Circular Motion

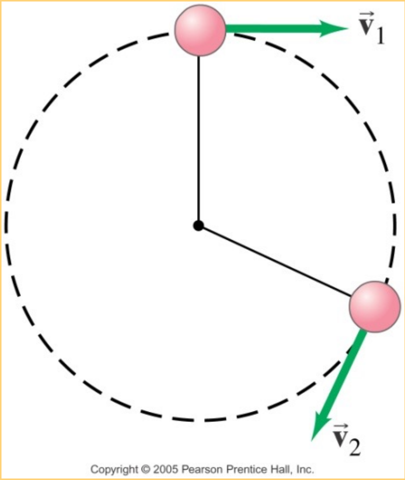
We need a net force to change the velocity
its *magnitude* or its ***direction***

Uniform Circular Motion:

motion in a circle of constant radius at constant speed

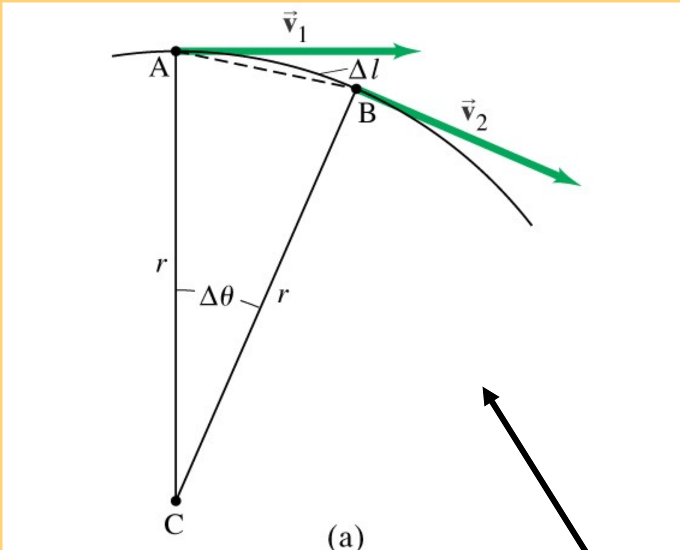
direction is continuously changing

Instantaneous velocity is always tangent to circle.



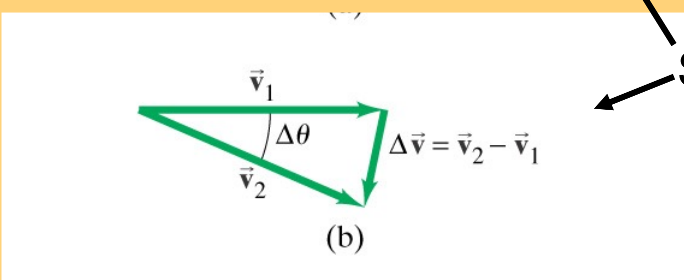
Attention!!! The material covered here is in Sec.4.7 and in Ch.6 of your book.

Centripetal Acceleration



$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

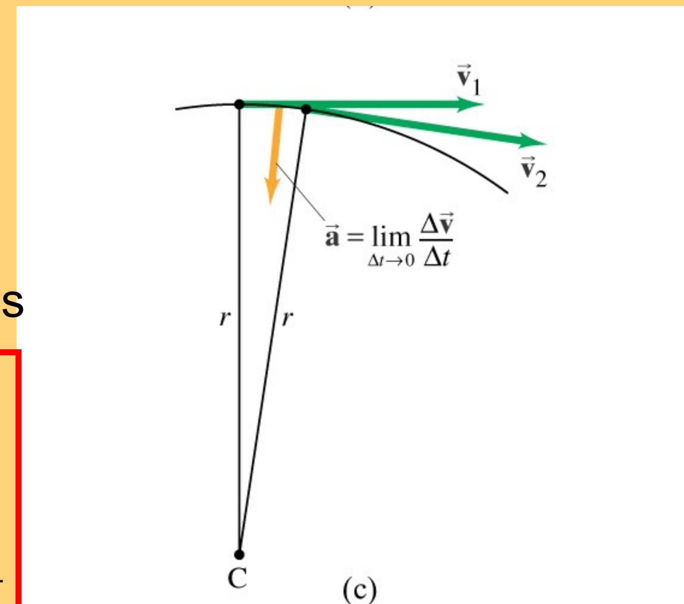
Acceleration points towards the center
centripetal or radial



Similar triangles

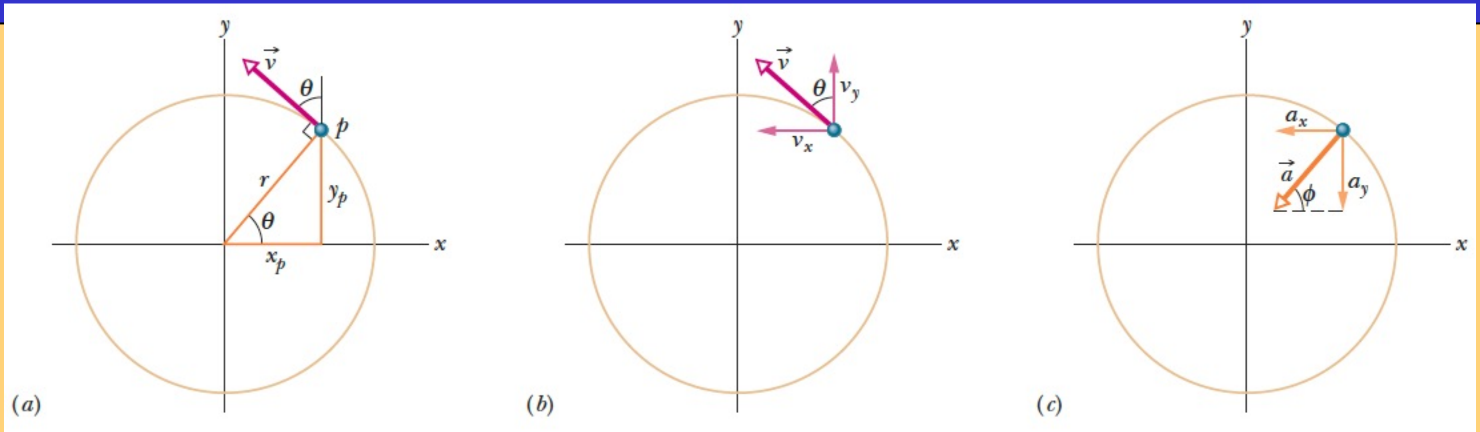
$$\frac{\Delta v}{v} = \frac{\Delta l}{r}$$

$$a_R = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$



Even though the magnitude of the velocity may not change, the direction changes, so there is an acceleration.

