Galileo Galilei

http://en.wikipedia.org/wiki/Galileo_Galilei

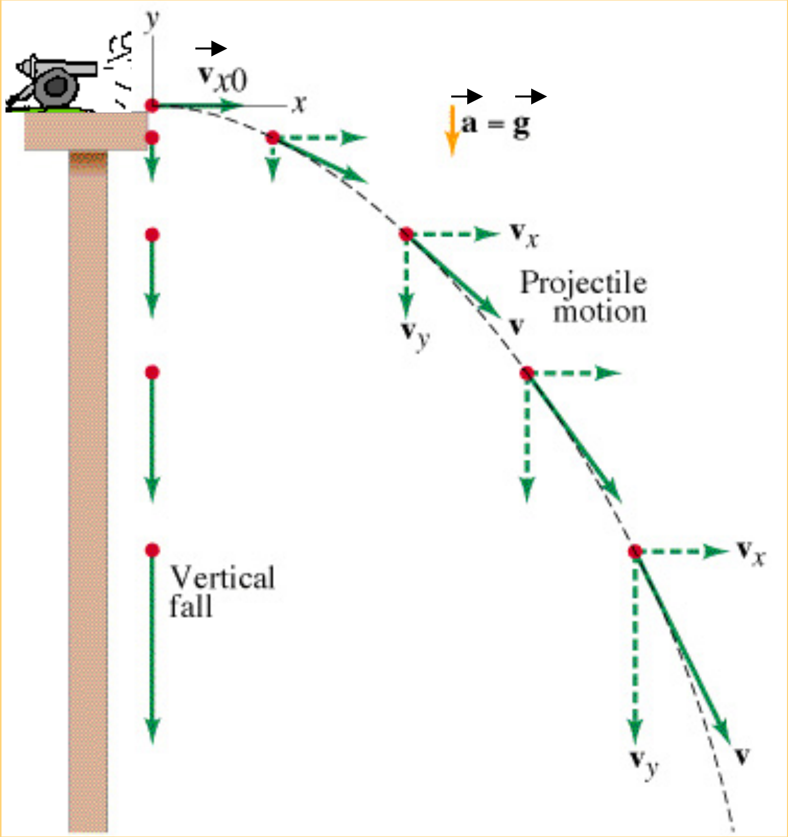
http://en.wikipedia.org/wiki/Dialogue_Concerning_the_Two_Chief_World_Systems
The ***Dialogue Concerning the Two Chief World Systems*** was a 1632 book by Galileo, comparing the Copernican system, and the traditional Ptolemaic system. In the Copernican system the Earth and other planets orbit the Sun, while in the Ptolemaic system everything in the Universe circles around the Earth. The *Dialogue* was published in Florence under a formal license from the Inquisition. In 1633, Galileo was convicted of "grave suspicion of heresy" based on the book, which was then placed on the Index of Forbidden Books, from which it was not removed until 1835.

http://en.wikipedia.org/wiki/Two_New_Sciences
The ***Discourses and Mathematical Demonstrations Relating to Two New Sciences*** (*Discorsi e dimostrazioni matematiche, intorno a due nuove scienze*, 1638) was Galileo's final book and a sort of scientific testament covering much of his work in physics over the preceding thirty years. Unlike the ***Dialogue Concerning the Two Chief World Systems***, it was not published with a license from the Inquisition; after the heresy trial based on the earlier book, the Roman Inquisition had banned publication of any work by Galileo, including any he might write in the future. After the failure of attempts to publish the work in France, Germany, or Poland, it was picked up by Lowys Elsevier in Leiden, The Netherlands, where the writ of the Inquisition was of little account.
The same three men as in the *Dialogue* carry on the discussion, but they have changed. **Simplicio**, in particular, is no longer the stubborn and rather dense Aristotelian; to some extent he represents the thinking of Galileo's early years, as **Sagredo** represents his middle period. **Salviati** remains the spokesman for Galileo.

The whole book ***Discourses and Mathematical Demonstrations Relating to Two New Sciences*** can be found at http://oll.libertyfund.org//files/753/0416_Bk.pdf

Projectile Motion

Galileo: projectile motion can be understood by analyzing the **horizontal and vertical** components of the motion **separately**.



$\vec{\mathbf{v}}$ is tangent to the path

For convenience: $t_0 = 0, \; x_0 = 0, \; y_0 = 0,$

According to the reference frame of the figure:

vertical motion
constant acceleration

$$v_{y0} = 0 \qquad a_y = -g$$

$$v_y = -gt \qquad y = -\frac{g}{2}t^2$$

horizontal motion (no air resistance)
constant velocity

$$v_x = v_{x0} \qquad a_x = 0$$

$$x = v_{x0}t$$

An object projected horizontally will reach the ground in the same time as an object dropped vertically.

