N d(0) = 0. \qquad \text{(Answer)}$$

We should have known this result. Because these forces are perpendicular to the displacement of the safe, they do zero work on the safe and do not transfer any energy to or from it.

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

KEY IDEA

The speed of the safe changes because its kinetic energy is changed when energy is transferred to it by $\vec{F}_1$ and $\vec{F}_2$.

Calculations: We relate the speed to the work done by combining Eqs. 7-10 and 7-1:

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

The initial speed v_i is zero, and we now know that the work done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\text{ J})}{225\text{ kg}}} = 1.17\text{ m/s}. \qquad \text{(Answer)}$$

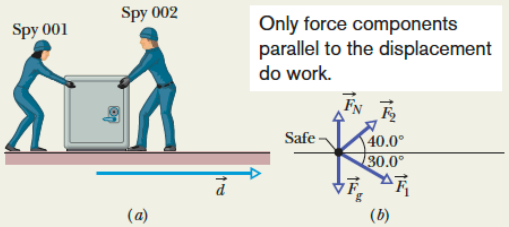


Fig. 7-4 (*a*) Two spies move a floor safe through a displacement $\vec{d}$. (*b*) A free-body diagram for the safe.

Exercise

Work done by a constant force in unit-vector notation

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0\text{ m})\hat{\text{i}}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0\text{ N})\hat{\text{i}} + (-6.0\text{ N})\hat{\text{j}}$. The situation and coordinate axes are shown in Fig. 7-5.

(a) How much work does this force do on the crate during the displacement?

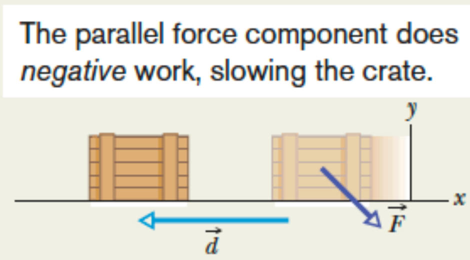


Fig. 7-5 Force $\vec{F}$ slows a crate during displacement $\vec{d}$.

KEY IDEA

Because we can treat the crate as a particle and because the wind force is constant ("steady") in both magnitude and direction during the displacement, we can use either Eq. 7-7 ($W = Fd\cos\phi$) or Eq. 7-8 ($W = \vec{F}\cdot\vec{d}$) to calculate the work. Since we know $\vec{F}$ and $\vec{d}$ in unit-vector notation, we choose Eq. 7-8.

Calculations: We write

$$W = \vec{F}\cdot\vec{d} = [(2.0\text{ N})\hat{\text{i}} + (-6.0\text{ N})\hat{\text{j}}]\cdot[(-3.0\text{ m})\hat{\text{i}}].$$

Of the possible unit-vector dot products, only $\hat{\text{i}}\cdot\hat{\text{i}}$, $\hat{\text{j}}\cdot\hat{\text{j}}$, and $\hat{\text{k}}\cdot\hat{\text{k}}$ are nonzero (see Appendix E). Here we obtain

$$W = (2.0\text{ N})(-3.0\text{ m})\hat{\text{i}}\cdot\hat{\text{i}} + (-6.0\text{ N})(-3.0\text{ m})\hat{\text{j}}\cdot\hat{\text{i}}$$
$$= (-6.0\text{ J})(1) + 0 = -6.0\text{ J}. \qquad \text{(Answer)}$$

Thus, the force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate.

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

KEY IDEA

Because the force does negative work on the crate, it reduces the crate's kinetic energy.

Calculation: Using the work–kinetic energy theorem in the form of Eq. 7-11, we have

$$K_f = K_i + W = 10\text{ J} + (-6.0\text{ J}) = 4.0\text{ J}. \qquad \text{(Answer)}$$

Less kinetic energy means that the crate has been slowed.

Work by Gravitational Force

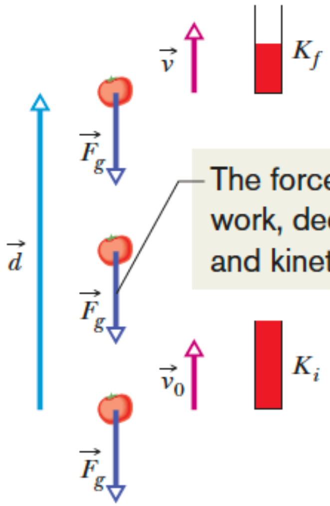


Fig. 7-6 Because the gravitational force $\vec{F}_g$ acts on it, a particle-like tomato of mass m thrown upward slows from velocity $\vec{v}_0$ to velocity $\vec{v}$ during displacement $\vec{d}$. A kinetic energy gauge indicates the resulting change in the kinetic energy of the tomato, from $K_i (= \frac{1}{2}mv_0^2)$ to $K_f (= \frac{1}{2}mv^2)$.