Centripetal Acceleration I



The scalar components of $\vec{v}$ are shown in Fig. 4-17*b*. With them, we can write the velocity $\vec{v}$ as

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} = (-v \sin\theta)\hat{\mathrm{i}} + (v\cos\theta)\hat{\mathrm{j}}. \quad (4\text{-}36)$$

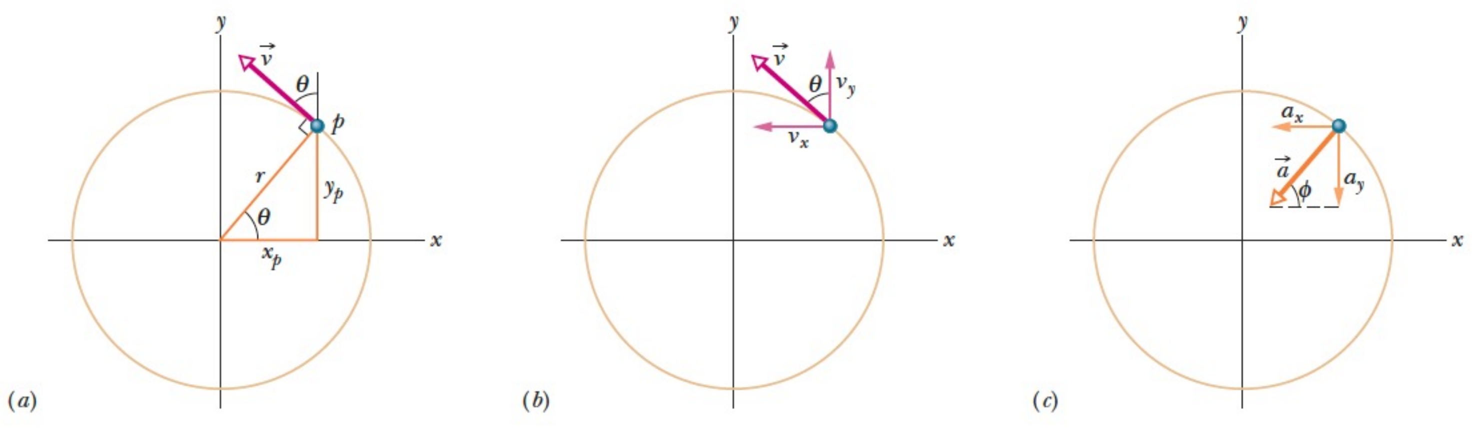
Now, using the right triangle in Fig. 4-17*a*, we can replace $\sin\theta$ with y_p/r and $\cos\theta$ with x_p/r to write

$$\vec{v} = \left(-\frac{vy_p}{r}\right)\hat{\mathrm{i}} + \left(\frac{vx_p}{r}\right)\hat{\mathrm{j}}. \quad (4\text{-}37)$$

To find the acceleration $\vec{a}$ of particle *p*, we must take the time derivative of this equation. Noting that speed *v* and radius *r* do not change with time, we obtain

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(-\frac{v}{r}\frac{dy_p}{dt}\right)\hat{\mathrm{i}} + \left(\frac{v}{r}\frac{dx_p}{dt}\right)\hat{\mathrm{j}}. \quad (4\text{-}38)$$

Centripetal Acceleration II



Now note that the rate dy_p/dt at which y_p changes is equal to the velocity component v_y. Similarly, $dx_p/dt = v_x$, and, again from Fig. 4-17*b*, we see that $v_x = -v \sin\theta$ and $v_y = v\cos\theta$. Making these substitutions in Eq. 4-38, we find

$$\vec{a} = \left(-\frac{v^2}{r}\cos\theta\right)\hat{\mathrm{i}} + \left(-\frac{v^2}{r}\sin\theta\right)\hat{\mathrm{j}}. \qquad (4\text{-}39)$$

This vector and its components are shown in Fig. 4-17*c*. Following Eq. 3-6, we find

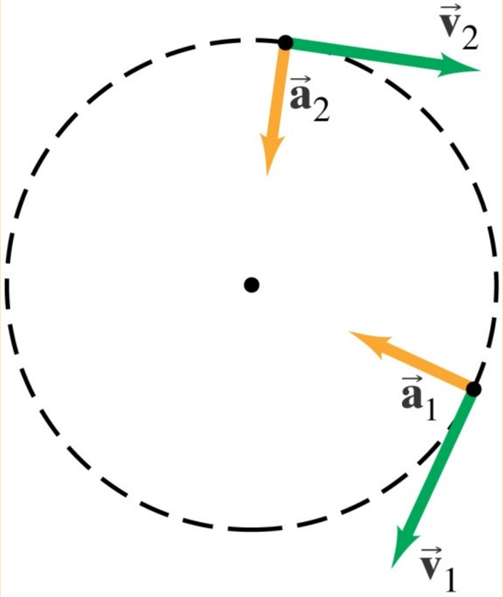
$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}\sqrt{(\cos\theta)^2 + (\sin\theta)^2} = \frac{v^2}{r}\sqrt{1} = \frac{v^2}{r},$$

as we wanted to prove. To orient $\vec{a}$, we find the angle ϕ shown in Fig. 4-17*c*:

$$\tan\phi = \frac{a_y}{a_x} = \frac{-(v^2/r)\sin\theta}{-(v^2/r)\cos\theta} = \tan\theta.$$

Thus, $\phi = \theta$, which means that $\vec{a}$ is directed along the radius r of Fig. 4-17*a*, toward the circle's center, as we wanted to prove.

Period and Frequency



Period is the time to complete a revolution

Frequency is the number of revolutions per second

$$T = \frac{1}{f}$$

$$v = \frac{2\pi r}{T}$$

Ex. 01 A ball at the end of a string is revolving uniformly in a ***horizontal*** circle of radius 0.600 m. The ball makes 2 revolutions per second. What is its centripetal acceleration?

Ex.02 The Moon's circular orbit about the Earth has a radius of 384000km and a period T of 27.3 days. What is the acceleration of the Moon toward the Earth

Exercise

Top gun pilots in turns

"Top gun" pilots have long worried about taking a turn too tightly. As a pilot's body undergoes centripetal acceleration, with the head toward the center of curvature, the blood pressure in the brain decreases, leading to loss of brain function.

There are several warning signs. When the centripetal acceleration is $2g$ or $3g$, the pilot feels heavy. At about $4g$, the pilot's vision switches to black and white and narrows to "tunnel vision." If that acceleration is sustained or increased, vision ceases and, soon after, the pilot is unconscious—a condition known as g-LOC for "g-induced loss of consciousness."

What is the magnitude of the acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_i = (400\hat{\text{i}} + 500\hat{\text{j}})$ m/s and 24.0 s later leaves the turn with a velocity of $\vec{v}_f = (-400\hat{\text{i}} - 500\hat{\text{j}})$ m/s?

KEY IDEAS

We assume the turn is made with uniform circular motion. Then the pilot's acceleration is centripetal and has magnitude a given by Eq. 4-34 ($a = v^2/R$), where R is the circle's radius. Also, the time required to complete a full circle is the period given by Eq. 4-35 ($T = 2\pi R/v$).

Calculations: Because we do not know radius R, let's solve Eq. 4-35 for R and substitute into Eq. 4-34. We find

$$a = \frac{2\pi v}{T}.$$

Speed v here is the (constant) magnitude of the velocity during the turning. Let's substitute the components of the initial velocity into Eq. 3-6:

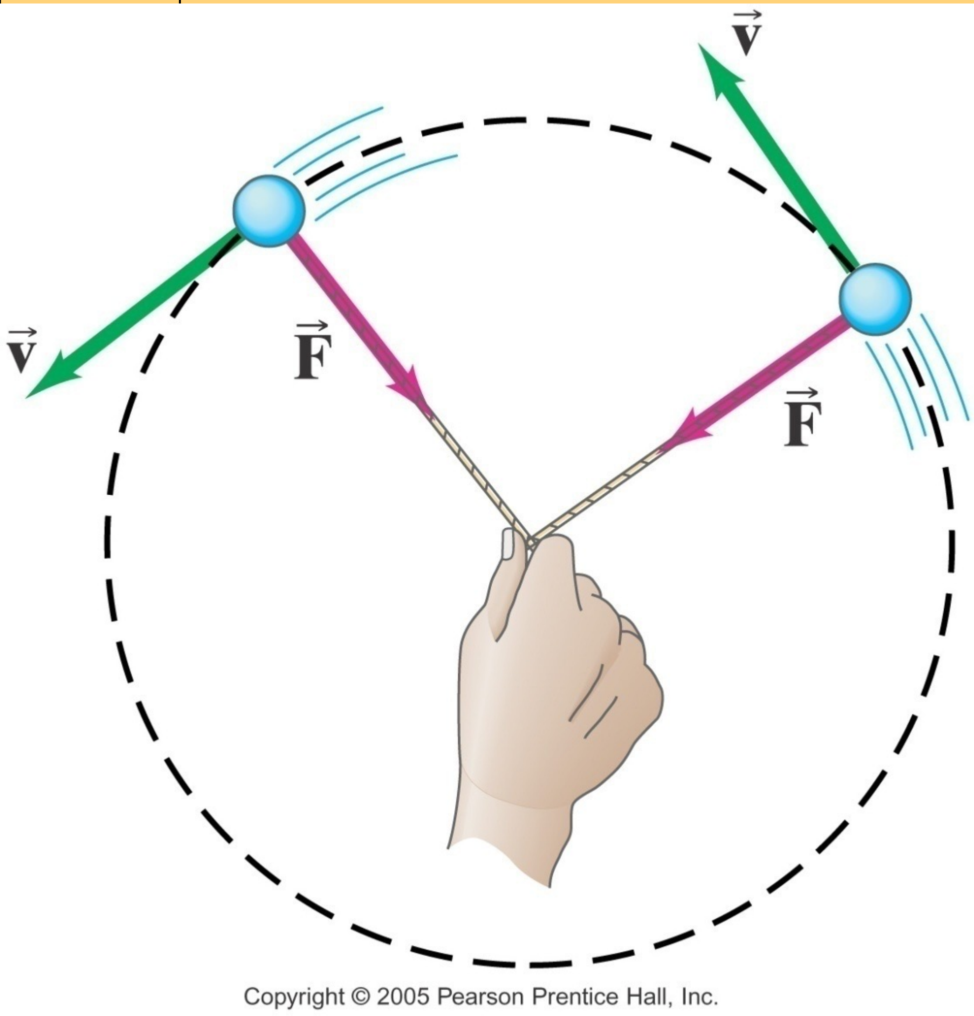
$$v = \sqrt{(400 \text{ m/s})^2 + (500 \text{ m/s})^2} = 640.31 \text{ m/s}.$$

To find the period T of the motion, first note that the final velocity is the reverse of the initial velocity. This means the aircraft leaves on the opposite side of the circle from the initial point and must have completed half a circle in the given 24.0 s. Thus a full circle would have taken $T = 48.0$ s. Substituting these values into our equation for a, we find

$$a = \frac{2\pi(640.31 \text{ m/s})}{48.0 \text{ s}} = 83.81 \text{ m/s}^2 \approx 8.6g. \qquad \text{(Answer)}$$

Dynamics of Uniform Circular Motion

For an object to be in uniform circular motion, there must be a net force acting on it.



HORIZONTAL motion

We already know the acceleration, so we can immediately write the force:

$$\Sigma F_R = ma_R = m\frac{v^2}{r}$$

Ex.-03 What is the force a person has to exert in Ex.5-1 if m=150g?

What happens if the ball is released?

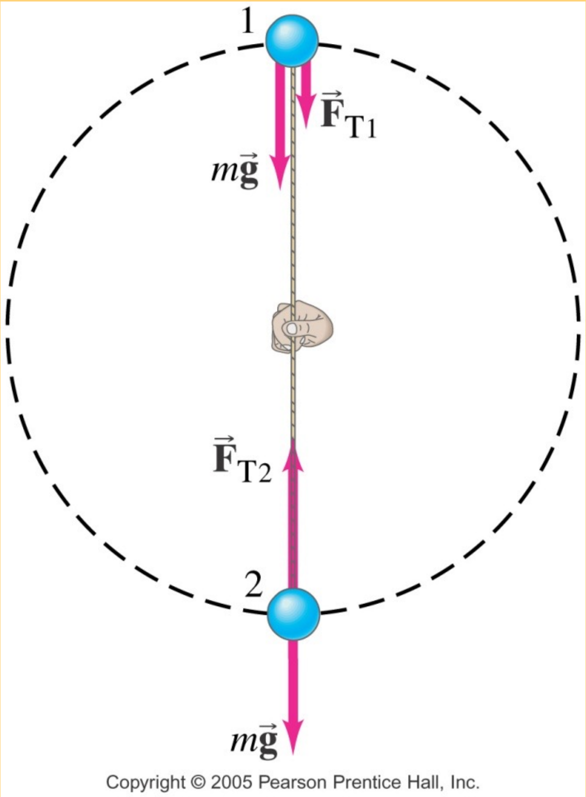
It flies off tangentially

Example (Vertical Circle)

A 0.150-kg ball on the end of a 1.10 m-long cord is swung in a ***VERTICAL*** circle.

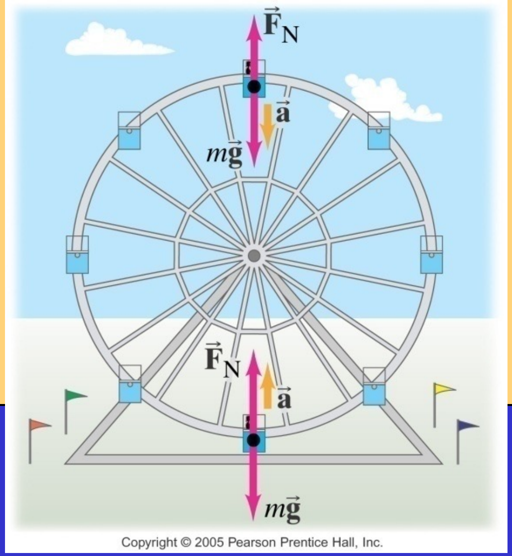
(a) Determine the minimum speed the ball must have at the top to keep the circular motion.

(b) Calculate the tension in the cord at the bottom assuming the ball is moving at twice the speed of part (a)



(a) $v_a = \sqrt{gr}$

(b) $v_b = 2\sqrt{gr} \Rightarrow F_T = 5mg$



Ex05. (Ferris wheel) Normal force at the top is less, more, or equal to the normal force at the bottom?

Example (Vertical Circular Loop)

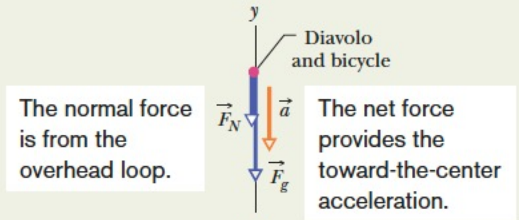
Vertical circular loop, Diavolo

In a 1901 circus performance, Allo "Dare Devil" Diavolo introduced the stunt of riding a bicycle in a loop-the-loop (Fig. 6-9*a*). Assuming that the loop is a circle with radius $R = 2.7$ m, what is the least speed v that Diavolo and his bicycle could have at the top of the loop to remain in contact with it there?

KEY IDEA

We can assume that Diavolo and his bicycle travel through the top of the loop as a single particle in uniform circular motion. Thus, at the top, the acceleration $\vec{a}$ of this particle must have the magnitude $a = v^2/R$ given by Eq. 6-17 and be directed downward, toward the center of the circular loop.