Two Sciences – page 244

FOURTH DAY

SALVIATI. Once more, Simplicio is here on time; so let us without delay take up the question of motion. The text of our Author is as follows:

THE MOTION OF PROJECTILES

In the preceding pages we have discussed the properties of uniform motion and of motion naturally accelerated along planes of all inclinations. I now propose to set forth those properties which belong to a body whose motion is compounded of two other motions, namely, one uniform and one naturally accelerated; these properties, well worth knowing, I propose to demonstrate in a rigid manner. This is the kind of motion seen in a moving projectile; its origin I conceive to be as follows:

Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits. But if the plane is limited and elevated, then the moving particle, which we imagine to be a heavy one, will on passing over the edge of the plane acquire, in addition to its previous uniform and perpetual motion, a downward propensity due to its own weight; so that the resulting motion which I call projection [*projectio*], is compounded of one which is uniform and horizontal and of another which is vertical and naturally accelerated. We now proceed to demonstrate

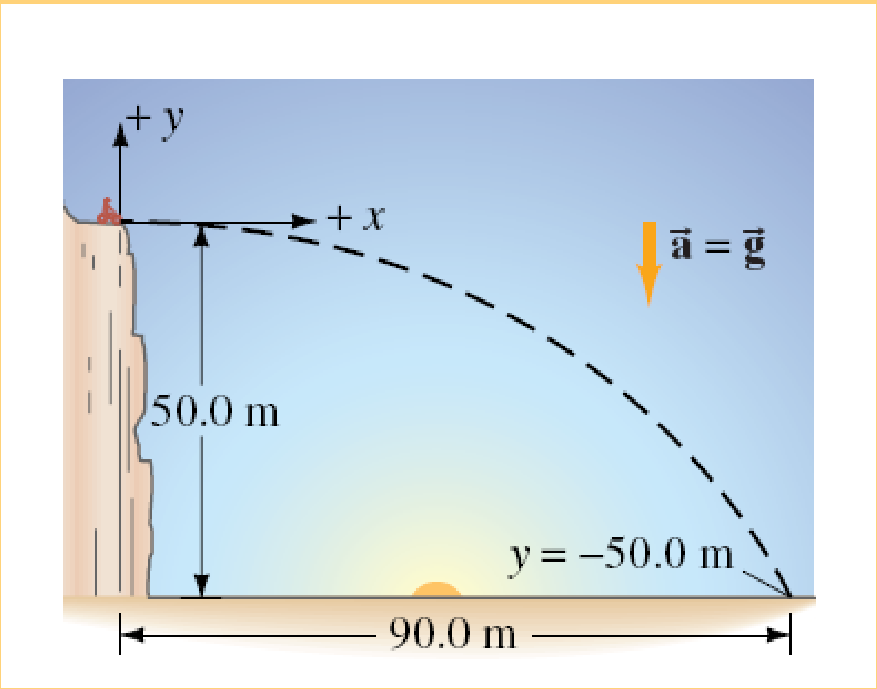
Example

Two balls having different speeds roll off the edge of a horizontal table at the same time. Which hits the floor sooner, the faster or the slower ball?

They hit at the same time

Example

Ex. 3-4 A motorcycle speeds horizontally off a 50 m high cliff. How fast must it leave the cliff top to land on level ground 90 m from the base of the cliff?