7-6 Work Done by the Gravitational Force

We next examine the work done on an object by the gravitational force acting on it. Figure 7-6 shows a particle-like tomato of mass m that is thrown upward with initial speed v_0 and thus with initial kinetic energy $K_i = \frac{1}{2}mv_0^2$. As the tomato rises, it is slowed by a gravitational force $\vec{F}_g$; that is, the tomato's kinetic energy decreases because $\vec{F}_g$ does work on the tomato as it rises. Because we can treat the tomato as a particle, we can use Eq. 7-7 ($W = Fd\cos\phi$) to express the work done during a displacement $\vec{d}$. For the force magnitude F, we use mg as the magnitude of $\vec{F}_g$. Thus, the work W_g done by the gravitational force $\vec{F}_g$ is

$$W_g = mgd\cos\phi \quad \text{(work done by gravitational force).} \tag{7-12}$$

For a rising object, force $\vec{F}_g$ is directed opposite the displacement $\vec{d}$, as indicated in Fig. 7-6. Thus, $\phi = 180°$ and

$W_G = - mg(y_f - y_i)$

$$W_g = mgd\cos 180° = mgd(-1) = -mgd. \tag{7-13}$$

The minus sign tells us that during the object's rise, the gravitational force acting on the object transfers energy in the amount mgd from the kinetic energy of the object. This is consistent with the slowing of the object as it rises.

After the object has reached its maximum height and is falling back down, the angle ϕ between force $\vec{F}_g$ and displacement $\vec{d}$ is zero. Thus,

$$W_g = mgd\cos 0° = mgd(+1) = +mgd. \tag{7-14}$$

Lifting an Object

Work Done in Lifting and Lowering an Object

Now suppose we lift a particle-like object by applying a vertical force $\vec{F}$ to it. During the upward displacement, our applied force does positive work W_a on the object while the gravitational force does negative work W_g on it. Our applied force tends to transfer energy to the object while the gravitational force tends to transfer energy from it. By Eq. 7-10, the change ΔK in the kinetic energy of the object due to these two energy transfers is

$$\Delta K = K_f - K_i = W_a + W_g, \tag{7-15}$$

in which K_f is the kinetic energy at the end of the displacement and K_i is that at the start of the displacement. This equation also applies if we lower the object, but then the gravitational force tends to transfer energy *to* the object while our force tends to transfer energy *from* it.

In one common situation, the object is stationary before and after the lift—for example, when you lift a book from the floor to a shelf. Then K_f and K_i are both zero, and Eq. 7-15 reduces to

$$W_a + W_g = 0$$

or

$$W_a = -W_g. \tag{7-16}$$

Note that we get the same result if K_f and K_i are not zero but are still equal.

Work done on an accelerating elevator cab

An elevator cab of mass $m = 500$ kg is descending with speed $v_i = 4.0$ m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$ (Fig. 7-8*a*).

(a) During the fall through a distance $d = 12$ m, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

KEY IDEA

We can treat the cab as a particle and thus use Eq. 7-12 ($W_g = mgd \cos \phi$) to find the work W_g.

Calculation: From Fig. 7-8*b*, we see that the angle between the directions of $\vec{F}_g$ and the cab's displacement $\vec{d}$ is 0°. Then, from Eq. 7-12, we find

$$W_g = mgd \cos 0° = (500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m})(1)$$
$$= 5.88 \times 10^4 \text{ J} \approx 59 \text{ kJ}. \qquad \text{(Answer)}$$

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

KEY IDEAS

(1) We can calculate work W_T with Eq. 7-7 ($W = Fd \cos \phi$) if we first find an expression for the magnitude T of the cable's pull. (2) We can find that expression by writing Newton's second law for components along the y axis in Fig. 7-8*b* ($F_{\text{net},y} = ma_y$).

