Chapter 33: Geometric Optics

We see an object in one of two ways:
(1) The object may be a *source* of light, it emits light (lightbulb, flame, star)
(2) Light *reflected* by the object, so we can see it.

Ray model of light:
We assume that light travels in straight-line paths called light rays.
This is true in various circumstances. Example: beam from
a laser pointer appears to be a straight line.

When we see an object,
light reaches our eyes from each point
on the object.

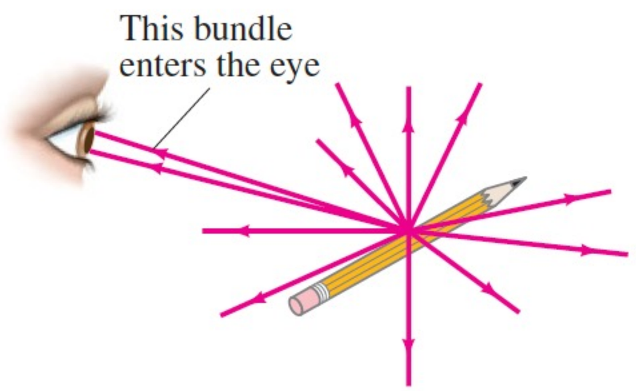
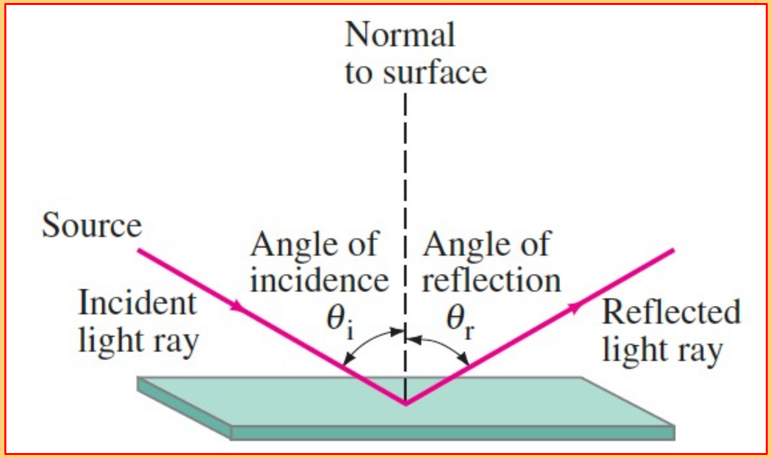


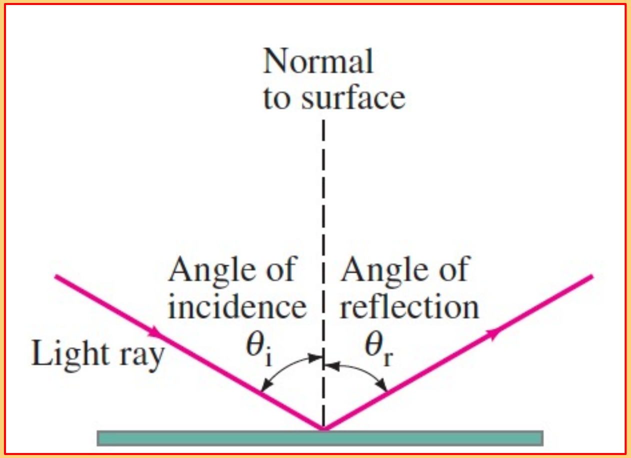
FIGURE 23–1 Light rays come from each single point on an object. A small bundle of rays leaving one point is shown entering a person's eye.

Geometric optics

Reflection

When light strikes the surface of an object, some of the light is reflected. The rest can be absorbed by the object (and transformed to thermal energy) or, if the object is transparent like glass or water, part can be transmitted through.



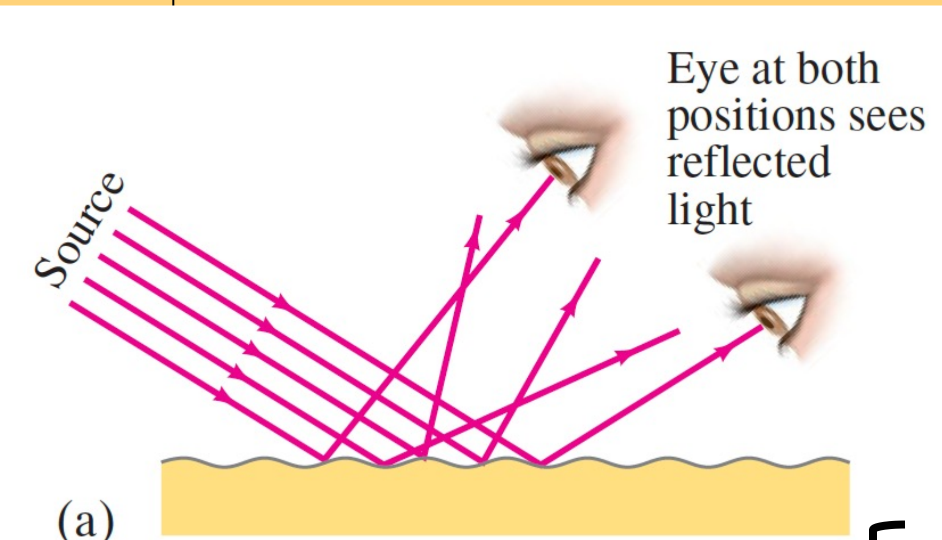


LAW OF REFLECTION:

the angle of reflection equals the angle of incidence, $\boxed{\theta_r = \theta_i}$

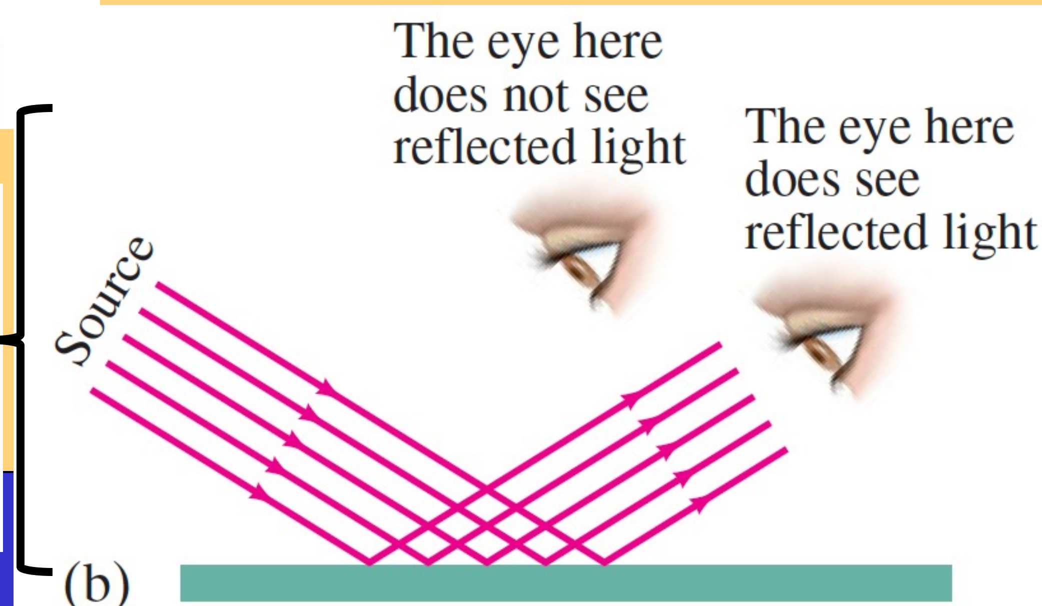
Diffusive vs Specular Reflection

When light is incident upon a rough surface, even microscopically rough, it is reflected in many directions (**diffuse reflection**). The law of reflection still holds, however, at each small section.



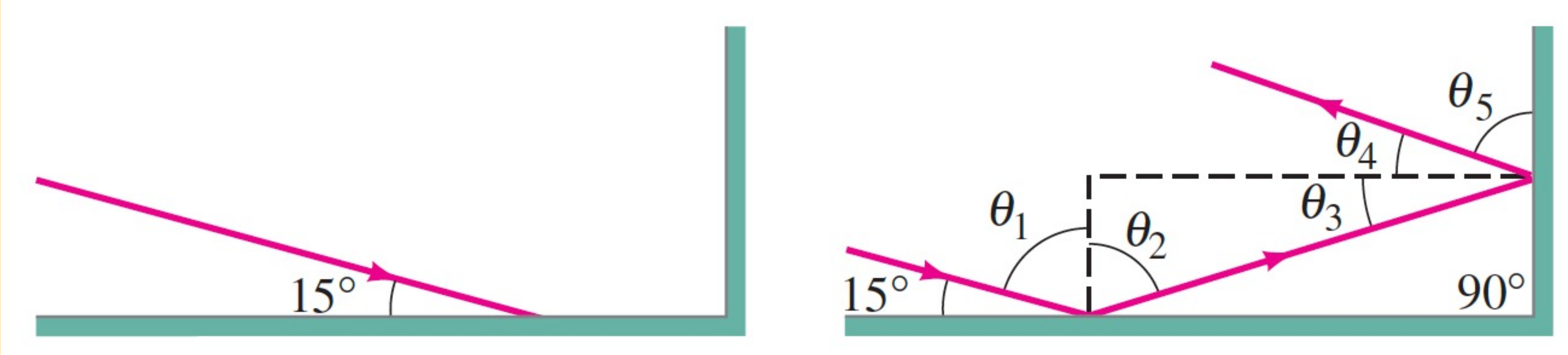
Because of **diffuse reflection** in all directions, an ordinary object can be seen at many different angles by the light reflected from it.

Specular reflection: When a narrow beam of light shines on a mirror, the light will not reach your eye unless your eye is positioned at just the right place where the law of reflection is satisfied.