+y

$x_0 = 0, \quad y_0 = 0,$

+x

$$y = -\frac{g}{2}t^2 \Rightarrow -50 = -\frac{9.8}{2}t^2$$

$$x = v_{x0}t \Rightarrow 90 = v_{x0}t$$

-y

$x_0 = 0, \quad y_0 = 0,$

+x

$$y = \frac{g}{2}t^2 \Rightarrow 50 = \frac{9.8}{2}t^2$$

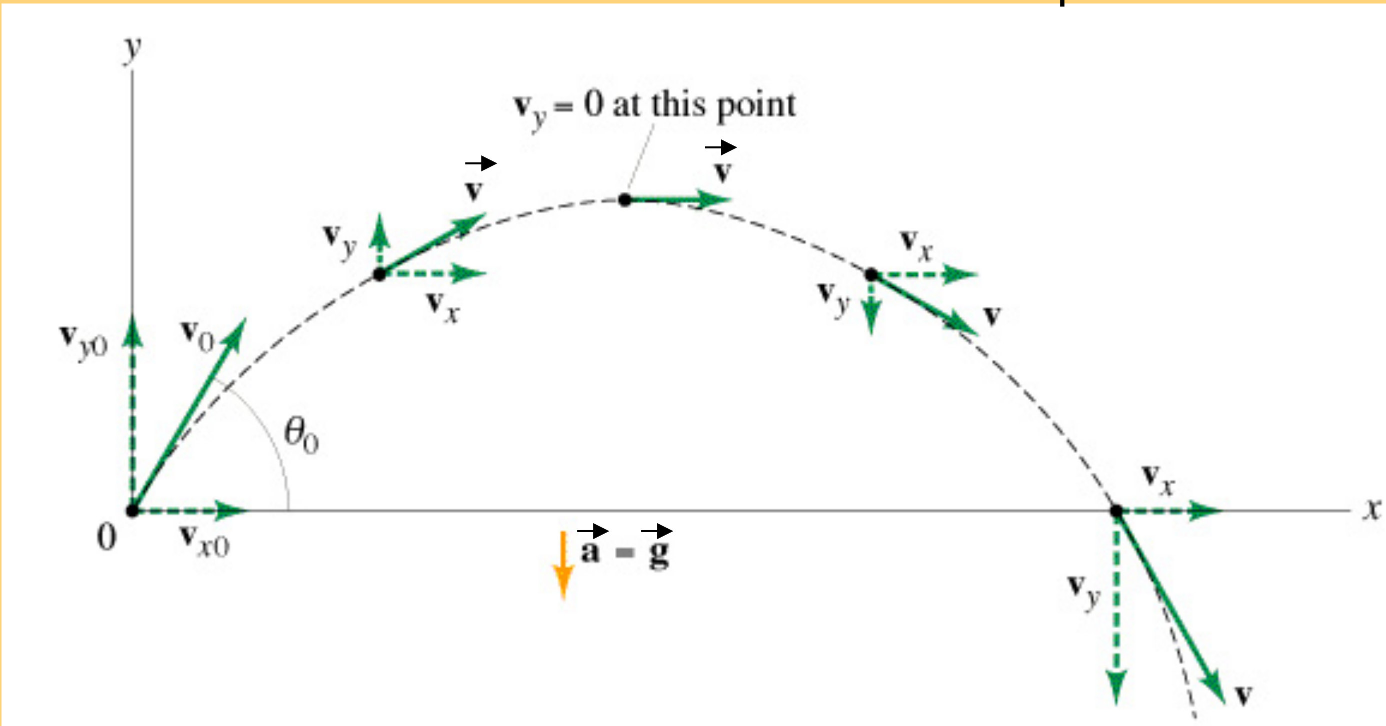
$$x = v_{x0}t \Rightarrow 90 = v_{x0}t$$

+y

When we choose $x_0 = 0, \quad y_0 = 0,$ then x and y give the displacements in these directions

Object projected upward

Now there are initial vertical and horizontal components of velocity



$$v_{x0} = v_0 \cos\theta$$
$$v_{y0} = v_0 \sin\theta$$

vertical motion
constant acceleration

$$v_{y0} \neq 0 \qquad a_y = -g$$
$$v_y = v_{y0} - gt \qquad y = v_{y0}t - \frac{g}{2}t^2$$

horizontal motion (no air resistance)
constant velocity

$$v_x = v_{x0} \qquad a_x = 0$$
$$x = v_{x0}t$$

Notation and Notes

The projectile is launched with initial velocity

$$\vec{v}_0 = v_{0x}\hat{\mathrm{i}} + v_{0y}\hat{\mathrm{j}}.$$

The components are:

$$v_{0x} = v_0 \cos\theta_0 \quad \text{and} \quad v_{0y} = v_0 \sin\theta_0.$$

During its two-dimensional motion, the projectile's position vector $\vec{r}$ and velocity vector $\vec{v}$ change continuously, but its acceleration vector $\vec{a}$ is constant and *always* directed vertically downward. The projectile has *no* horizontal acceleration.

$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}}$$

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}.$$

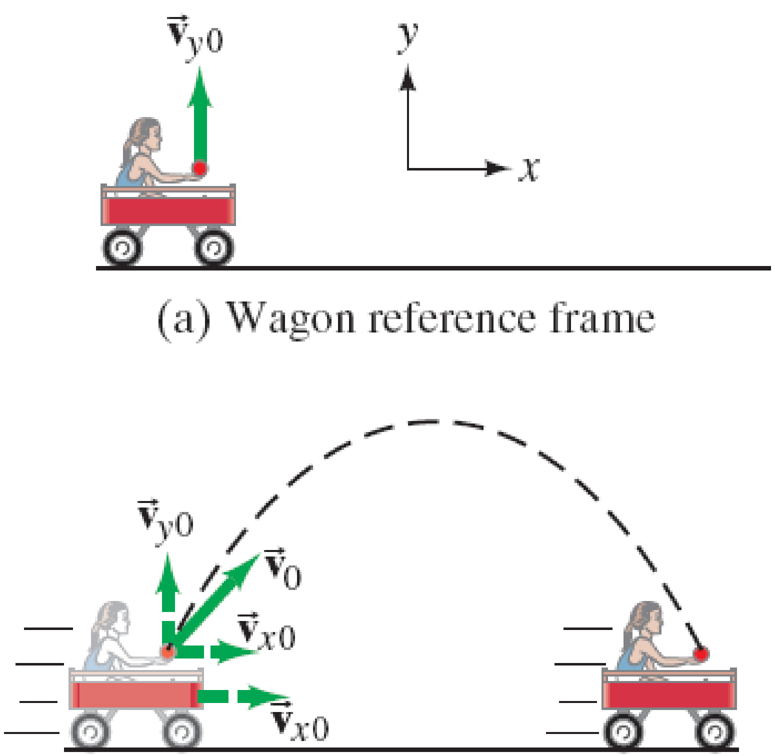
$$\vec{v} = \frac{d\vec{r}}{dt}.$$

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}}$$

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

Example

Where will the ball land?