Calculations: We get

$$T - F_g = ma. \qquad (7\text{-}18)$$

Solving for T, substituting mg for F_g, and then substituting the result in Eq. 7-7, we obtain

$$W_T = Td \cos \phi = m(a + g)d \cos \phi. \qquad (7\text{-}19)$$

Next, substituting $-g/5$ for the (downward) acceleration a and then 180° for the angle ϕ between the directions of forces $\vec{T}$ and $m\vec{g}$, we find

$$W_T = m\left(-\frac{g}{5} + g\right) d \cos \phi = \frac{4}{5} mgd \cos \phi$$
$$= \frac{4}{5} (500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m}) \cos 180°$$
$$= -4.70 \times 10^4 \text{ J} \approx -47 \text{ kJ}. \qquad \text{(Answer)}$$

Caution: Note that W_T is not simply the negative of W_g. The reason is that, because the cab accelerates during the fall, its speed changes during the fall, and thus its kinetic energy also changes. Therefore, Eq. 7-16 (which assumes that the initial and final kinetic energies are equal) does *not* apply here.

(c) What is the net work W done on the cab during the fall?

Calculation: The net work is the sum of the works done by the forces acting on the cab:

$$W = W_g + W_T = 5.88 \times 10^4 \text{ J} - 4.70 \times 10^4 \text{ J}$$
$$= 1.18 \times 10^4 \text{ J} \approx 12 \text{ kJ}. \qquad \text{(Answer)}$$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

KEY IDEA

The kinetic energy changes *because* of the net work done on the cab, according to Eq. 7-11 ($K_f = K_i + W$).

Calculation: From Eq. 7-1, we can write the kinetic energy at the start of the fall as $K_i = \frac{1}{2}mv_i^2$. We can then write Eq. 7-11 as

$$K_f = K_i + W = \tfrac{1}{2}mv_i^2 + W$$
$$= \tfrac{1}{2}(500 \text{ kg})(4.0 \text{ m/s})^2 + 1.18 \times 10^4 \text{ J}$$
$$= 1.58 \times 10^4 \text{ J} \approx 16 \text{ kJ}. \qquad \text{(Answer)}$$

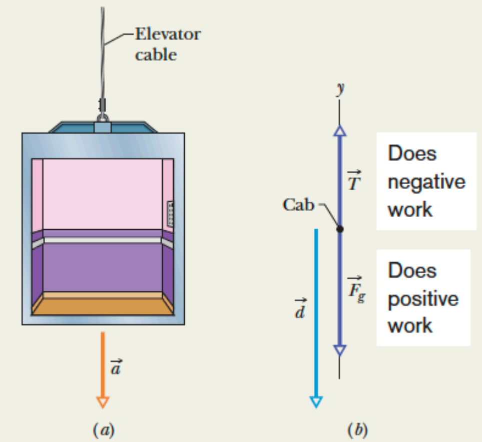


Fig. 7-8 An elevator cab, descending with speed v_i, suddenly begins to accelerate downward. (*a*) It moves through a displacement $\vec{d}$ with constant acceleration $\vec{a} = \vec{g}/5$. (*b*) A free-body diagram for the cab, displacement included.

Hooke's law

To a good approximation for many springs, the force $\vec{F}_s$ from a spring is proportional to the displacement $\vec{d}$ of the free end from its position when the spring is in the relaxed state. The *spring force* is given by

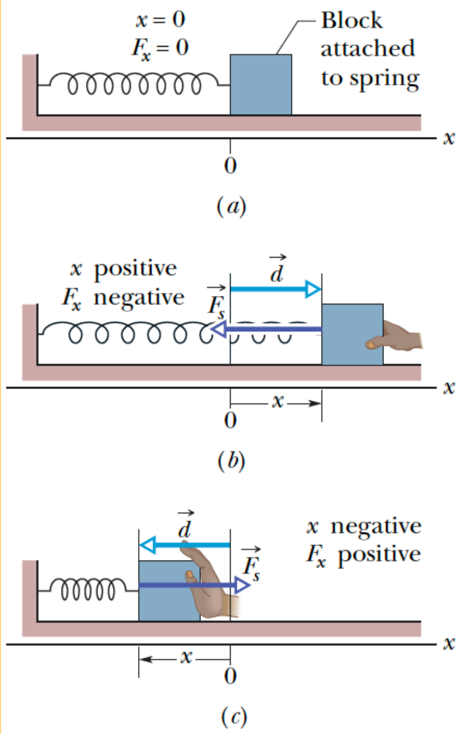
$$\vec{F}_s = -k\vec{d} \qquad \text{(Hooke's law)}, \tag{7-20}$$

which is known as **Hooke's law** after Robert Hooke, an English scientist of the late 1600s. The minus sign in Eq. 7-20 indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end. The constant k is called the **spring constant** (or **force constant**) and is a measure of the stiffness of the spring. The larger k is, the stiffer the spring; that is, the larger k is, the stronger the spring's pull or push for a given displacement. The SI unit for k is the newton per meter.