Reflection from flat mirrors

Ex. Two flat mirrors are perpendicular to each other. An incoming beam of light makes an angle of 15° with the first mirror. What angle will the outgoing beam make with the second mirror, that is what is θ_5?

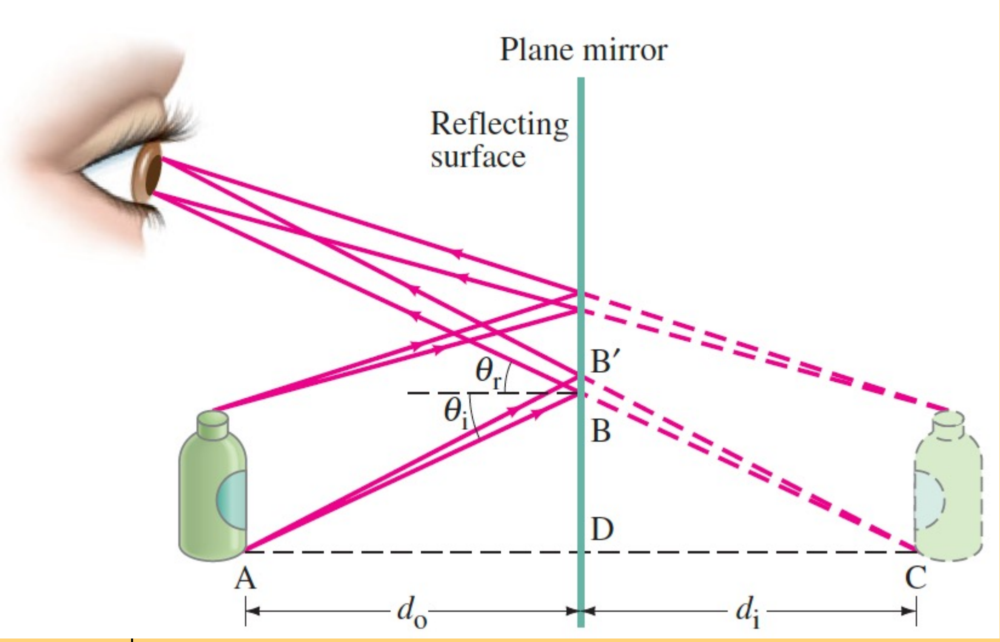


SOLUTION In Fig. 23–5b, $\theta_1 + 15° = 90°$, so $\theta_1 = 75°$; by the law of reflection $\theta_2 = \theta_1 = 75°$ too. Using the fact that the sum of the three angles of a triangle is always 180°, and noting that the two normals to the two mirrors are perpendicular to each other, we have $\theta_2 + \theta_3 + 90° = 180°$. Thus $\theta_3 = 180° - 90° - 75° = 15°$. By the law of reflection, $\theta_4 = \theta_3 = 15°$, so $\theta_5 = 75°$ is the angle the reflected ray makes with the second mirror surface.

The outgoing ray is parallel to the incoming ray.

Plane Mirror

When you look straight into a mirror, your face and the other objects look as if they are in front of you, *beyond the mirror*. What you see is an **image.**
Left and right appear reversed in the image.

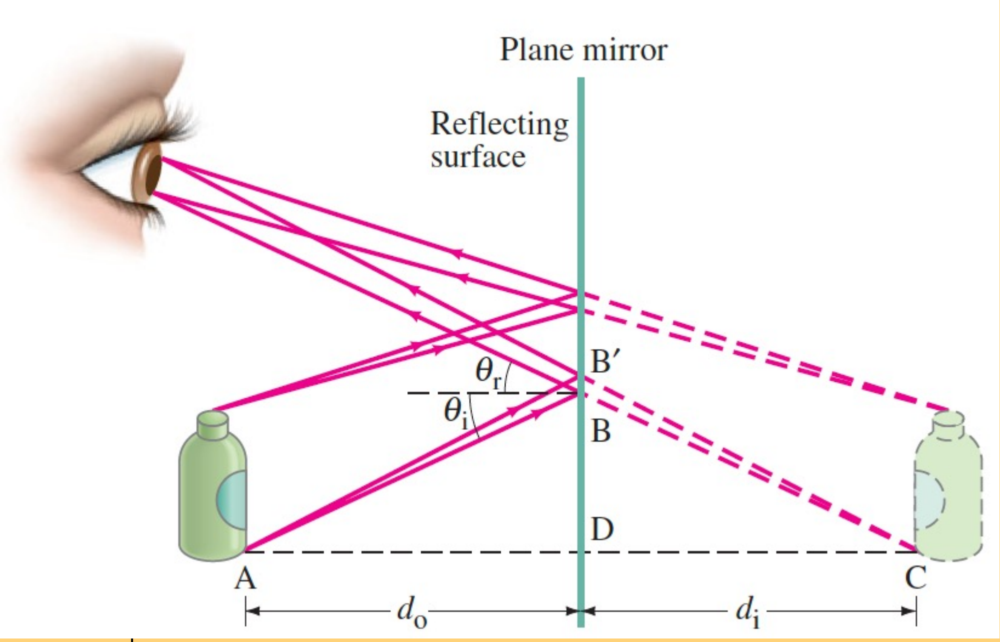


Each set of diverging rays that reflect from the mirror and enter the eye appear to come from a single point behind the mirror, called the **image point.**
Our eyes and brain interpret any rays that enter an eye as having traveled straightline paths, but the light rays do not actually pass through the image location itself (dashed lines). The image is **virtual** not **real**.

Virtual image does not appear on paper or film placed at the location of the image. In contrast, a movie projector lens, for example, produces a real image that is visible on the screen.

Plane Mirror: Distance

When you look straight into a mirror, your face and the other objects look as if they are in front of you, *beyond the mirror*. What you see is an **image.**
Left and right appear reversed in the image.



Let's look at the two rays that leave point A on the object and strike the mirror at points B.
The angles ADB and CDB are right angles; and because of the law of reflection, $\theta_i = \theta_r$ at point B.
Therefore, angles ABD and CBD are equal. The two triangles ABD and CBD are **congruent**, and the length

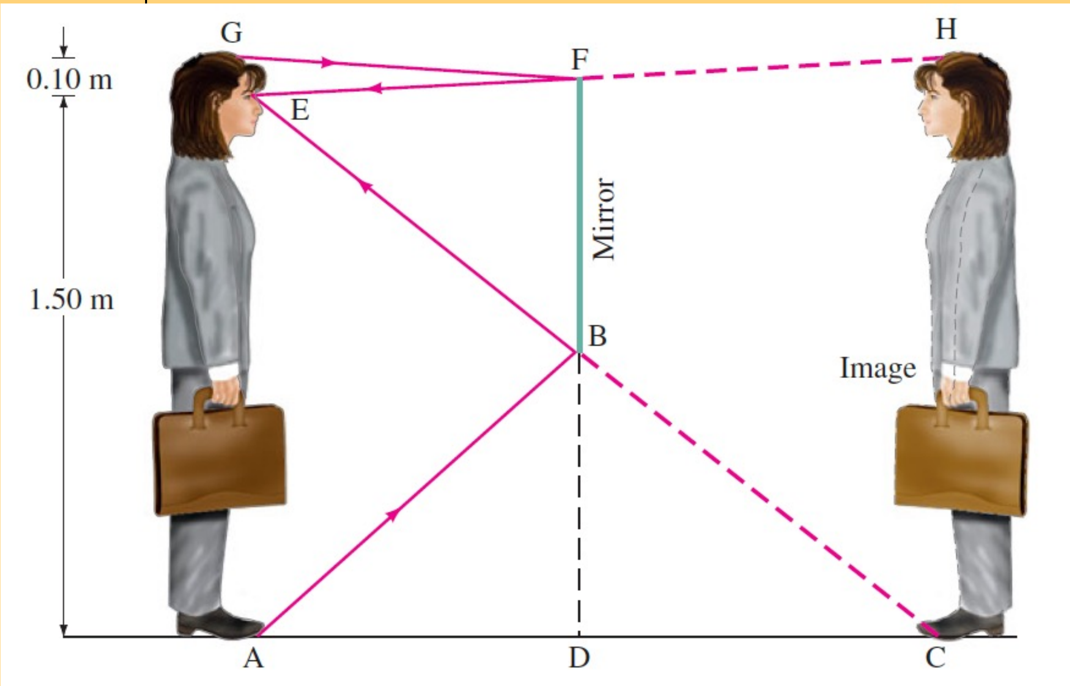
AD=CD

That is, the image appears as far behind the mirror as the object is in front.

The image distance, d_i (perpendicular distance from mirror to image), equals the object distance, d_o (perpendicular distance from object to mirror).
The height of the image is the same as that of the object.

How tall must a full-length mirror be?

A woman 1.60 m tall stands in front of a vertical plane mirror. What is the minimum height of the mirror, and how high must its lower edge be above the floor, if she is to be able to see her whole body? Assume that her eyes are 10 cm below the top of her head.



For her to see her whole body, light rays from the top of her head (point G) and from the bottom of her foot (A) must reflect from the mirror and enter her eye.

The mirror needs to extend no lower than B. The angle of reflection equals the angle of incidence, so the height BD=AE/2.