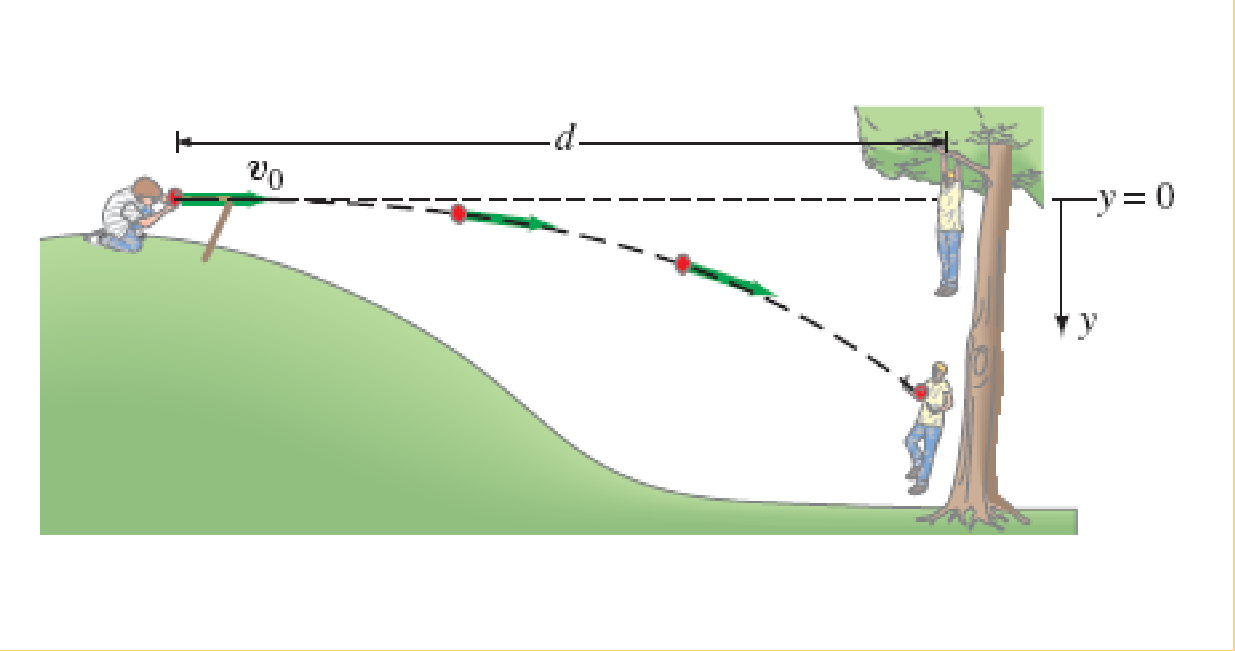


(a) Wagon reference frame

(b) Ground reference frame

Example

What should the boy on the tree do avoid being hit by the bullet?



Example

Where will the boy land?

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

Examples

A football is kicked at an angle 37 degrees with velocity 20 m/s.
Calculate: (a) the maximum height; (b) the time of travel before it hits the ground; (c) how far away it hits the ground; (d) the velocity vector at maximum height; (e) the acceleration vector at maximum height

$$a)\ \ v_y^2 = v_{y0}^2 - 2gy \Rightarrow y = \frac{v_{y0}^2}{2g} \qquad v_{x0} = v_0 \cos 37^o;\ \ v_{y0} = v_0 \sin 37^o$$

$$b)\ \ v_y = v_{y0} - gt \Rightarrow t = \frac{v_{y0}}{g} \Rightarrow 2t \qquad c)\ \ x = v_{x0}t$$

(d) there is no vertical component, only horizontal: 16m/s
(e) Acceleration is the same throughout, pointing downward

Range

Derive a formula for the horizontal range R of a projectile in terms of its initial velocity v_0 and angle theta$_0$. The horizontal range is defined as the horizontal distance the projectile travels before returning to ***its original height***.