In Fig. 7-9 an x axis has been placed parallel to the length of the spring, with the origin ($x = 0$) at the position of the free end when the spring is in its relaxed state. For this common arrangement, we can write Eq. 7-20 as

$$F_x = -kx \qquad \text{(Hooke's law)}, \tag{7-21}$$

where we have changed the subscript. If x is positive (the spring is stretched toward the right on the x axis), then F_x is negative (it is a pull toward the left). If x is negative (the spring is compressed toward the left), then F_x is positive (it is a push toward the right). Note that a spring force is a *variable force* because it is a function of x, the position of the free end. Thus F_x can be symbolized as $F(x)$. Also note that Hooke's law is a *linear* relationship between F_x and x.



Work BY Elastic/Spring Force

$$W_s = \int_{x_i}^{x_f} -F_x\, dx. \qquad (7\text{-}23)$$

From Eq. 7-21, the force magnitude F_x is kx. Thus, substitution leads to

$$W_s = \int_{x_i}^{x_f} -kx\, dx = -k \int_{x_i}^{x_f} x\, dx$$

$$= (-\tfrac{1}{2}k)[x^2]_{x_i}^{x_f} = (-\tfrac{1}{2}k)(x_f^2 - x_i^2). \qquad (7\text{-}24)$$

Multiplied out, this yields

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{(work by a spring force).} \qquad (7\text{-}25)$$

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

If $x_i = 0$ and if we call the final position x, then Eq. 7-25 becomes

$$W_s = -\tfrac{1}{2}kx^2 \qquad \text{(work by a spring force).} \qquad (7\text{-}26)$$

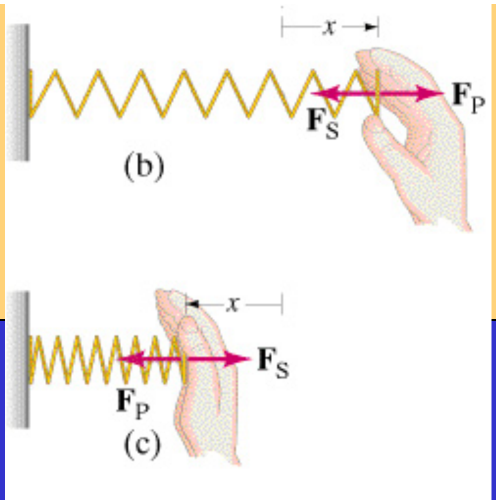
Work ON the spring

Now suppose that we displace the block along the x axis while continuing to apply a force $\vec{F}_a$ to it. During the displacement, our applied force does work W_a on the block while the spring force does work W_s. By Eq. 7-10, the change ΔK in the kinetic energy of the block due to these two energy transfers is

$$\Delta K = K_f - K_i = W_a + W_s, \tag{7-27}$$

in which K_f is the kinetic energy at the end of the displacement and K_i is that at the start of the displacement. If the block is stationary before and after the displacement, then K_f and K_i are both zero and Eq. 7-27 reduces to

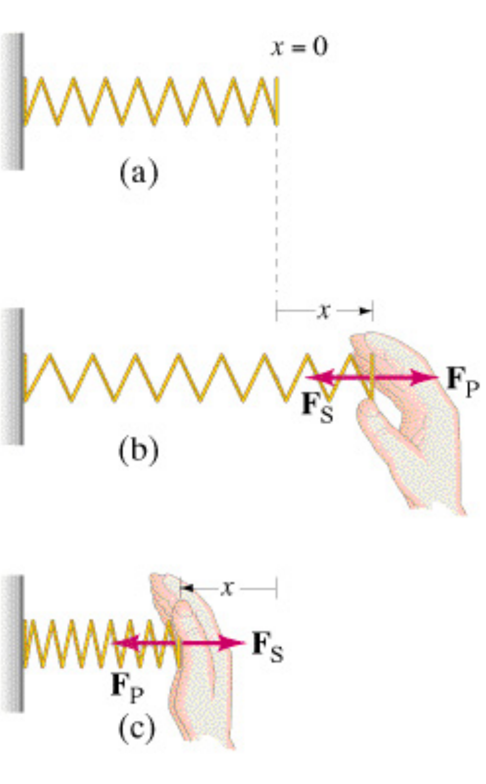
$$W_a = -W_s. \tag{7-28}$$



If the kinetic energy **CHANGES**, we need to consider BOTH works. They are NOT equal.