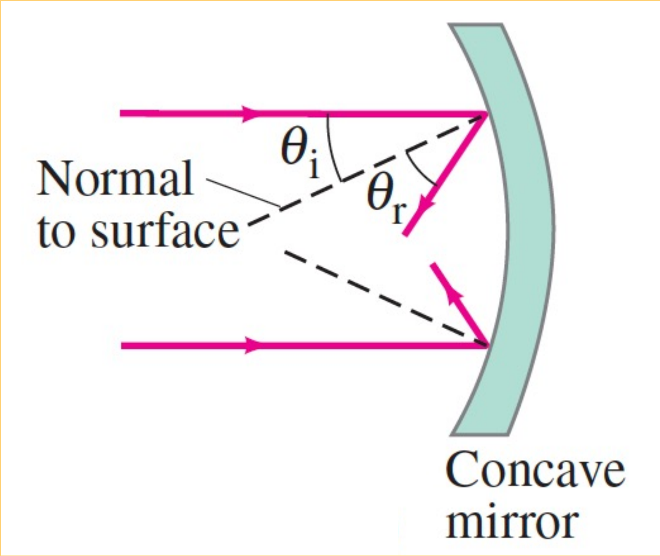
AE = 1.60 m - 0.10 m = 1.50 m,
BD = 0.75 m (above the floor)

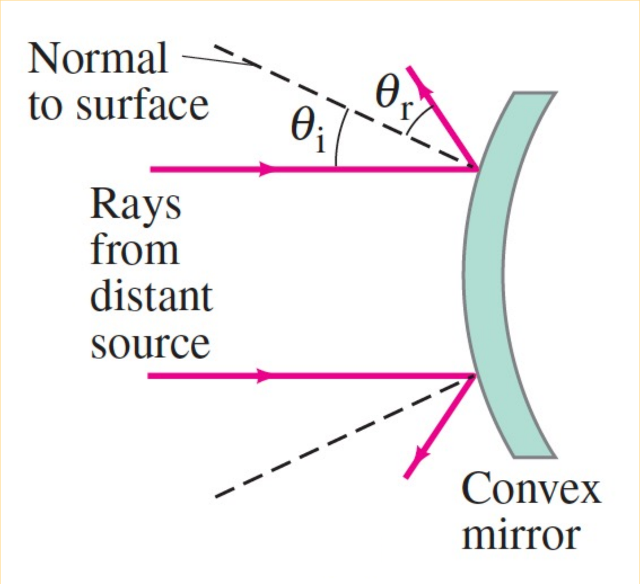
To see the top of her head, the top edge of the mirror only needs to reach point F, which is 5 cm below the top of her head. Thus DF=1.55m.

The mirror needs to have a vertical height of only (1.55 m - 0.75 m) = **0.80 m.**

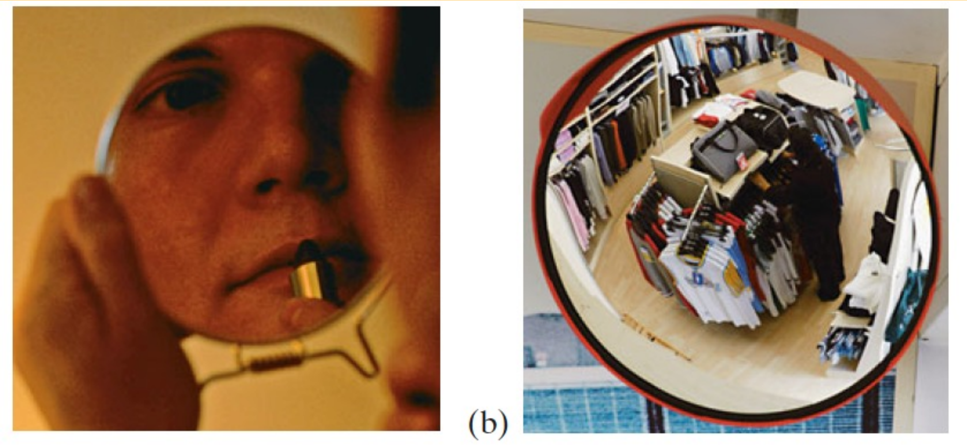
(half as tall as the person)

Spherical Mirrors





Concave mirrors are used as shaving or cosmetic mirrors (magnifying mirrors). Convex mirrors are sometimes used on cars and trucks (rearview mirrors) and in shops (to watch for theft), because they take in a wide field of view,



Focal Point and Focal Length

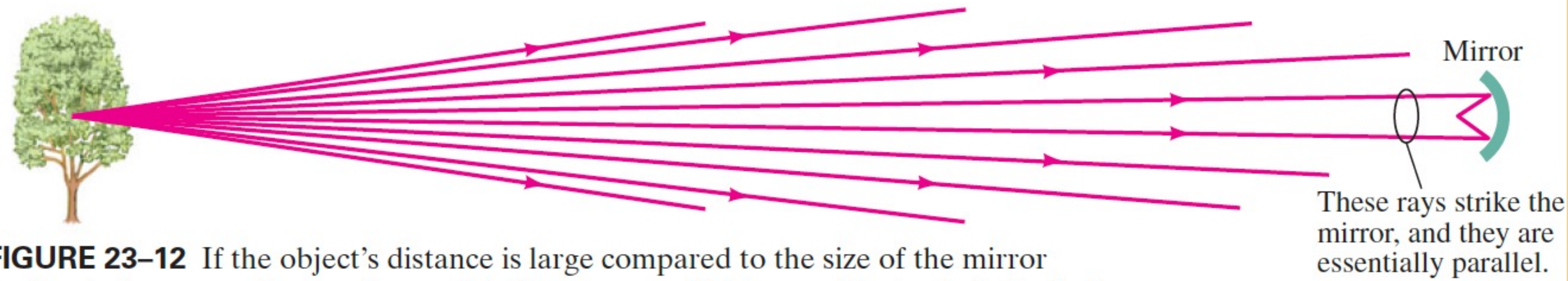
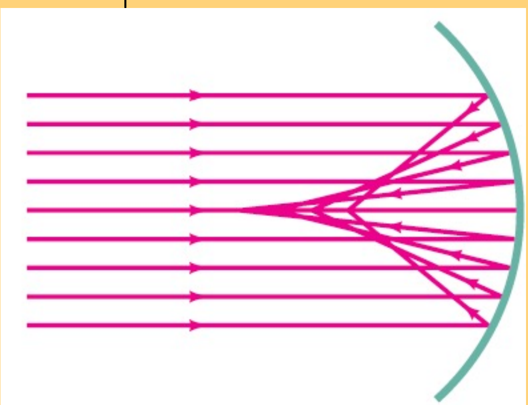
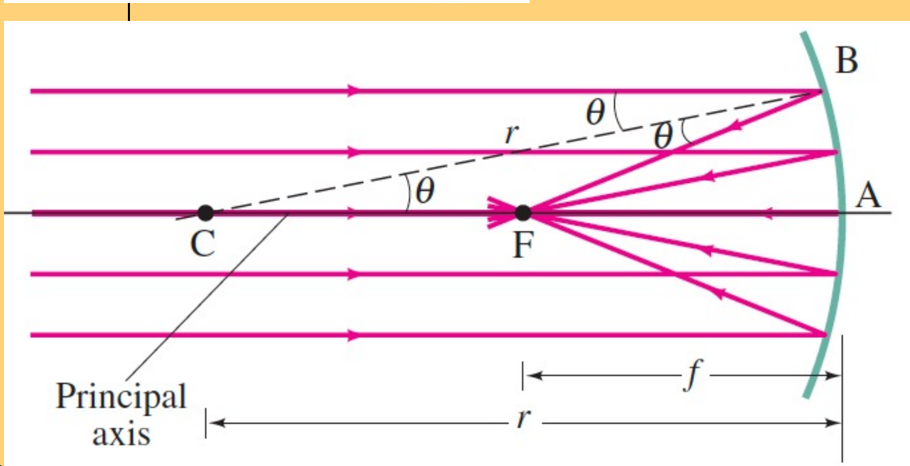


FIGURE 23–12 If the object's distance is large compared to the size of the mirror (or lens), the rays arrive nearly parallel. They are parallel for an object at infinity (∞).

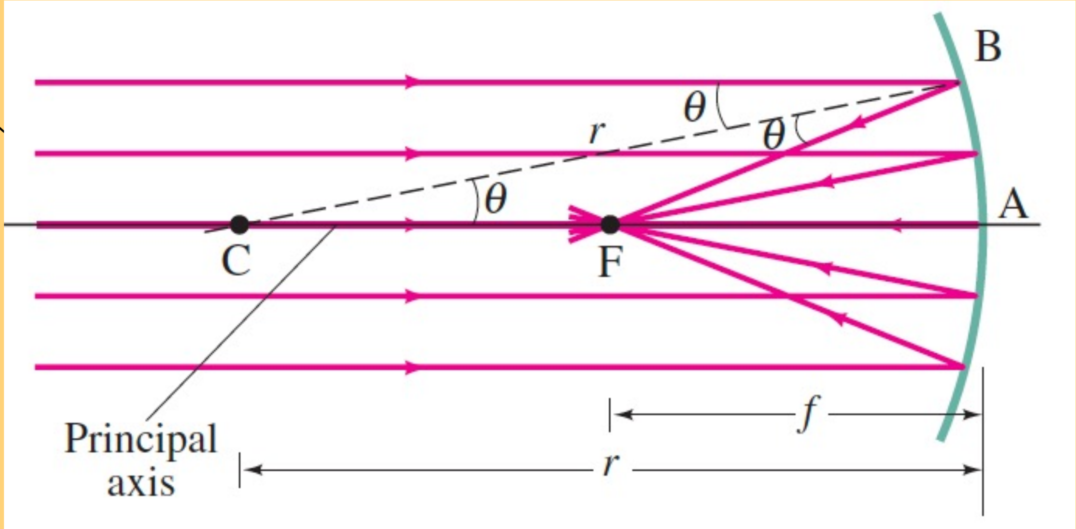


The reflected rays are not all brought to a single point, so the image is not sharp.
But if the mirror is small compared to its radius of curvature, the rays will cross each other almost at a single point, or **focus**.



CA= **principal axis**
(straight line perpendicular to the curved surface at its center)
F = **focal point** of the mirror.
FA = the **focal length**, *f*, of the mirror
C = **center of curvature** of the mirror
(the center of the sphere of which the mirror is a part).

Focal Length



For a mirror whose reflecting surface is small compared to its radius of curvature, the rays very nearly meet at a common point, F.