$$x_0 = 0, y_0 = 0, R = x = x_0 + v_{x0}t$$

$$\boxed{R \Rightarrow v_{x0}t = v_0 \cos\theta_0 t}$$

$$y = y_0 + v_0 \sin\theta_0 t - \frac{g}{2}t^2$$

$$0 = v_0 \sin\theta_0 t - \frac{g}{2}t^2$$

$$v_0 \sin\theta_0 t = \frac{g}{2}t^2$$

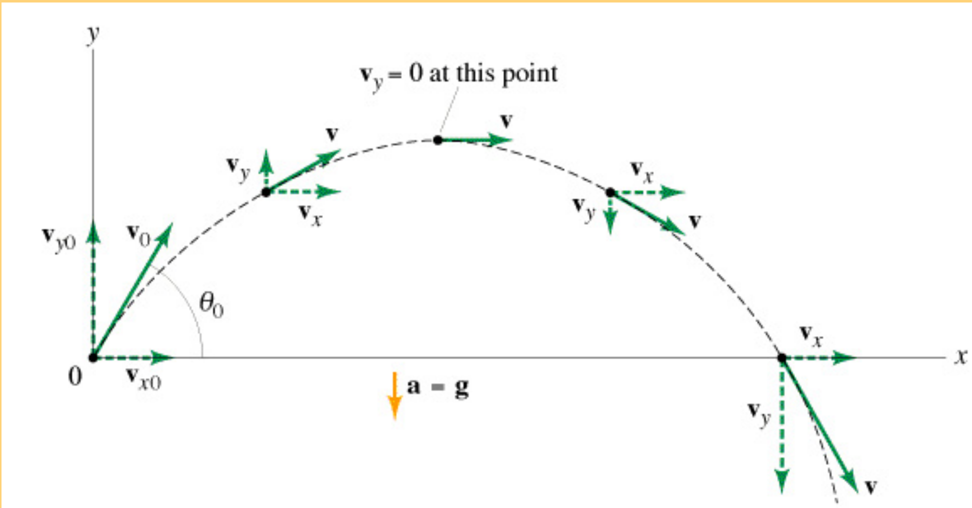
$$t = \frac{2v_0 \sin\theta_0}{g}$$

$$R = v_0 \cos\theta_0 \frac{2v_0 \sin\theta_0}{g} = \frac{v_0^2 2\cos\theta_0 \sin\theta_0}{g}$$

$$\boxed{R = \frac{v_0^2 \sin(2\theta_0)}{g}}$$

The horizontal range R is maximum for a launch angle of 45°.

Projectile Motion is Parabolic



$$y = v_{y0}t - \tfrac{1}{2}gt^2$$

$$x = v_{x0}t$$

$$t = \frac{x}{v_{x0}}$$

$$y = \frac{v_{y0}}{v_{x0}}x - \tfrac{1}{2}g\left(\frac{1}{v_{x0}}\right)^2 x^2 \qquad \frac{v_{y0}}{v_{x0}} = \tan\theta_0; \;\; v_{x0} = v_0\cos\theta_0$$

$$y = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2 = Ax - Bx^2$$

Two Sciences – page 153


Y purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small; nevertheless I have discovered by experiment some properties of it which are worth knowing and which have not hitherto been either observed or demonstrated. Some superficial observations have been made, as, for instance, that the free motion [*naturalem motum*] of a heavy falling body is continuously accelerated;* but to just what extent this acceleration occurs has not yet been announced; for so far as I know, no one has yet pointed out that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity.†

It has been observed that missiles and projectiles describe a curved path of some sort; however no one has pointed out the fact that this path is a parabola. But this and other facts, not few in number or less worth knowing, I have succeeded in proving; and what I consider more important, there have been opened up to this vast and most excellent science, of which my

* "Natural motion" of the author has here been translated into "free motion"—since this is the term used to-day to distinguish the "natural" from the "violent" motions of the Renaissance. [*Trans.*]

† A theorem demonstrated on p. 175 below. [*Trans.*]

Problems

Projectile dropped from airplane

In Fig. 4-14, a rescue plane flies at 198 km/h (= 55.0 m/s) and constant height h = 500 m toward a point directly over a victim, where a rescue capsule is to land.

(a) What should be the angle ϕ of the pilot's line of sight to the victim when the capsule release is made?

KEY IDEAS

Once released, the capsule is a projectile, so its horizontal and vertical motions can be considered separately (we need not consider the actual curved path of the capsule).

Calculations: In Fig. 4-14, we see that ϕ is given by

$$\phi = \tan^{-1}\frac{x}{h}, \tag{4-27}$$

where x is the horizontal coordinate of the victim (and of the capsule when it hits the water) and h = 500 m. We should be able to find x with Eq. 4-21:

$$x - x_0 = (v_0 \cos \theta_0)t. \tag{4-28}$$

Here we know that $x_0 = 0$ because the origin is placed at the point of release. Because the capsule is *released* and not shot from the plane, its initial velocity $\vec{v}_0$ is equal to the plane's velocity. Thus, we know also that the initial velocity has magnitude v_0 = 55.0 m/s and angle θ_0 = 0° (measured relative to the positive direction of the x axis). However, we do not know the time t the capsule takes to move from the plane to the victim.