(*a*)



(*b*)

Fig. 6-9 (*a*) Contemporary advertisement for Diavolo and (*b*) free-body diagram for the performer at the top of the loop. *(Photograph in part a reproduced with permission of Circus World Museum)*

Calculations: The forces on the particle when it is at the top of the loop are shown in the free-body diagram of Fig 6-9*b*. The gravitational force $\vec{F}_g$ is downward along a y axis; so is the normal force $\vec{F}_N$ on the particle from the loop; so also is the centripetal acceleration of the particle. Thus, Newton's second law for y components ($F_{net,y} = ma_y$) gives us

$$-F_N - F_g = m(-a)$$

and

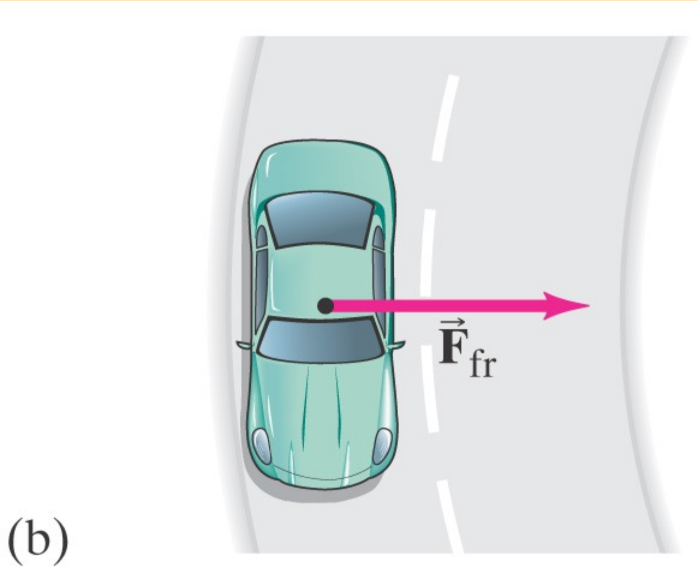
$$-F_N - mg = m\left(-\frac{v^2}{R}\right). \qquad (6\text{-}19)$$

If the particle has the *least speed v* needed to remain in contact, then it is on the *verge of losing contact* with the loop (falling away from the loop), which means that $F_N = 0$ at the top of the loop (the particle and loop touch but without any normal force). Substituting 0 for F_N in Eq. 6-19, solving for v, and then substituting known values give us

$$v = \sqrt{gR} = \sqrt{(9.8\ \text{m/s}^2)(2.7\ \text{m})}$$
$$= 5.1\ \text{m/s}. \qquad \text{(Answer)}$$

Comments: Diavolo made certain that his speed at the top of the loop was greater than 5.1 m/s so that he did not lose contact with the loop and fall away from it. Note that this speed requirement is independent of the mass of Diavolo and his bicycle. Had he feasted on, say, pierogies before his performance, he still would have had to exceed only 5.1 m/s to maintain contact as he passed through the top of the loop.

Highway Curves



It is the ***FRICTION*** force that allows a car to round a curve.
It points toward the center of the curve.

If the tires roll without sliding, the bottom of the tire is at rest against the road
Static friction force

If the static friction is not enough to keep the circular motion, the car slides.
The friction force becomes ***kinetic***

Ex. 06 A 1000-kg car rounds a curve on a flat road of radius 50 m at a speed of 14 m/s. Will the car follow the curve or skid? Assume:
(a) Pavement is dry, coefficient of static friction = 0.60
(b) Pavement is wet, coefficient of static friction = 0.25

Negative Lift I

Car in flat circular turn

Upside-down racing: A modern race car is designed so that the passing air pushes down on it, allowing the car to travel much faster through a flat turn in a Grand Prix without friction failing. This downward push is called *negative lift*. Can a race car have so much negative lift that it could be driven upside down on a long ceiling, as done fictionally by a sedan in the first *Men in Black* movie?

Figure 6-10*a* represents a Grand Prix race car of mass $m = 600$ kg as it travels on a flat track in a circular arc of radius $R = 100$ m. Because of the shape of the car and the wings on it, the passing air exerts a negative lift $\vec{F}_L$ downward on the car. The coefficient of static friction between the tires and the track is 0.75. (Assume that the forces on the four tires are identical.)

(a) If the car is on the verge of sliding out of the turn when its speed is 28.6 m/s, what is the magnitude of the negative lift $\vec{F}_L$ acting downward on the car?

KEY IDEAS

1. A centripetal force must act on the car because the car is moving around a circular arc; that force must be directed toward the center of curvature of the arc (here, that is horizontally).
2. The only horizontal force acting on the car is a frictional force on the tires from the road. So the required centripetal force is a frictional force.
3. Because the car is not sliding, the frictional force must be a *static* frictional force $\vec{f}_s$ (Fig. 6-10*a*).
4. Because the car is on the verge of sliding, the magnitude f_s is equal to the maximum value $f_{s,max} = \mu_s F_N$, where F_N is the magnitude of the normal force $\vec{F}_N$ acting on the car from the track.

Radial calculations: The frictional force $\vec{f}_s$ is shown in the free-body diagram of Fig. 6-10*b*. It is in the negative direc-

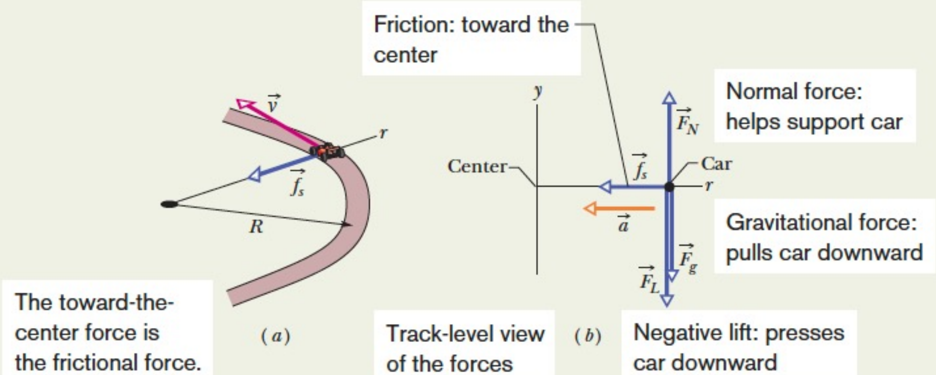


Fig. 6-10 (*a*) A race car moves around a flat curved track at constant speed *v*. The frictional force $\vec{f}_s$ provides the necessary centripetal force along a radial axis *r*. (*b*) A free-body diagram (not to scale) for the car, in the vertical plane containing *r*.

Negative Lift II

tion of a radial axis r that always extends from the center of curvature through the car as the car moves. The force produces a centripetal acceleration of magnitude v^2/R. We can relate the force and acceleration by writing Newton's second law for components along the r axis ($F_{net,r} = ma_r$) as

$$-f_s = m\left(-\frac{v^2}{R}\right). \tag{6-20}$$

Substituting $f_{s,max} = \mu_s F_N$ for f_s leads us to

$$\mu_s F_N = m\left(\frac{v^2}{R}\right). \tag{6-21}$$

Vertical calculations: Next, let's consider the vertical forces on the car. The normal force $\vec{F}_N$ is directed up, in the positive direction of the y axis in Fig. 6-10*b*. The gravitational force $\vec{F}_g = m\vec{g}$ and the negative lift $\vec{F}_L$ are directed down. The acceleration of the car along the y axis is zero. Thus we can write Newton's second law for components along the y axis ($F_{net,y} = ma_y$) as

$$F_N - mg - F_L = 0,$$

or

$$F_N = mg + F_L. \tag{6-22}$$

Combining results: Now we can combine our results along the two axes by substituting Eq. 6-22 for F_N in Eq. 6-21. Doing so and then solving for F_L lead to

$$F_L = m\left(\frac{v^2}{\mu_s R} - g\right)$$
$$= (600 \text{ kg})\left(\frac{(28.6 \text{ m/s})^2}{(0.75)(100 \text{ m})} - 9.8 \text{ m/s}^2\right)$$
$$= 663.7 \text{ N} \approx 660 \text{ N}. \quad \text{(Answer)}$$

(b) The magnitude F_L of the negative lift on a car depends on the square of the car's speed v^2, just as the drag force does (Eq. 6-14). Thus, the negative lift on the car here is greater when the car travels faster, as it does on a straight section of track. What is the magnitude of the negative lift for a speed of 90 m/s?