The rays make a small angle with the principal axis (**<u>paraxial rays</u>**)

CB is normal to the mirror's surface at B.
CB = r = radius of curvature.
The incoming ray that hits the mirror at B makes an angle θ with this normal (CB),
The reflected ray, BF, also makes an angle θ with the normal (law of reflection).
The angle BCF is also θ. Thus, the triangle CBF is isosceles, so CF = FB.
We assume the mirror surface is small compared to the mirror's radius of curvature, so the angles are small, and the length **FB is nearly equal to length FA**.
In this approximation, **FA = FC**.
But FA = f and CA = 2xFA = r.

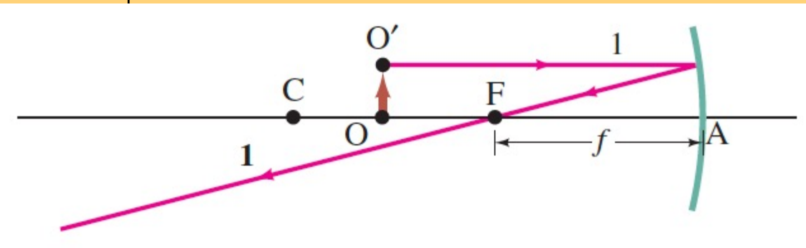
$$f = \frac{r}{2}$$

Spherical aberration = the "defect" of **spherical** mirrors.
A **parabolic reflector** reflect the rays to a perfect focus.

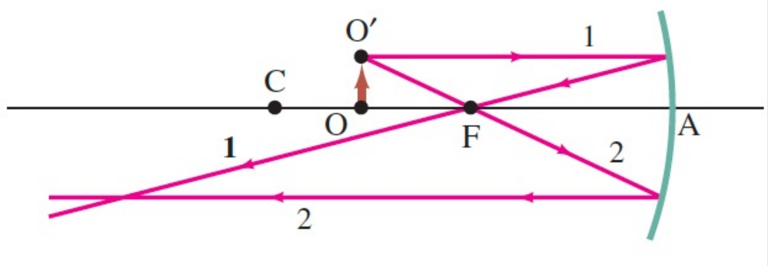
Image Formation - Ray Diagrams

For an object **at infinity**, the image is located at the focal point of a concave spherical mirror, where *f=r/2*.

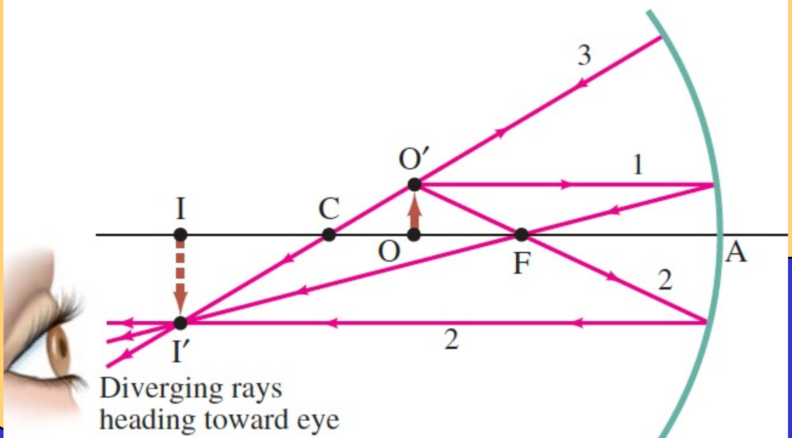
Where does the image lie for an object not at infinity?
Determining the image position is faster if we deal with **three** particular rays.



Ray 1 leaving O' is parallel to the axis; therefore after reflection it must pass along a line through F.



Ray 2 goes through F and then reflects back parallel to the axis.

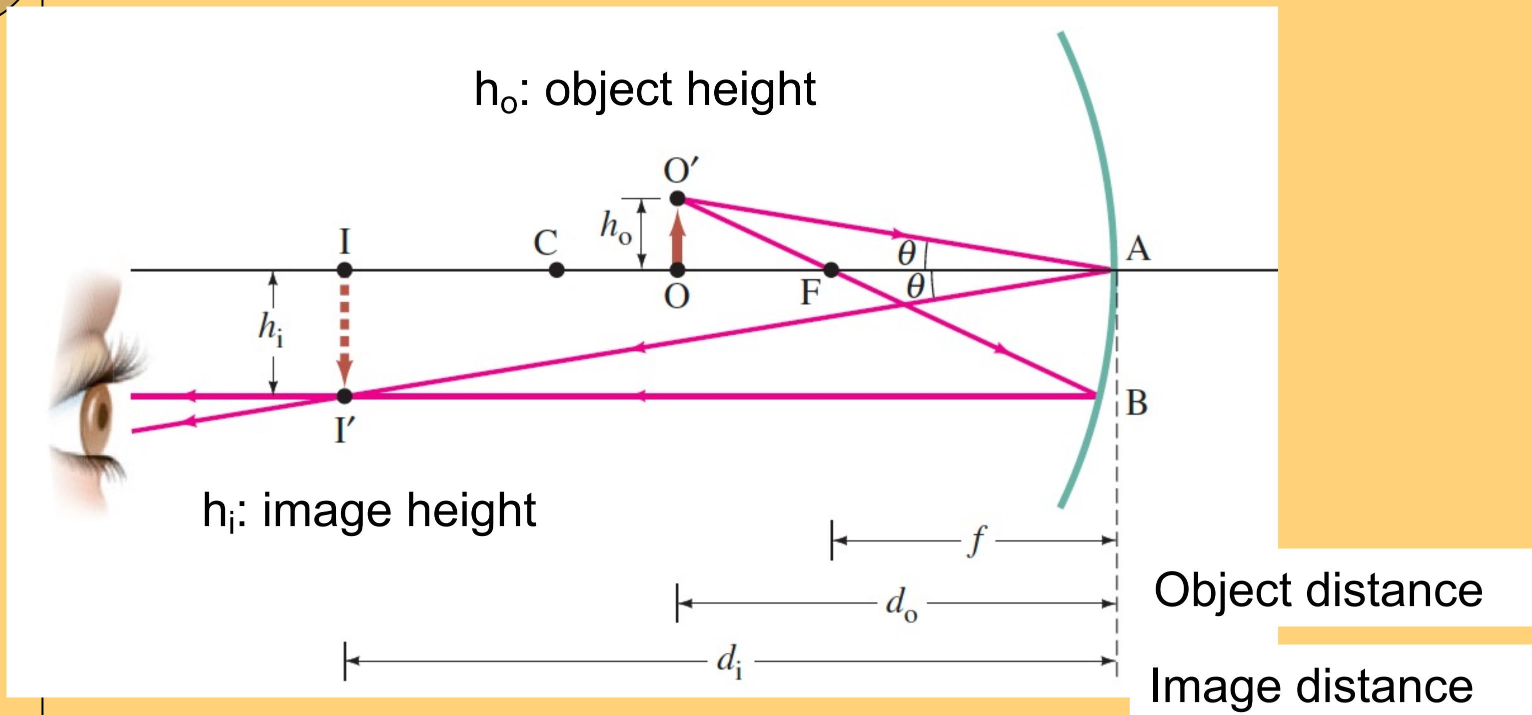


Ray 3 is perpendicular to mirror, and so must reflect back on itself and go through C

Only two of these rays are needed, but the third serves as a check.
Because the light passes through the image, this is a **real image.** It can be seen by the eye only when the eye is placed to the left of the image.

Mirror Equation and Magnification

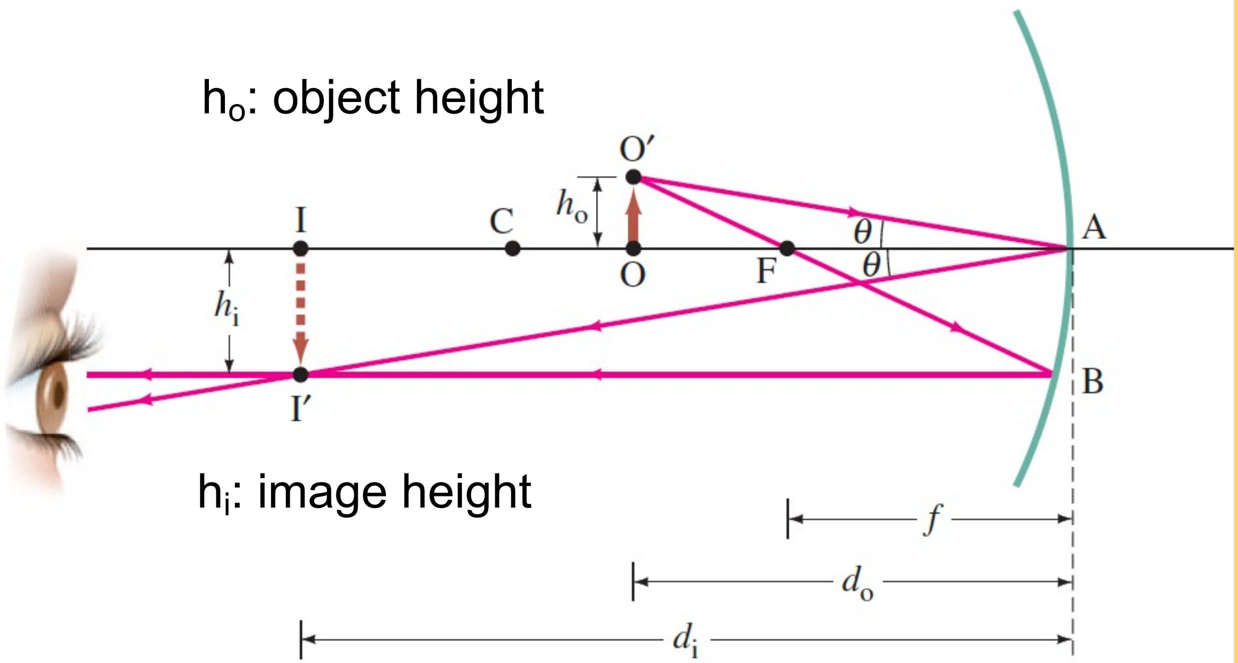
Equation that gives the image distance if the object distance and radius of curvature of the mirror are known.