Force and Work ON the spring

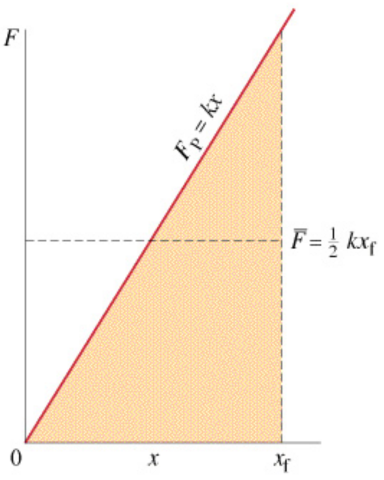
If the kinetic energy does **NOT** change, the applied force is EQUAL to the force by the spring, as in a stationary case.



Force **by the hand** on the spring $F_P = kx$
(k – spring stiffness constant)

Work done BY the hand to compress or stretch the spring. This force is NOT constant!

$$W = \tfrac{1}{2}\left(kx_f\right)x_f = \tfrac{1}{2}kx_f^{\;2}$$



Example

Work done by spring to change kinetic energy

In Fig. 7-10, a cumin canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

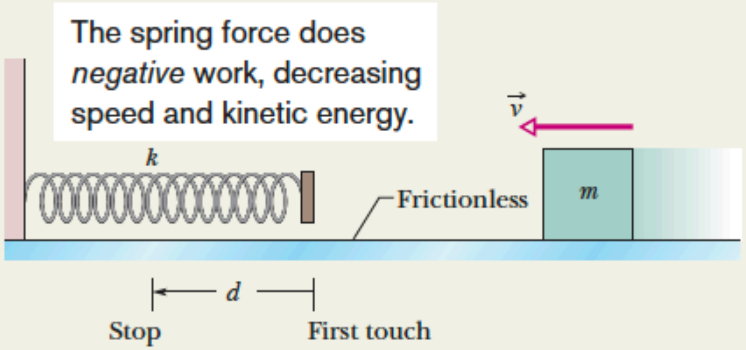


Fig. 7-10 A canister of mass m moves at velocity $\vec{v}$ toward a spring that has spring constant k.

KEY IDEAS

1. The work W_s done on the canister by the spring force is related to the requested distance d by Eq. 7-26 ($W_s = -\frac{1}{2}kx^2$), with d replacing x.
2. The work W_s is also related to the kinetic energy of the canister by Eq. 7-10 ($K_f - K_i = W$).
3. The canister's kinetic energy has an initial value of $K = \frac{1}{2}mv^2$ and a value of zero when the canister is momentarily at rest.

Calculations: Putting the first two of these ideas together, we write the work–kinetic energy theorem for the canister as

$$K_f - K_i = -\tfrac{1}{2}kd^2.$$

Substituting according to the third key idea gives us this expression

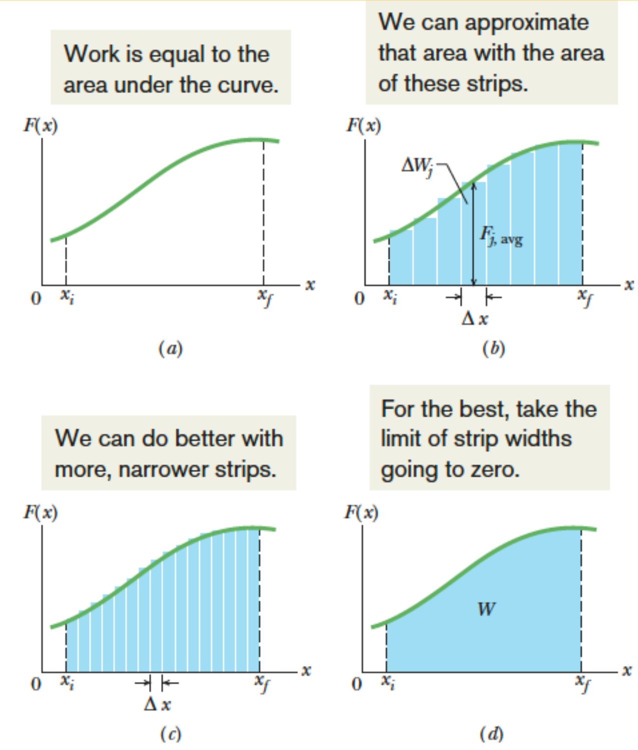
$$0 - \tfrac{1}{2}mv^2 = -\tfrac{1}{2}kd^2.$$