KEY IDEA

F_L is proportional to v^2.

Calculations: Thus we can write a ratio of the negative lift $F_{L,90}$ at $v = 90$ m/s to our result for the negative lift F_L at $v = 28.6$ m/s as

$$\frac{F_{L,90}}{F_L} = \frac{(90 \text{ m/s})^2}{(28.6 \text{ m/s})^2}.$$

Substituting our known negative lift of $F_L = 663.7$ N and solving for $F_{L,90}$ give us

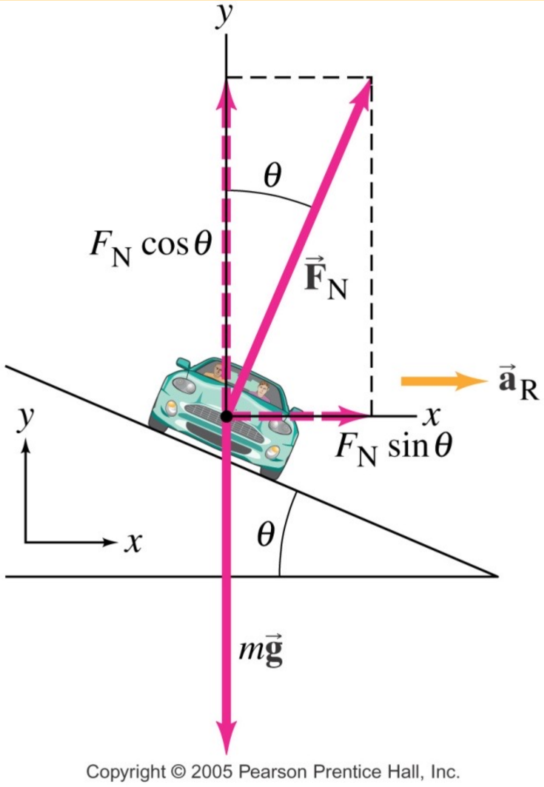
$$F_{L,90} = 6572 \text{ N} \approx 6600 \text{ N}. \quad \text{(Answer)}$$

Upside-down racing: The gravitational force is, of course, the force to beat if there is a chance of racing upside down:

$$F_g = mg = (600 \text{ kg})(9.8 \text{ m/s}^2)$$
$$= 5880 \text{ N}.$$

With the car upside down, the negative lift is an *upward* force of 6600 N, which exceeds the downward 5880 N. Thus, the car could run on a long ceiling *provided* that it moves at about 90 m/s (= 324 km/h = 201 mi/h). However, moving that fast while right side up on a horizontal track is dangerous enough, so you are not likely to see upside-down racing except in the movies.

Banked Curves



The banking of curves reduce the chance of skidding

For a given angle, there is one speed for which no friction is required to keep the circular motion

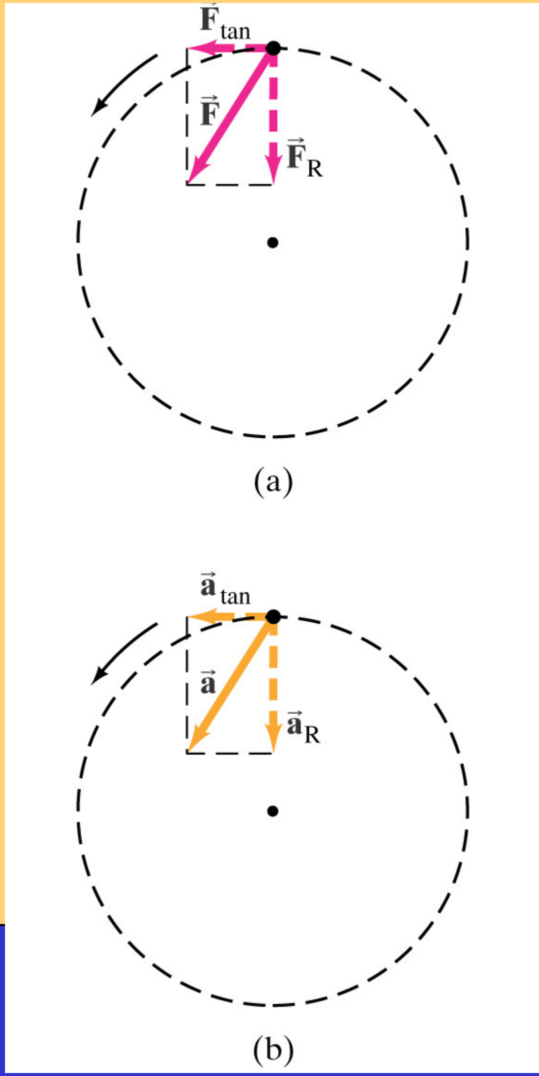
$$F_N \sin\theta = m\frac{v^2}{r}$$

Ex. 07 For a car traveling at speed v around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required

$$\tan\theta = \frac{v^2}{rg}$$

NONuniform Circular Motion

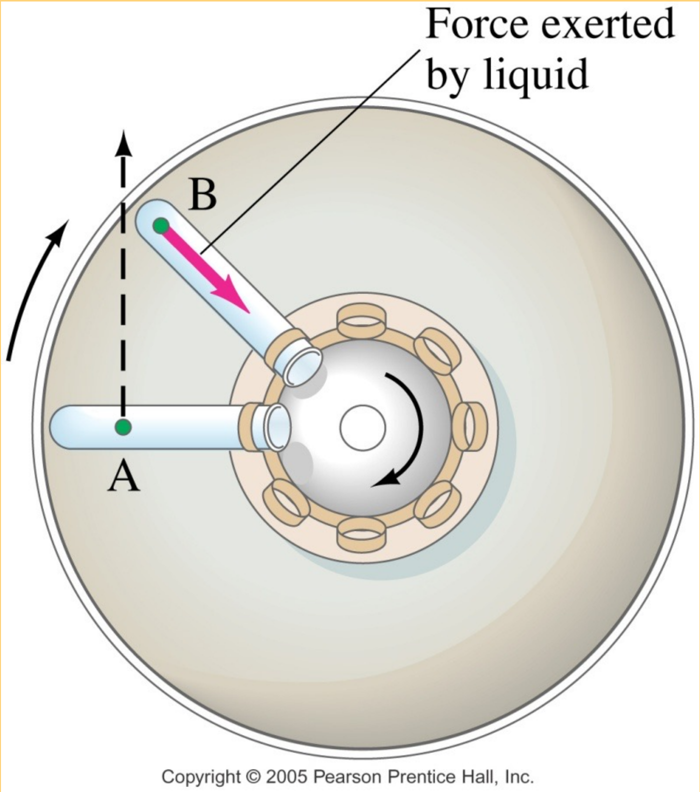
The speed of a object moving in a circle changes if the force on it has a tangential component



$$a_R = \frac{v^2}{r} \qquad a_{tan} = \frac{\Delta v}{\Delta t}$$

$$a = \sqrt{a_R^{\ 2} + a_{tan}^{\ 2}}$$

Centrifugation



A centrifuge works by spinning very fast. This means there must be a very large centripetal force.