Two rays leaving O’ are shown: O’FBI’ and O’AI’ which is a fourth type of ray that reflects at the center of the mirror and can also be used to find an image point.

Mirror Equation

h_o: object height



h_i: image height

The 2 right triangles O'AO and IAI' are similar, so

$$\frac{h_o}{h_i} = \frac{d_o}{d_i}$$

The triangles O'FO and AFB are also similar (angles at F are equal)

$$\frac{h_o}{AB} = \frac{OF}{FA}$$

We use the approximation **AB=*h*** (mirror small compared to its radius).
We also have that AF =*f* (focal length)

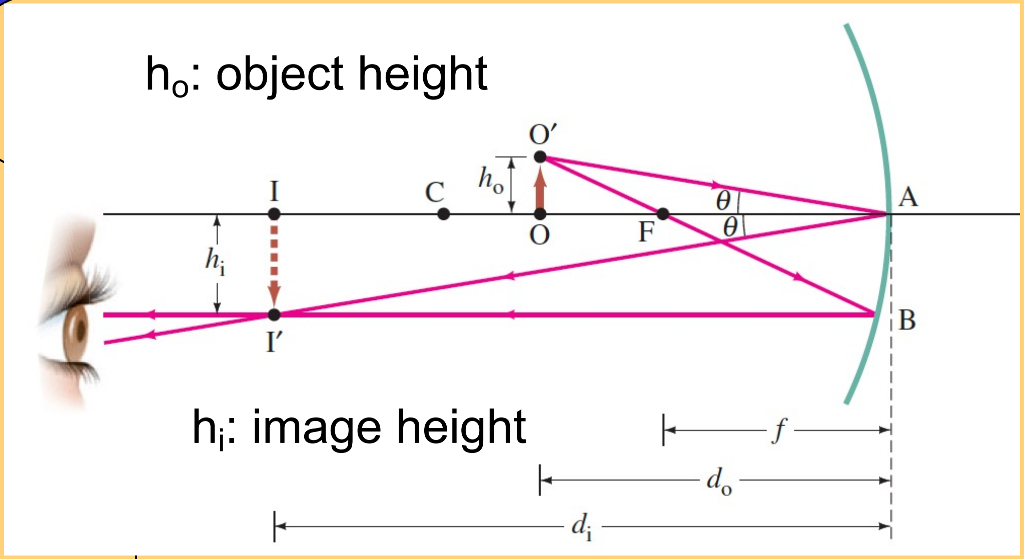
$$\frac{h_o}{h_i} = \frac{\mathrm{OF}}{\mathrm{FA}} = \frac{d_o - f}{f}$$

$$\frac{d_o}{d_i} = \frac{d_o - f}{f}$$

Divide by d_o

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Magnification



The 2 right triangles O'AO and IAI' are similar, so

$$\frac{h_o}{h_i} = \frac{d_o}{d_i}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

The minus sign is inserted as a convention.

1. the image height h_i is positive if the image is upright, and negative if inverted, relative to the object;
2. d_o or d_i is positive if image or object is in front of the mirror; if either image or object is behind the mirror, the corresponding distance is negative.

Thus, the magnification is positive for an upright image and negative for an inverted image (upside down).

Exercise 1

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Ex: A 1.50-cm-high object is placed 20.0 cm from a concave mirror with radius of curvature 30.0 cm. Determine (a) the position of the image, and (b) its size.

(a)

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$f = r/2 = 15.0 \text{ cm}$$

So d_i = 60.0 cm

Because d_i is positive, the image is 60.0 cm in front of the mirror, on the same side as the object.

(b)

$$m = -\frac{d_i}{d_o} = -\frac{60.0 \text{ cm}}{20.0 \text{ cm}} = -3.00$$

$$h_i = mh_o = (-3.00)(1.5 \text{ cm}) = -4.5 \text{ cm}$$

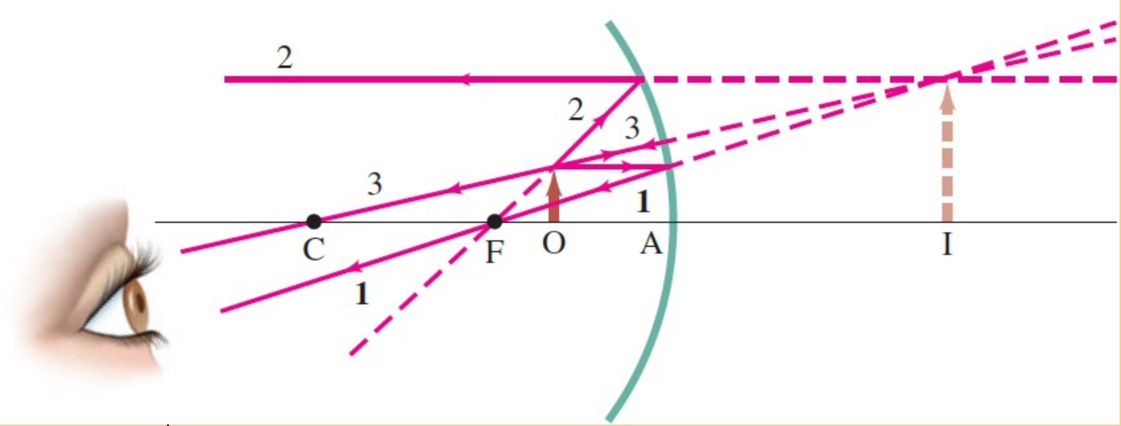
The minus sign reminds us that the image is inverted

Exercise 2

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Ex: **Object closer to concave mirror than focal point**. A 1.00-cm-high object is placed 10.0 cm from a concave mirror whose radius of curvature is 30.0 cm. (a) Draw a ray diagram to locate (approximately) the position of the image. (b) Determine the position of the image and the magnification analytically.



Since $f=r/2$=15.0 cm, the object is between the mirror and the focal point.
The rays reflected from the mirror diverge. They appear to be coming from a point behind the mirror (dashed lines). The image is **virtual**.

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15.0\text{ cm}} - \frac{1}{10.0\text{ cm}} = \frac{2-3}{30.0\text{ cm}} = -\frac{1}{30.0\text{ cm}}$$

Minus sign means the image is behind the mirror

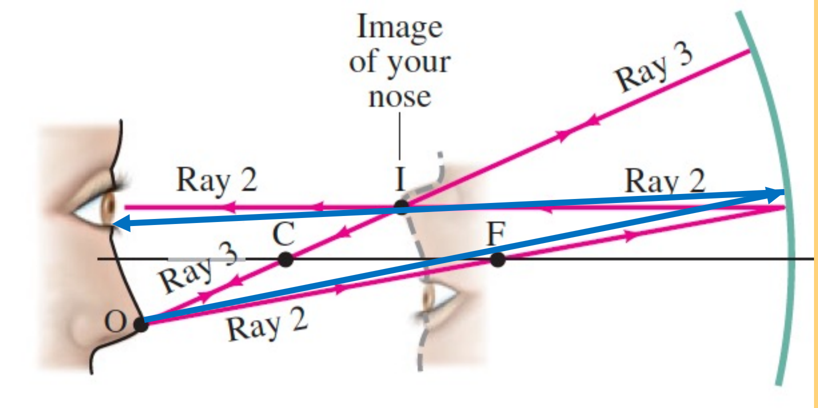
$$m = -d_i/d_o = -(-30.0\text{ cm})/(10.0\text{ cm}) = +3.00$$

The plus sign indicates that the image is upright

This is how a shaving or cosmetic mirror is used—you must place your head closer to the mirror than the focal point if you are to see yourself right-side up

Seeing the Image

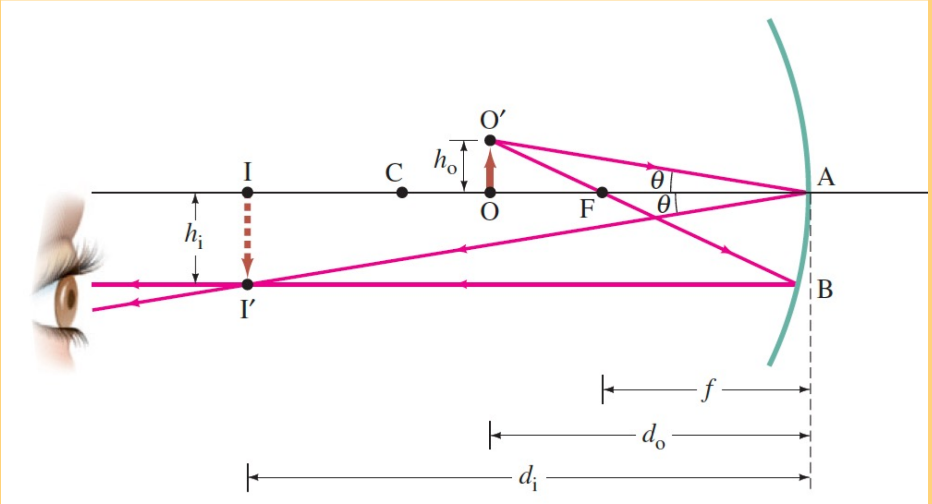
You can see a clear inverted image of your face in a concave mirror when you are beyond C ($d_o > 2f$) because the rays that arrive at your eye are diverging. Ray 2 (and other nearby rays) enters your eye and they are diverging as they move to the left of image point I.