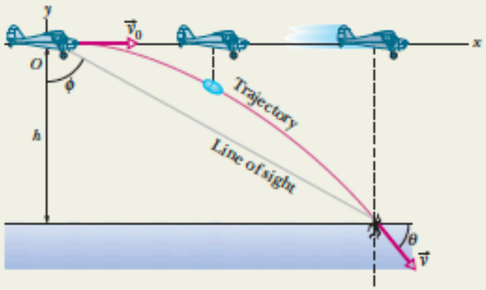


Fig. 4-14 A plane drops a rescue capsule while moving at constant velocity in level flight. While falling, the capsule remains under the plane.

To find t, we next consider the *vertical* motion and specifically Eq. 4-22:

$$y - y_0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2. \tag{4-29}$$

Here the vertical displacement $y - y_0$ of the capsule is −500 m (the negative value indicates that the capsule moves *downward*). So,

$$-500 \text{ m} = (55.0 \text{ m/s})(\sin 0°)t - \tfrac{1}{2}(9.8 \text{ m/s}^2)t^2. \tag{4-30}$$

Solving for t, we find t = 10.1 s. Using that value in Eq. 4-28 yields

$$x - 0 = (55.0 \text{ m/s})(\cos 0°)(10.1 \text{ s}), \tag{4-31}$$

or

$$x = 555.5 \text{ m}.$$

Then Eq. 4-27 gives us

$$\phi = \tan^{-1}\frac{555.5 \text{ m}}{500 \text{ m}} = 48.0°. \qquad \text{(Answer)}$$

(b) As the capsule reaches the water, what is its velocity $\vec{v}$ in unit-vector notation and in magnitude-angle notation?

KEY IDEAS

(1) The horizontal and vertical components of the capsule's velocity are independent. (2) Component v_x does not change from its initial value $v_{0x} = v_0 \cos \theta_0$ because there is no horizontal acceleration. (3) Component v_y changes from its initial value $v_{0y} = v_0 \sin \theta_0$ because there is a vertical acceleration.

Calculations: When the capsule reaches the water,

$$v_x = v_0 \cos \theta_0 = (55.0 \text{ m/s})(\cos 0°) = 55.0 \text{ m/s}.$$

Using Eq. 4-23 and the capsule's time of fall t = 10.1 s, we also find that when the capsule reaches the water,

$$\begin{aligned} v_y &= v_0 \sin \theta_0 - gt \\ &= (55.0 \text{ m/s})(\sin 0°) - (9.8 \text{ m/s}^2)(10.1 \text{ s}) \\ &= -99.0 \text{ m/s}. \end{aligned} \tag{4-32}$$

Thus, at the water

$$\vec{v} = (55.0 \text{ m/s})\hat{i} - (99.0 \text{ m/s})\hat{j}. \qquad \text{(Answer)}$$

Using Eq. 3-6 as a guide, we find that the magnitude and the angle of $\vec{v}$ are

$$v = 113 \text{ m/s} \quad \text{and} \quad \theta = -60.9°. \qquad \text{(Answer)}$$

Problems

Cannonball to pirate ship

Figure 4-15 shows a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires balls at initial speed $v_0 = 82$ m/s.

(a) At what angle θ_0 from the horizontal must a ball be fired to hit the ship?

KEY IDEAS

(1) A fired cannonball is a projectile. We want an equation that relates the launch angle θ_0 to the ball's horizontal displacement as it moves from cannon to ship. (2) Because the cannon and the ship are at the same height, the horizontal displacement is the range.

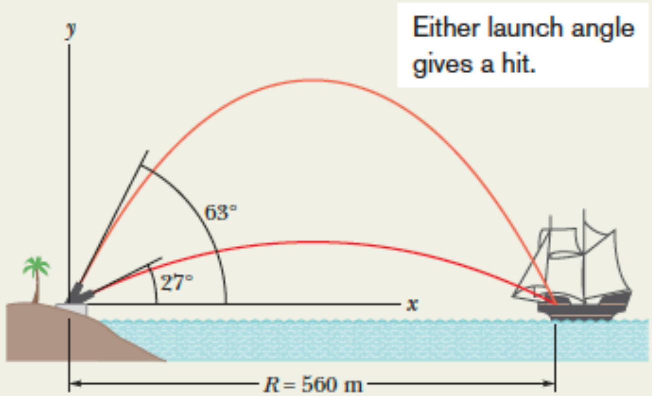


Fig. 4-15 A pirate ship under fire.

Calculations: We can relate the launch angle θ_0 to the range R with Eq. 4-26 which, after rearrangement, gives

$$\theta_0 = \frac{1}{2}\sin^{-1}\frac{gR}{v_0^2} = \frac{1}{2}\sin^{-1}\frac{(9.8\text{ m/s}^2)(560\text{ m})}{(82\text{ m/s})^2}$$

$$= \frac{1}{2}\sin^{-1} 0.816. \qquad (4\text{-}33)$$

One solution of $\sin^{-1}$ (54.7°) is displayed by a calculator; we subtract it from 180° to get the other solution (125.3°). Thus, Eq. 4-33 gives us

$$\theta_0 = 27° \quad \text{and} \quad \theta_0 = 63°. \qquad \text{(Answer)}$$

(b) What is the maximum range of the cannonballs?