Simplifying, solving for d, and substituting known data then give us

$$d = v\sqrt{\frac{m}{k}} = (0.50\ \text{m/s})\sqrt{\frac{0.40\ \text{kg}}{750\ \text{N/m}}}$$

$$= 1.2 \times 10^{-2}\ \text{m} = 1.2\ \text{cm}. \qquad \text{(Answer)}$$

Work done by a general variable force



In one dimension

$$W = \int_{x_i}^{x_f} F(x)\,dx \qquad \text{(work: variable force).}$$

In three dimensions

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz.$$

Work-Kinetic Energy Theorem

$$W = \int_{x_i}^{x_f} F(x)\, dx = \int_{x_i}^{x_f} ma\, dx,$$

$$ma\, dx = m \frac{dv}{dt}\, dx.$$

From the chain rule of calculus, we have

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx} v,$$

$$ma\, dx = m \frac{dv}{dx} v\, dx = mv\, dv.$$

$$W = \int_{v_i}^{v_f} mv\, dv = m \int_{v_i}^{v_f} v\, dv$$
$$= \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2.$$

$$W = K_f - K_i = \Delta K,$$

which is the work–kinetic energy theorem.

Work calculated by graphical integration

In an epidural procedure, as used in childbirth, a surgeon or an anesthetist must run a needle through the skin on the patient's back, through various tissue layers and into a narrow region called the epidural space that lies within the spinal canal surrounding the spinal cord. The needle is intended to deliver an anesthetic fluid. This tricky procedure requires much practice so that the doctor knows when the needle has reached the epidural space and not overshot it, a mistake that could result in serious complications.

The feel a doctor has for the needle's penetration is the variable force that must be applied to advance the needle through the tissues. Figure 7-12*a* is a graph of the force magnitude F versus displacement x of the needle tip in a typical epidural procedure. (The line segments have been straightened somewhat from the original data.) As x increases from 0, the skin resists the needle, but at $x = 8.0$ mm the force is finally great enough to pierce the skin, and then the required force decreases. Similarly, the needle finally pierces the interspinous ligament at $x = 18$ mm and the relatively tough ligamentum flavum at $x = 30$ mm. The needle then enters the epidural space (where it is to deliver the anesthetic fluid), and the force drops sharply. A new doctor must learn this pattern of force versus displacement to recognize when to stop pushing on the needle. (This is the pattern to be programmed into a virtual-reality simulation of an epidural procedure.) How much work W is done by the force exerted on the needle to get the needle to the epidural space at $x = 30$ mm?

KEY IDEAS

(1) We can calculate the work W done by a variable force $F(x)$ by integrating the force versus position x. Equation 7-32 tells us that

$$W = \int_{x_i}^{x_f} F(x)\,dx.$$

We want the work done by the force during the displacement from $x_i = 0$ to $x_f = 0.030$ m. (2) We can evaluate the integral by finding the area under the curve on the graph of Fig. 7-12*a*.

$$W = \begin{pmatrix}\text{area between force curve} \\ \text{and } x \text{ axis, from } x_i \text{ to } x_f\end{pmatrix}.$$

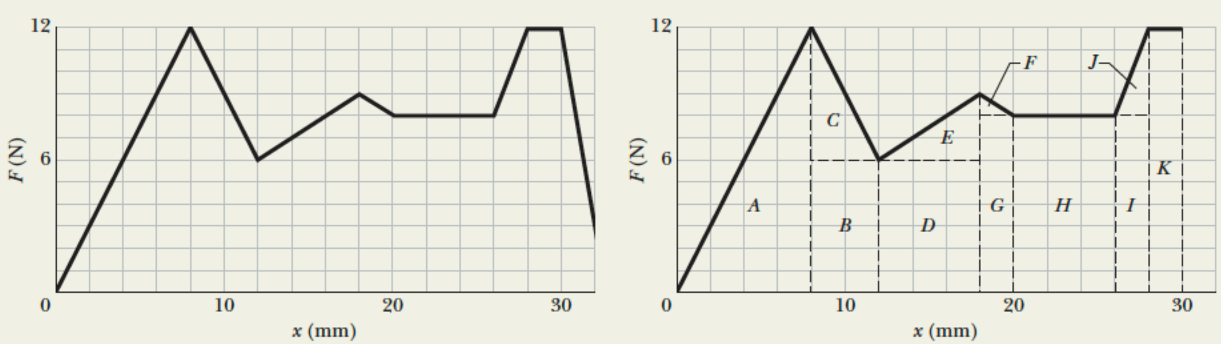
Calculations: Because our graph consists of straight-line segments, we can find the area by splitting the region below the curve into rectangular and triangular regions, as shown in Fig. 7-12*b*. For example, the area in triangular region A is