Seeing the Image

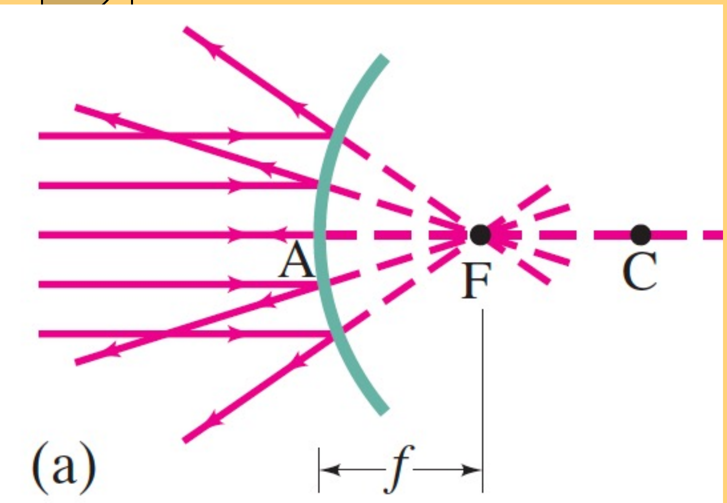
For a person's eye to see a sharp image, the eye must be at a place where it intercepts **diverging** rays from points on the image.

If the rays are **converging**, you see a blurry image. Example: If you placed your eye between points O and I.



Convex Mirrors

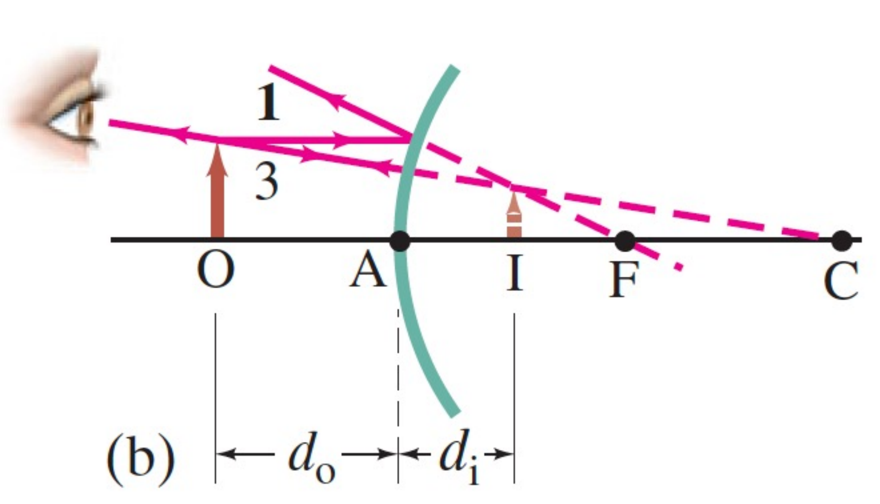
The mirror equation holds for convex mirrors, but the focal length f and radius of curvature must be considered negative. The magnification is also valid.



Again, spherical aberration is significant, unless we assume the mirror is small compared to its radius of curvature.

The equation *f=r/2* is valid also for a convex mirror.

No matter where the object is placed on the reflecting side of a convex mirror, the image will be **virtual** and **upright**.



Sign Conventions

(a) When the object, image, or focal point is on the reflecting side of the mirror (on the left in our drawings), the corresponding distance is positive.

If any of these points is behind the mirror (on the right) the corresponding distance is negative.

(b) The image height h_i is positive if the image is upright, and negative if inverted, relative to the object (h_o is always taken as positive).

Exercise

Ex. An external rearview car mirror is convex with a radius of curvature of 16.0 m. Determine the location of the image and its magnification for an object 10.0 m from the mirror.

We have a convex mirror, so *r* is negative.

The center of curvature of a convex mirror is behind the mirror, as is its focal point, so we set *r* = -16.0 m, so that the focal length is *f* = -8.0 m. The object is in front of the mirror, d_o = 10.0 m.

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-8.0\text{ m}} - \frac{1}{10.0\text{ m}}$$

$$d_i = -80.0\text{ m}/18 = -4.4\text{ m}$$

$$m = -\frac{d_i}{d_o} = -\frac{(-4.4\text{ m})}{(10.0\text{ m})} = +0.44$$

The image distance is negative,-4.4m, so the image is behind the mirror.
The magnification is +0.44 so the image is upright and about half what it would be in a plane mirror.

Convex rearview mirrors on vehicles sometimes come with a warning that objects are closer than they appear in the mirror. The fact that d_i is smaller than d_o contradicts this observation. The real reason the object seems farther away is that its image in the convex mirror is smaller than it would be in a plane mirror, and we judge distance of ordinary objects such as other cars mostly by their size.

Index of Refraction

The ratio of the speed of light in vacuum, $c = 3.00 \times 10^8\ \mathrm{m/s}$ to the speed v in a given material is called the index of refraction.

TABLE 23–1 Indices of Refraction[†]

Material	$n = \dfrac{c}{v}$
Vacuum	1.0000
Air (at STP)	1.0003
Water	1.33
Ethyl alcohol	1.36
Glass	
Fused quartz	1.46
Crown glass	1.52
Light flint	1.58
Plastic	
Acrylic, Lucite, CR-39	1.50
Polycarbonate	1.59
"High-index"	1.6–1.7
Sodium chloride	1.53
Diamond	2.42

[†] $\lambda = 589$ nm.

$$n = \frac{c}{v}$$

$$v = \frac{c}{n} = \frac{(3.00 \times 10^8\ \mathrm{m/s})}{1.33} = 2.26 \times 10^8\ \mathrm{m/s}.$$

n varies somewhat with the wavelength of the light—except in vacuum—so a particular wavelength is specified in the Table.

Refraction: Snell's Law

When light passes from one transparent medium into another with a different index of refraction, some or all of the incident light is reflected at the boundary. The rest passes into the new medium. If a ray of light is incident at an angle to the surface (other than perpendicular), the ray changes direction as it enters the new medium. This change in direction, or bending, of the light ray is called **refraction**.

(a) Light refracted when passing from air (n_1) into water (n_2): $n_2 > n_1$.

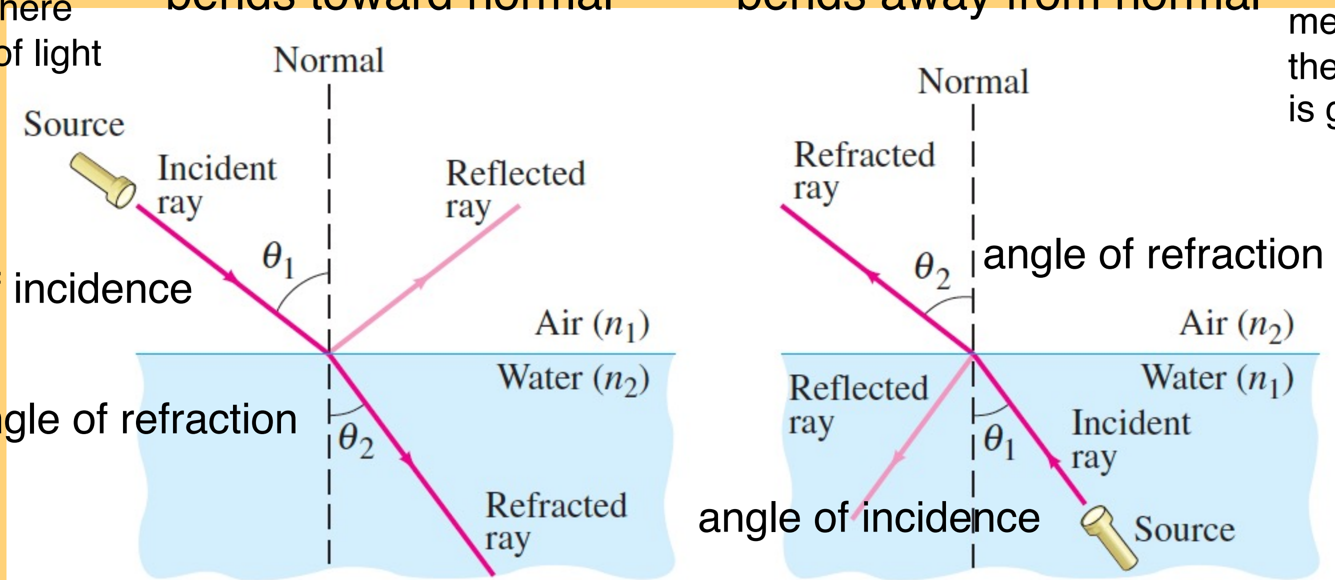
(b) Light refracted when passing from water (n_1) into air (n_2): $n_1 > n_2$.

ray enters a medium, where the speed of light is less

bends toward normal

bends away from normal

ray enters a medium, where the speed of light is greater



(a) $n_2 > n_1$: Ray bends toward $\perp$

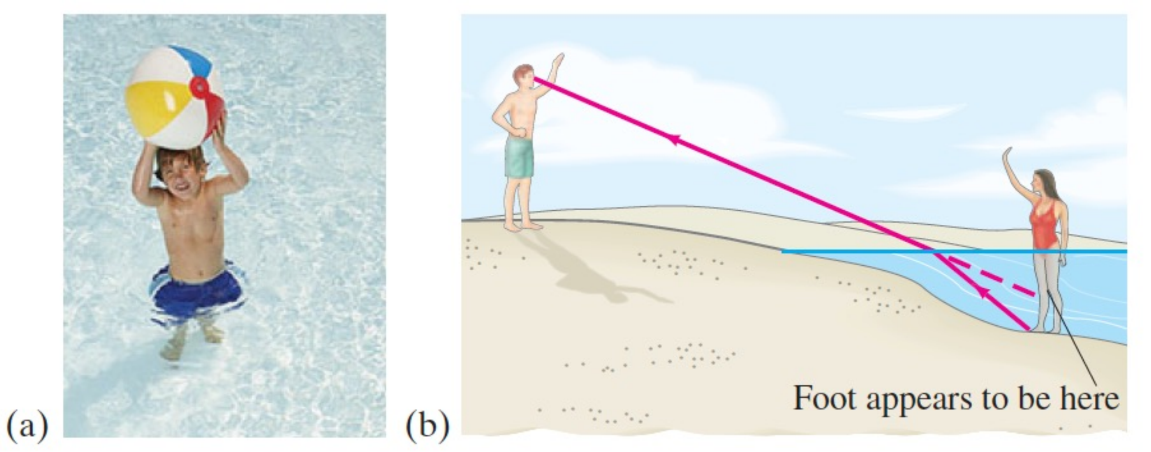
(b) $n_1 > n_2$: Ray bends away from $\perp$