Calculations: We have seen that maximum range corresponds to an elevation angle θ_0 of 45°. Thus,

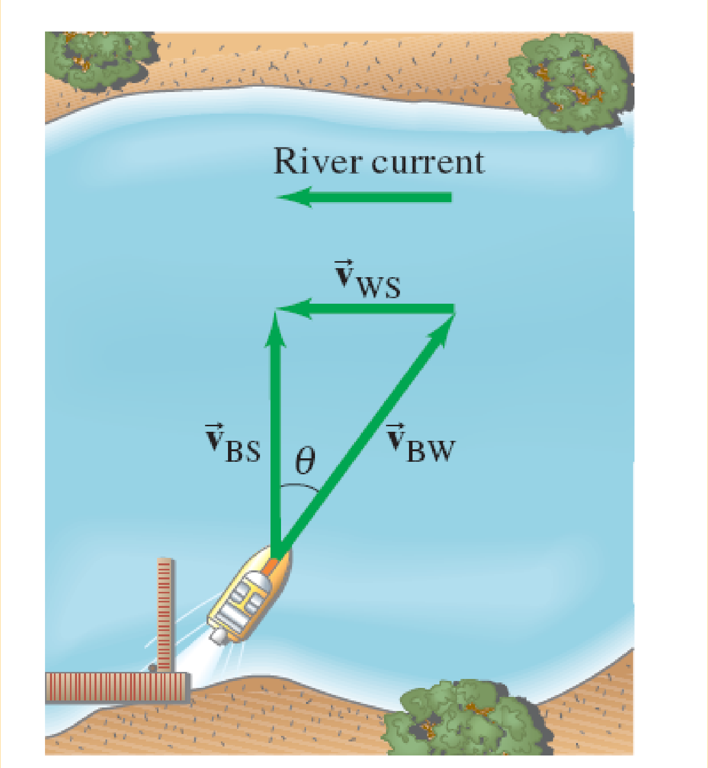
$$R = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{(82\text{ m/s})^2}{9.8\text{ m/s}^2}\sin(2\times 45°)$$

$$= 686\text{ m} \approx 690\text{ m}. \qquad \text{(Answer)}$$

As the pirate ship sails away, the two elevation angles at which the ship can be hit draw together, eventually merging at $\theta_0 = 45°$ when the ship is 690 m away. Beyond that distance the ship is safe. However, the cannonballs could go farther if the cannon were higher.

Relative Motion

A man in a small boat is trying to cross a river that flows due west with a current. The man starts on the south bank and is trying to reach the north bank directly north from his starting point. In which direction should he head?



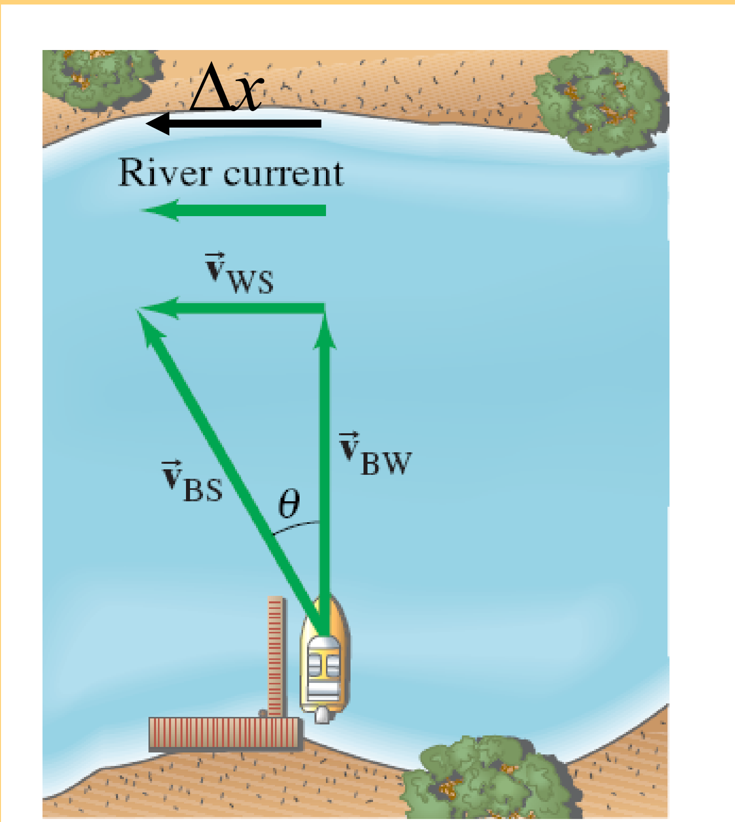
BW- boat with respect to the water
WS – water with respect to the shore

BS- boat with respect to the shore

Example

A boat heads directly north from the south shore. The river has a current from east to west. Find the boat velocity relative to the shore. How far downstream will the boat arrive?