$$\text{area}_A = \tfrac{1}{2}(0.0080\text{ m})(12\text{ N}) = 0.048\text{ N}\cdot\text{m} = 0.048\text{ J}.$$

Once we've calculated the areas for all the labeled regions in Fig. 7-12*b*, we find that the total work is

$$\begin{aligned} W &= (\text{sum of the areas of regions } A \text{ through } K) \\ &= 0.048 + 0.024 + 0.012 + 0.036 + 0.009 + 0.001 \\ &\quad + 0.016 + 0.048 + 0.016 + 0.004 + 0.024 \\ &= 0.238\text{ J}. \end{aligned}$$

(Answer)



(*a*) (*b*)

Example

Work, two-dimensional integration

Force $\vec{F} = (3x^2 \text{ N})\hat{i} + (4 \text{ N})\hat{j}$, with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

KEY IDEA

The force is a variable force because its x component depends on the value of x. Thus, we cannot use Eqs. 7-7 and 7-8 to find the work done. Instead, we must use Eq. 7-36 to integrate the force.

Calculation: We set up two integrals, one along each axis:

$$W = \int_2^3 3x^2\,dx + \int_3^0 4\,dy = 3\int_2^3 x^2\,dx + 4\int_3^0 dy$$

$$= 3[\tfrac{1}{3}x^3]_2^3 + 4[y]_3^0 = [3^3 - 2^3] + 4[0 - 3]$$

$$= 7.0 \text{ J.} \qquad \text{(Answer)}$$

The positive result means that energy is transferred to the particle by force $\vec{F}$. Thus, the kinetic energy of the particle increases and, because $K = \frac{1}{2}mv^2$, its speed must also increase. If the work had come out negative, the kinetic energy and speed would have decreased.

Power

Power is the rate at which work is done

The time rate at which work is done by a force is said to be the **power** due to the force. If a force does an amount of work W in an amount of time Δt, the **average power** due to the force during that time interval is

$$P_{avg} = \frac{W}{\Delta t} \quad \text{(average power).} \tag{7-42}$$

The **instantaneous power** P is the instantaneous time rate of doing work, which we can write as

$$P = \frac{dW}{dt} \quad \text{(instantaneous power).} \tag{7-43}$$

Suppose we know the work $W(t)$ done by a force as a function of time. Then to get the instantaneous power P at, say, time $t = 3.0$ s during the work, we would first take the time derivative of $W(t)$ and then evaluate the result for $t = 3.0$ s.

Power

Power is the rate at which work is done

$$\bar{P} = \text{average power} = \frac{\text{work}}{\text{time}} = \frac{\text{energy transformed}}{\text{time}}$$

In the SI system, the units of power are watts:

$$1\text{ W} = 1\text{ J/s}$$

The difference between walking and running up these stairs is power – the change in gravitational potential energy is **the same.**

Ex. 6-14 Compare the power of a 60-kg person to climb 4.5 m in 2.0s and in 4.0 s

$$\bar{P} = \frac{W}{t} = \frac{mgy}{t}$$

in 4.0s: power=660W
in 2.0s: power=1320W

Power

Power is the rate at which work is done

$$\overline{P} = \text{average power} = \frac{\text{work}}{\text{time}} = \frac{\text{energy transformed}}{\text{time}}$$

In the SI system, the units of power are watts:

$$1\ \text{W} = 1\ \text{J/s}$$

Inspection of Eq. 7-42 shows that work can be expressed as power multiplied by time, as in the common unit kilowatt-hour. Thus,

$$\begin{aligned} 1\ \text{kilowatt-hour} &= 1\ \text{kW} \cdot \text{h} = (10^3\ \text{W})(3600\ \text{s}) \\ &= 3.60 \times 10^6\ \text{J} = 3.60\ \text{MJ}. \end{aligned} \qquad (7\text{-}46)$$