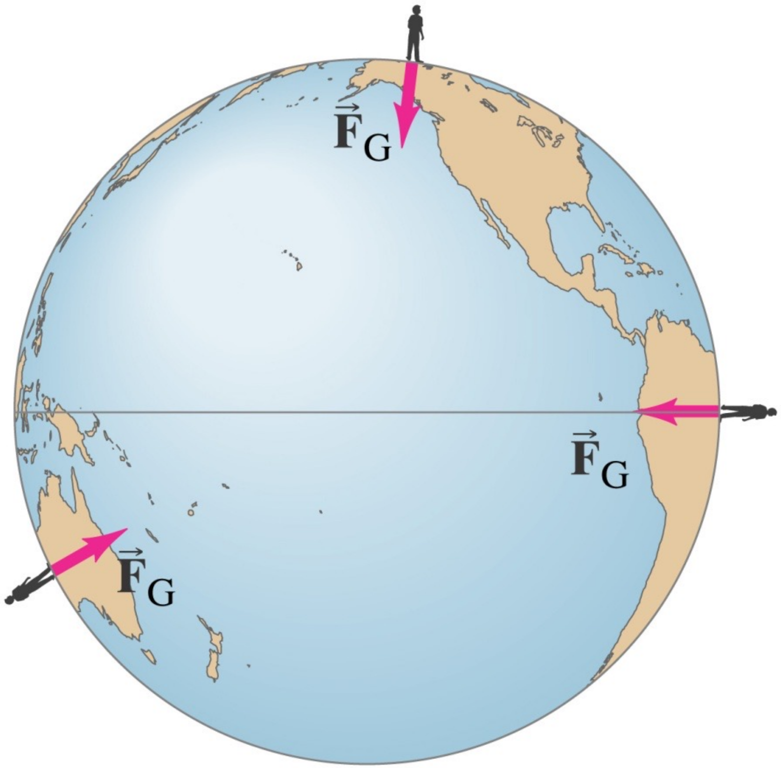
The resistance of the fluid does not equal the centripetal force and the particles eventually reach the bottom of the tube.

Gravitation

Gravitation is covered by your book in Chapter 13. We discuss here what is contained in Ch13.1-Ch.13.2 and a little bit, in a simplified language, what is In Ch.13.6-Ch.13.7.

Newton's Law of Universal Gravitation

What exerts the force of gravity? Every object on Earth feels it and it always points towards the center of the Earth.



Newton's concluded that it must be the Earth that exerts the gravitational force.
(legend: falling apple)

He further realized that this force must be what keeps the Moon in its orbit.

This forces decreases with the square of the distance from the Earth's center.
(Ex.5.2: a~g/3600)

Action and reaction – the force is proportional to both masses.

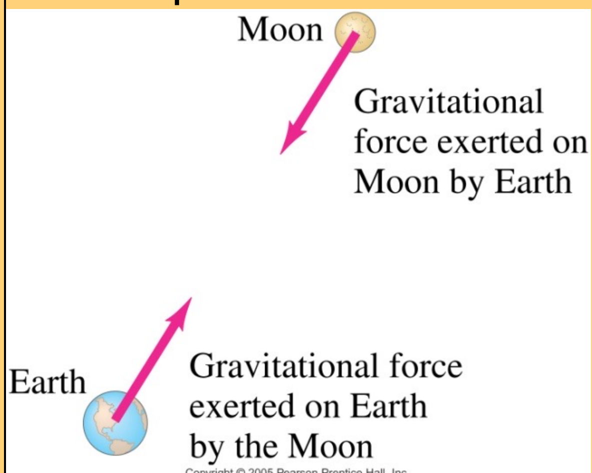
He further concluded that this force should also keep the planets in their orbits – therefore it should be a force between all objects!

Newton proposed a law of **universal** gravitation

Law of Universal Gravitation

Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.

Example:



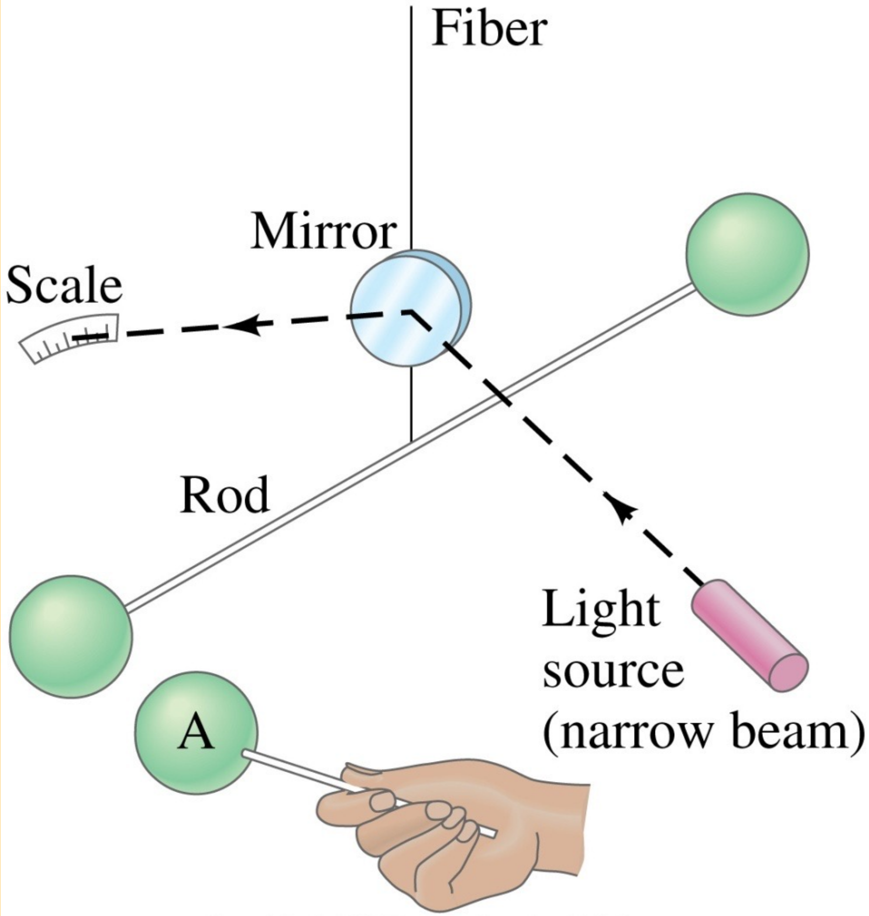
$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$$

Ex. A 50-kg person and a 75-kg person are sitting on a bench. Estimate the magnitude of the gravitational force each exerts on the other (r~0.5m).

$$F \approx 10^{-6}\,N$$

Cavendish Experiment



The magnitude of the gravitational constant *G* can be measured in the laboratory.

Gravity near the Earth's surface

Now we can relate the gravitational constant to the local acceleration of gravity. We know that, on the surface of the Earth:

$$mg = G\frac{mm_E}{r_E^2}$$

Solving for *g* gives:

$$g = G\frac{m_E}{r_E^2}$$

Now, knowing *g* and the radius of the Earth, the mass of the Earth can be calculated

$$m_E = \frac{gr_E^2}{G} = \frac{(9.80\ \mathrm{m/s^2})(6.38\times 10^6\ \mathrm{m})^2}{6.67\times 10^{-11}\ \mathrm{N\cdot m^2/kg^2}} = 5.98\times 10^{24}\ \mathrm{kg}$$

Ex. Estimate the value of g on the top of the Mt. Everest (8850 m above Sea level), given that the radius of Earth is 6380 km.