Snell's law is the law of refraction.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 .$$

Refraction

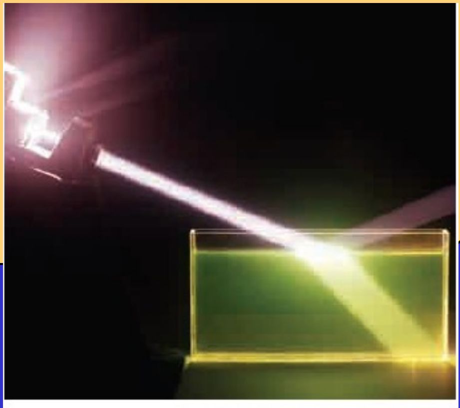
Refraction is responsible for a number of common optical illusions.
A person standing in waist-deep water appears to have shortened legs.
When you put a straw in water, it appears to be bent.



Exercise: Light passes from a medium with n=1.3 (water) into a medium with n = 1.5 (glass). Is the light bent toward or away from the perpendicular to the interface?

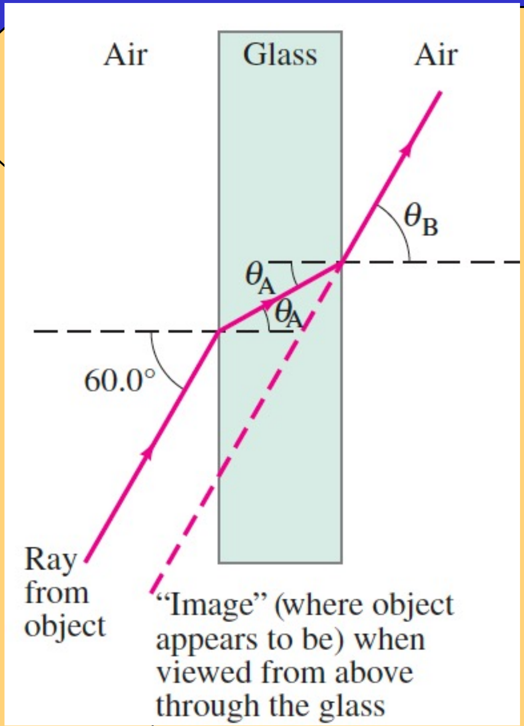
Toward.

A photograph showing an incident beam of light reflected and refracted by a horizontal water surface.



https://www.youtube.com/watch?v=sBb5WUw2_2I

Exercise 01



Ex: Light traveling in air strikes a flat piece of uniformly thick glass at an incident angle of 60.0°. If the index of refraction of the glass is 1.50, (a) what is the angle θ_A of refraction in the glass; (b) what is the angle θ_B at which the ray emerges from the glass?

(a) The incident ray is in air, so n_1 = 1.00 and n_2 = 1.50.

$$\theta_A = 35.3°$$

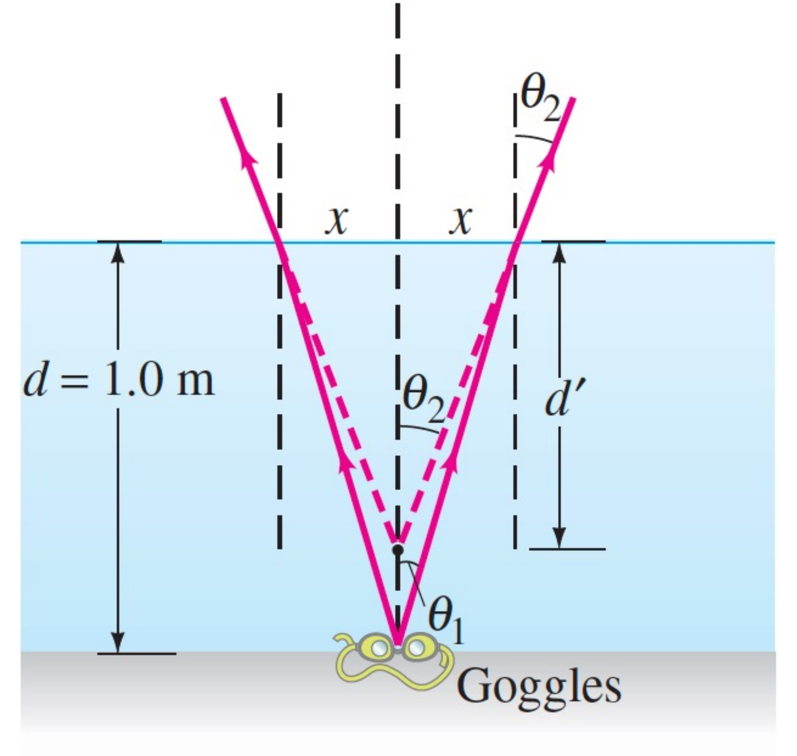
(b) Since the faces of the glass are parallel, the incident angle at the second surface is also θ_A

$$\theta_B = 60.0°$$

The direction of a light ray is thus unchanged by passing through a flat piece of glass of uniform thickness.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 .$$

Exercise 02



Ex: A swimmer has dropped her goggles to the bottom of a pool at the shallow end, marked as 1.0 m deep. But the goggles don't look that deep. Why? How deep do the goggles appear to be when you look straight down into the water?

The two rays traveling upward from the goggles are refracted away from the normal as they exit the water, and so appear to be diverging from a point above the goggles (dashed lines), which is why the water seems less deep than it actually is.

$n_1 = 1.33$ for water and $n_2 = 1$ for air

$$\sin\theta_2 = n_1 \sin\theta_1$$

considering only small angles,

$$\sin\theta \approx \tan\theta \approx \theta \Rightarrow \theta_2 \approx n_1\theta_1$$

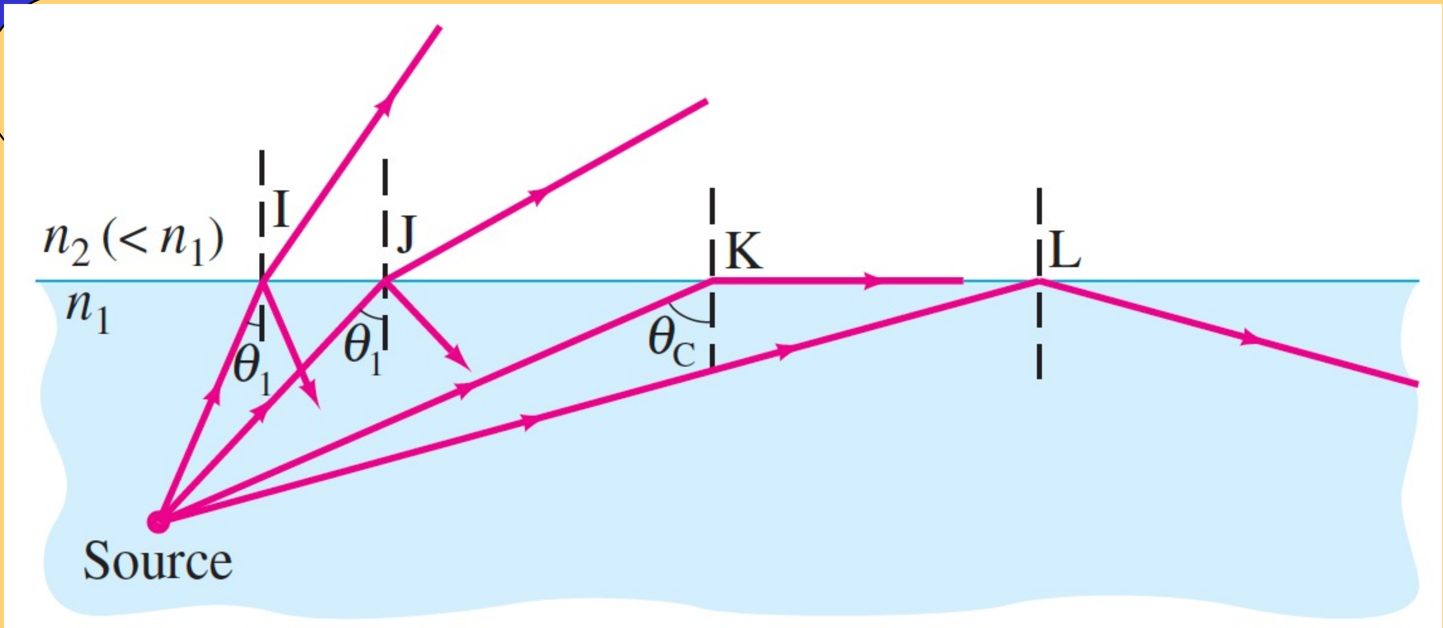
$$\theta_2 \approx \tan\theta_2 = x/d' \qquad \theta_1 \approx \tan\theta_1 = x/d$$

$$\frac{x}{d'} \approx n_1 \frac{x}{d}$$

$$d' \approx \frac{d}{n_1} = \frac{1.0\text{ m}}{1.33} = 0.75\text{ m}$$

Water in general is deeper than it looks

Total Internal Reflection



When light passes from one material into a second material where the index of refraction is less (say, from water into air), the refracted light ray bends away from the normal, as for rays I and J. At a particular incident angle, the angle of refraction will be 90°, and the refracted ray would skim the surface (ray K).