D



BW- boat with respect to the water
WS – water with respect to the shore

BS- boat with respect to the shore

$$v_{BW} = 1.85 m/s; \;\; v_{WS} = 1.20 m/s$$

$$D = 110 m;$$

$$v_{BS} = ?; \;\; \theta = ?$$

$$x = ?$$

$$v_{BS} = 2.21 m/s; \;\; \theta = 33.0^{o}$$

$$x = 72 m$$

Problem

Relative motion, two dimensional, airplanes

In Fig. 4-20*a*, a plane moves due east while the pilot points the plane somewhat south of east, toward a steady wind that blows to the northeast. The plane has velocity $\vec{v}_{PW}$ relative to the wind, with an airspeed (speed relative to the wind) of 215 km/h, directed at angle θ south of east. The wind has velocity $\vec{v}_{WG}$ relative to the ground with speed 65.0 km/h, directed 20.0° east of north. What is the magnitude of the velocity $\vec{v}_{PG}$ of the plane relative to the ground, and what is θ?

KEY IDEAS

The situation is like the one in Fig. 4-19. Here the moving particle P is the plane, frame A is attached to the ground (call it G), and frame B is "attached" to the wind (call it W). We need a vector diagram like Fig. 4-19 but with three velocity vectors.

Calculations: First we construct a sentence that relates the three vectors shown in Fig. 4-20*b*:

$$\underset{(PG)}{\text{velocity of plane relative to ground}} = \underset{(PW)}{\text{velocity of plane relative to wind}} + \underset{(WG)}{\text{velocity of wind relative to ground.}}$$

This relation is written in vector notation as

$$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}. \qquad (4\text{-}46)$$

We need to resolve the vectors into components on the coordinate system of Fig. 4-20*b* and then solve Eq. 4-46 axis by axis. For the y components, we find

$$v_{PG,y} = v_{PW,y} + v_{WG,y}$$

or $\quad 0 = -(215 \text{ km/h}) \sin\theta + (65.0 \text{ km/h})(\cos 20.0°).$

Solving for θ gives us

$$\theta = \sin^{-1}\frac{(65.0 \text{ km/h})(\cos 20.0°)}{215 \text{ km/h}} = 16.5°. \qquad \text{(Answer)}$$

Similarly, for the x components we find

$$v_{PG,x} = v_{PW,x} + v_{WG,x}.$$

Here, because $\vec{v}_{PG}$ is parallel to the x axis, the component $v_{PG,x}$ is equal to the magnitude v_{PG}. Substituting this notation and the value $\theta = 16.5°$, we find

$$v_{PG} = (215 \text{ km/h})(\cos 16.5°) + (65.0 \text{ km/h})(\sin 20.0°)$$
$$= 228 \text{ km/h}. \qquad \text{(Answer)}$$

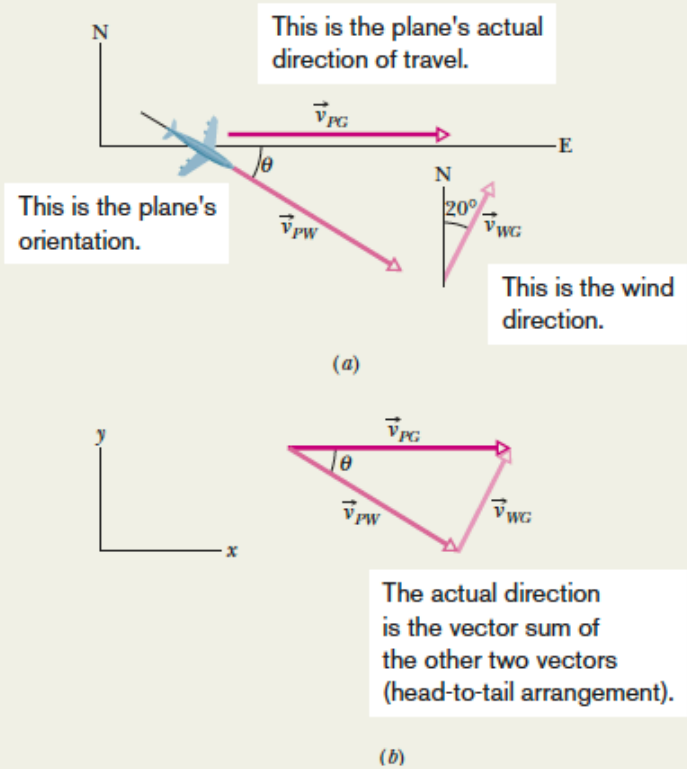


Fig. 4-20 A plane flying in a wind.