$r = 6380\ \text{km} + 8.9\ \text{km} = 6.389\times 10^6\ \text{m}$ $g = 9.77\ \text{m/s}^2$

The value of g varies locally on the Earth's surface – this is used by geophysicists to study the structure of the Earth's crust and in mineral and oil exploration.

Exercise

Difference in acceleration at head and feet

(a) An astronaut whose height h is 1.70 m floats "feet down" in an orbiting space shuttle at distance $r = 6.77 \times 10^6$ m away from the center of Earth. What is the difference between the gravitational acceleration at her feet and at her head?

KEY IDEAS

We can approximate Earth as a uniform sphere of mass M_E. Then, from Eq. 13-11, the gravitational acceleration at any distance r from the center of Earth is

$$a_g = \frac{GM_E}{r^2}. \qquad (13\text{-}15)$$

We might simply apply this equation twice, first with $r = 6.77 \times 10^6$ m for the location of the feet and then with $r = 6.77 \times 10^6\ \text{m} + 1.70\ \text{m}$ for the location of the head. However, a calculator may give us the same value for a_g twice, and thus a difference of zero, because h is so much smaller than r. Here's a more promising approach: Because we have a differential change dr in r between the astronaut's feet and head, we should differentiate Eq. 13-15 with respect to r.

Calculations: The differentiation gives us

$$da_g = -2\frac{GM_E}{r^3}\,dr, \qquad (13\text{-}16)$$

where da_g is the differential change in the gravitational acceleration due to the differential change dr in r. For the astronaut, $dr = h$ and $r = 6.77 \times 10^6$ m. Substituting data into Eq. 13-16, we find

$$da_g = -2\frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98 \times 10^{24}\ \text{kg})}{(6.77 \times 10^6\ \text{m})^3}(1.70\ \text{m})$$
$$= -4.37 \times 10^{-6}\ \text{m/s}^2, \qquad \text{(Answer)}$$

where the M_E value is taken from Appendix C. This result means that the gravitational acceleration of the astronaut's feet toward Earth is slightly greater than the gravitational acceleration of her head toward Earth. This difference in acceleration (often called a *tidal effect*) tends to stretch her body, but the difference is so small that she would never even sense the stretching, much less suffer pain from it.

(b) If the astronaut is now "feet down" at the same orbital radius $r = 6.77 \times 10^6$ m about a black hole of mass $M_h = 1.99 \times 10^{31}$ kg (10 times our Sun's mass), what is the difference between the gravitational acceleration at her feet and at her head? The black hole has a mathematical surface (*event horizon*) of radius $R_h = 2.95 \times 10^4$ m. Nothing, not even light, can escape from that surface or anywhere inside it. Note that the astronaut is well outside the surface (at $r = 229R_h$).

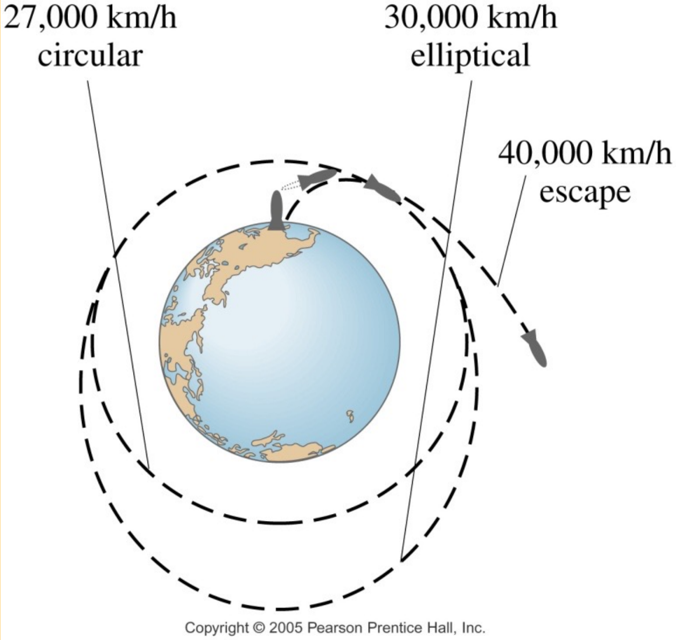
Calculations: We again have a differential change dr in r between the astronaut's feet and head, so we can again use Eq. 13-16. However, now we substitute $M_h = 1.99 \times 10^{31}$ kg for M_E. We find

$$da_g = -2\frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.99 \times 10^{31}\ \text{kg})}{(6.77 \times 10^6\ \text{m})^3}(1.70\ \text{m})$$
$$= -14.5\ \text{m/s}^2. \qquad \text{(Answer)}$$

This means that the gravitational acceleration of the astronaut's feet toward the black hole is noticeably larger than that of her head. The resulting tendency to stretch her body would be bearable but quite painful. If she drifted closer to the black hole, the stretching tendency would increase drastically.

Satellites

Satellites are routinely put into orbit around the Earth. The tangential speed must be high enough so that the satellite does not return to Earth, but not so high that it escapes Earth's gravity altogether.



Satellite in orbit:

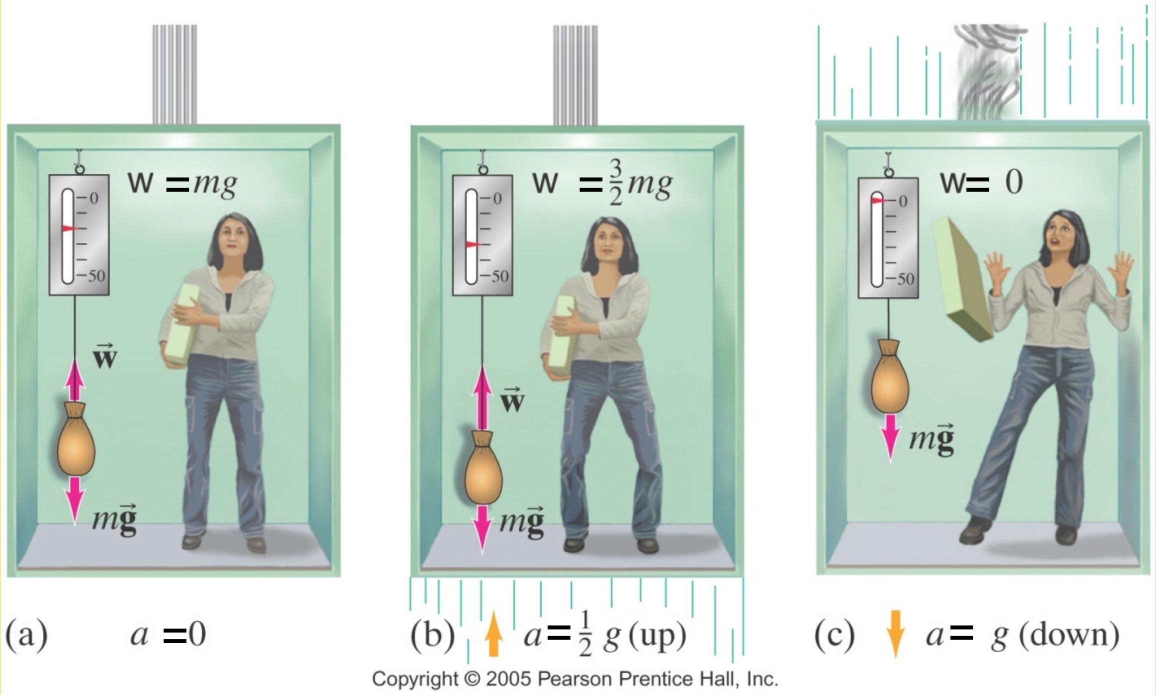
$$G\frac{m_{sat}m_E}{r^2} = m_{sat}\frac{v^2}{r}$$

Ex. 5-14 A geosynchronous satellite always stays above the same point on the Earth. It is used for TV, radio, weather forecasting,etc. Determine (a) the height above Earth; (b) such satellite's speed.