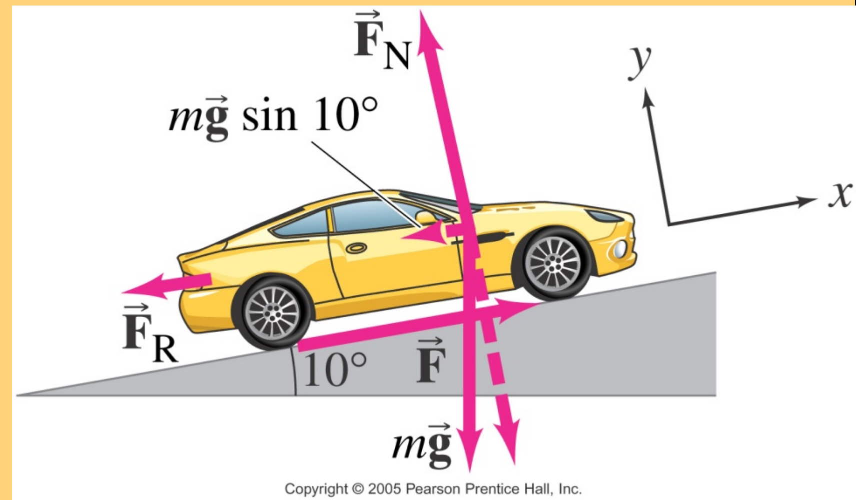
Average Power

Power is also needed for acceleration and for moving against the force of gravity.

The average power can be written in terms of the force and the average velocity:

$$\overline{P} = \frac{W}{t} = \frac{Fd}{t} = F\overline{v}.$$

Ex. 6-15 Calculate the power required for a 1400-kg car to climb a10 degree hill at a steady 22m/s. Assume the retarding force on the car F_R=700N.



$$F = 700N + mg\sin 10^o$$

$$\overline{P} = F\overline{v} = 6.80\times 10^4 W$$

Instantaneous Power

$$P = \frac{dW}{dt} = \frac{F\cos\phi\, dx}{dt} = F\cos\phi\left(\frac{dx}{dt}\right),$$

$$P = Fv\cos\phi.$$

$$P = \vec{F}\cdot\vec{v} \qquad \text{(instantaneous power).}$$

Power, force, and velocity

Figure 7-14 shows constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?

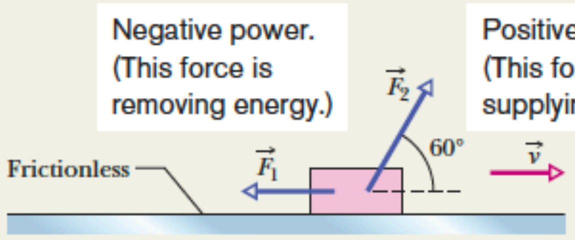


Fig. 7-14 Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a box that slides rightward across a frictionless floor. The velocity of the box is $\vec{v}$.

KEY IDEA

We want an instantaneous power, not an average power over a time period. Also, we know the box's velocity (rather than the work done on it).

Calculation: We use Eq. 7-47 for each force. For force $\vec{F}_1$, at angle $\phi_1 = 180°$ to velocity $\vec{v}$, we have

$$P_1 = F_1 v\cos\phi_1 = (2.0\text{ N})(3.0\text{ m/s})\cos 180° = -6.0\text{ W}. \qquad \text{(Answer)}$$

This negative result tells us that force $\vec{F}_1$ is transferring energy *from* the box at the rate of 6.0 J/s.

For force $\vec{F}_2$, at angle $\phi_2 = 60°$ to velocity $\vec{v}$, we have

$$P_2 = F_2 v\cos\phi_2 = (4.0\text{ N})(3.0\text{ m/s})\cos 60° = 6.0\text{ W}. \qquad \text{(Answer)}$$

This positive result tells us that force $\vec{F}_2$ is transferring energy *to* the box at the rate of 6.0 J/s.

The net power is the sum of the individual powers:

$$P_{net} = P_1 + P_2 = -6.0\text{ W} + 6.0\text{ W} = 0, \qquad \text{(Answer)}$$

which tells us that the net rate of transfer of energy to or from the box is zero. Thus, the kinetic energy ($K = \frac{1}{2}mv^2$) of the box is not changing, and so the speed of the box will remain at 3.0 m/s. With neither the forces $\vec{F}_1$ and $\vec{F}_2$ nor the velocity $\vec{v}$ changing, we see from Eq. 7-48 that P_1 and P_2 are constant and thus so is P_{net}.