T=24h=86400s

r=42300km a) Above Earth:42300-6380~36000km b) v=3070m/s

"Weightlessness"



$$\sum F = ma$$

$$w - mg = ma$$

$$w = mg + ma$$

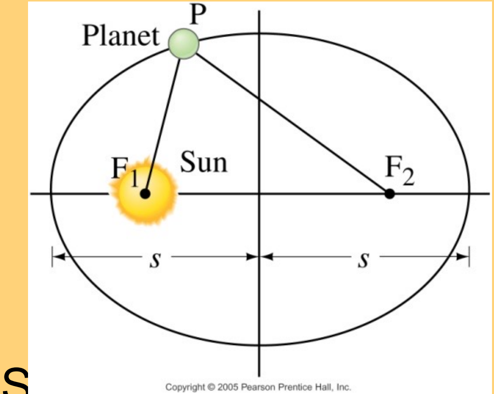
If a is + (elevator going up) the *APPARENT* weight is larger than mg

If a is – (elevator going down) the *APPARENT* weight is less than mg

If the elevator is in ***free fall***, a=-g and the scale reads zero
APPARENT WEIGHTLESSNESS: with respect to the elevator, things do not fall to the floor

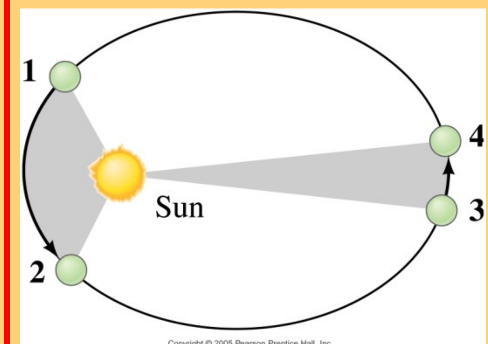
Kepler's Laws

(i) The orbit of each planet is an **ellipse**, with the **Sun at one focus.**



The Sleepwalkers – by Arthur Koestler

(ii) An imaginary line drawn from each planet to the Sun sweeps out equal areas in equal times.



(iii) $$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{s_1}{s_2}\right)^3$$

$$\sum F = ma$$

$$G\frac{m_1 M_S}{r_1^2} = m_1\frac{v^2}{r_1} = m_1\frac{(2\pi r_1)^2}{r_1 T_1^2}$$

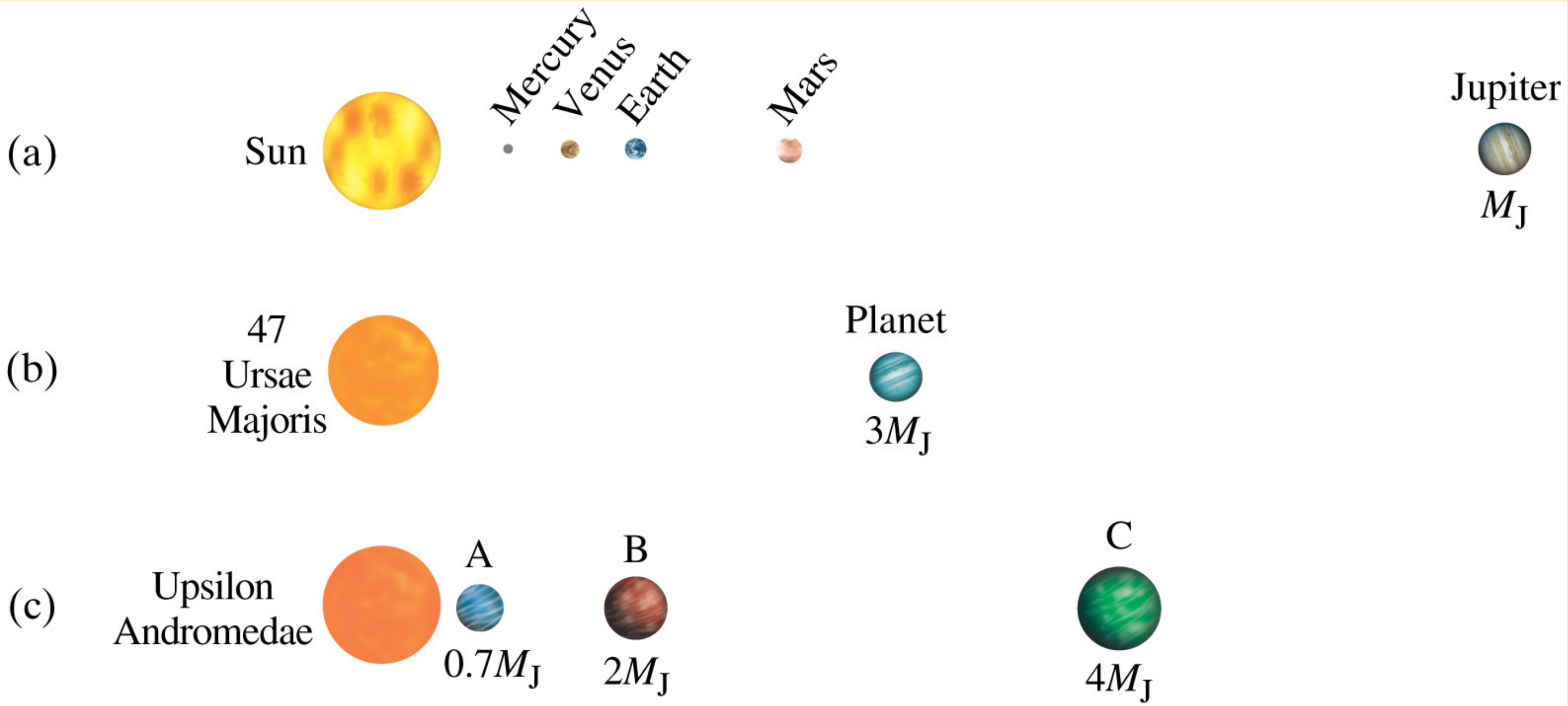
$$\frac{T_1^2}{r_1^3} = \frac{4\pi^2}{GM_S}$$

Ex. Mars' period is 687 days=1.88 years, Earth's period is 1 year, and the distance of Earth from the Sun is 1.50x10^11 m. How far is Mars from the Sun? $r_{Ms}=1.52r_{Es}$

Ex. Determine the mass of the Sun.

Kepler and Newton's laws

Kepler's laws can be derived from Newton's laws. Irregularities in planetary motion led to the discovery of Neptune, and irregularities in stellar motion have led to the discovery of many planets outside our Solar System.



Types of Forces in Nature

Modern physics now recognizes four fundamental forces:

1. Gravity
2. Electromagnetism
3. Weak nuclear force (responsible for some types of radioactive decay)
4. Strong nuclear force (binds protons and neutrons together in the nucleus)