The incident angle at which this occurs is called the critical angle

$$\sin\theta_C = \frac{n_2}{n_1}\sin 90^\circ = \frac{n_2}{n_1}$$

For incident angles greater than θ_C there is no refracted ray at all, and all of the light is reflected, as for ray L. This effect is called **total internal reflection**.

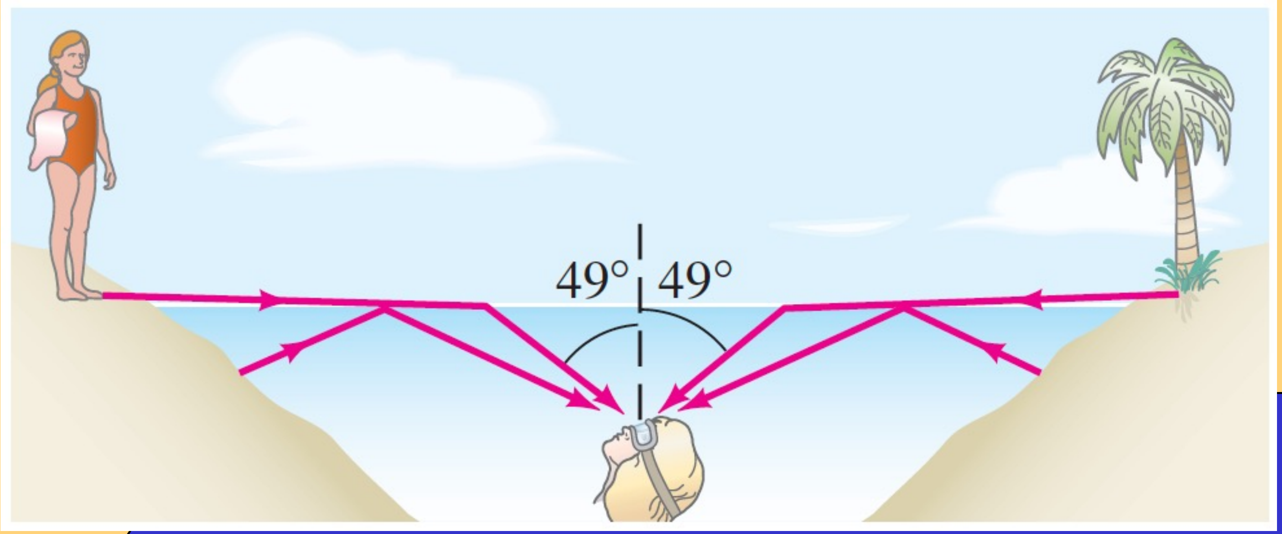
Total Internal Reflection

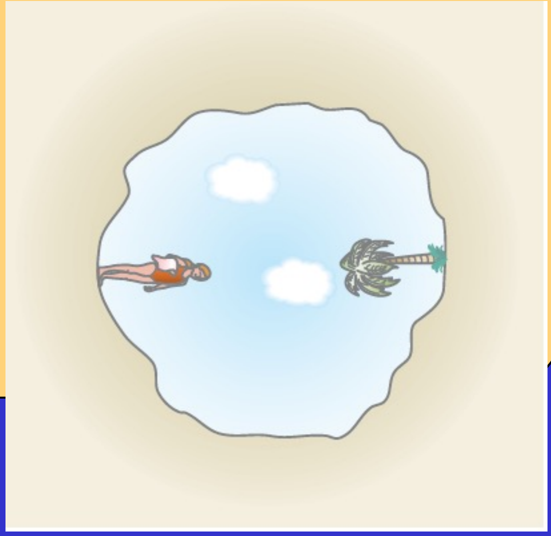
Ex. Describe what a person would see who looked up at the world from beneath the perfectly smooth surface of a lake or swimming pool.

Answer: For an air–water interface, the critical angle is given by

$$\sin \theta_C = \frac{1.00}{1.33} = 0.750 \quad \Rightarrow \quad \theta_C = 49^\circ$$

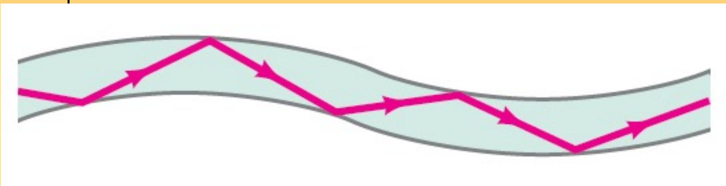
The person would see the outside world compressed into a circle whose edge makes a 49° angle with the vertical. Beyond this angle, the person would see reflections from the sides and bottom of the lake or pool.





Fiber Optics; Medical Instruments

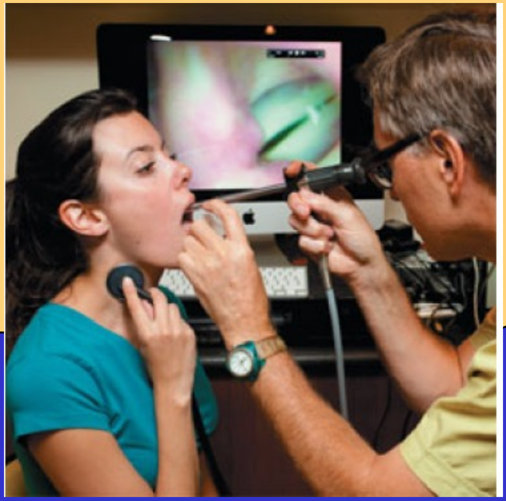
Total internal reflection is the principle behind fiber optics. Glass and plastic fibers as thin as a few micrometers in diameter are commonly used. A bundle of such slender transparent fibers is called a light pipe or fiber-optic cable.



Light (not only visible light but also infrared light, ultraviolet light, and microwaves) can be transmitted along the fiber with almost no loss because of total internal reflection.

Important applications of fiber-optic cables are in communications and medicine. They are used in place of wire to carry telephone calls, video signals, and computer data.

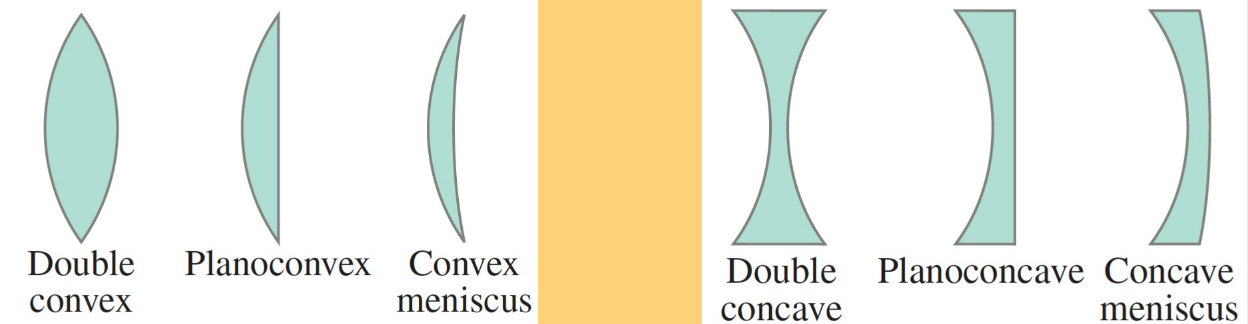
Instruments, including bronchoscopes (for viewing a patient's lungs), colonoscopes (for viewing the colon), and endoscopes (stomach or other organs), are extremely useful for examining hard-to-reach places.



Example of a fiber-optic device inserted through the mouth to view the vocal cords.

Thin Lenses

The most important simple optical device is the thin lens: eyeglasses, cameras, magnifying glasses, telescopes, binoculars, microscopes, and medical instruments. The two faces can be concave, convex, or plane.



(a) Converging lenses (thicker in the center than at the edges)

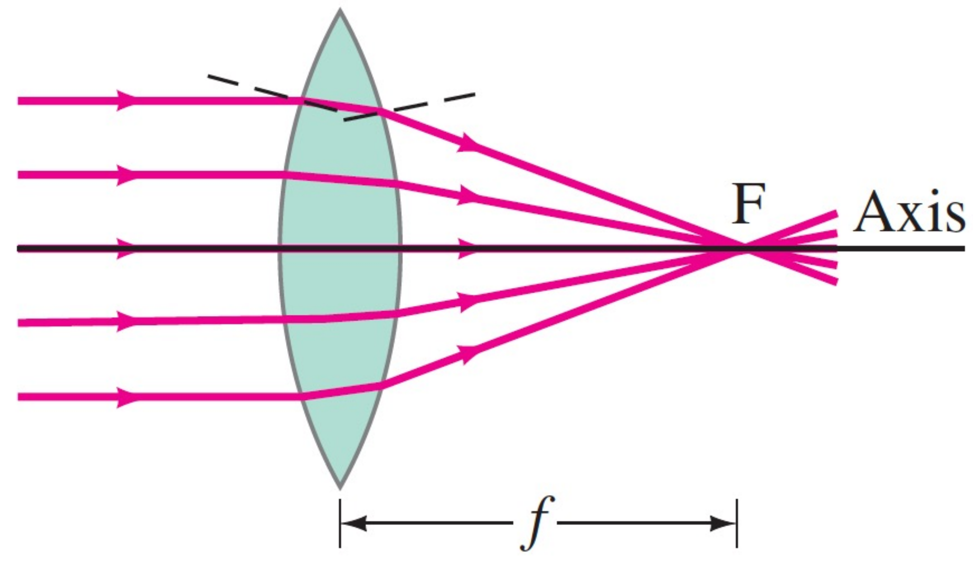
(b) Diverging lenses (thinner in the center than at the edges)

The **axis** of a lens is a straight line passing through the center of the lens and perpendicular to its two surfaces.

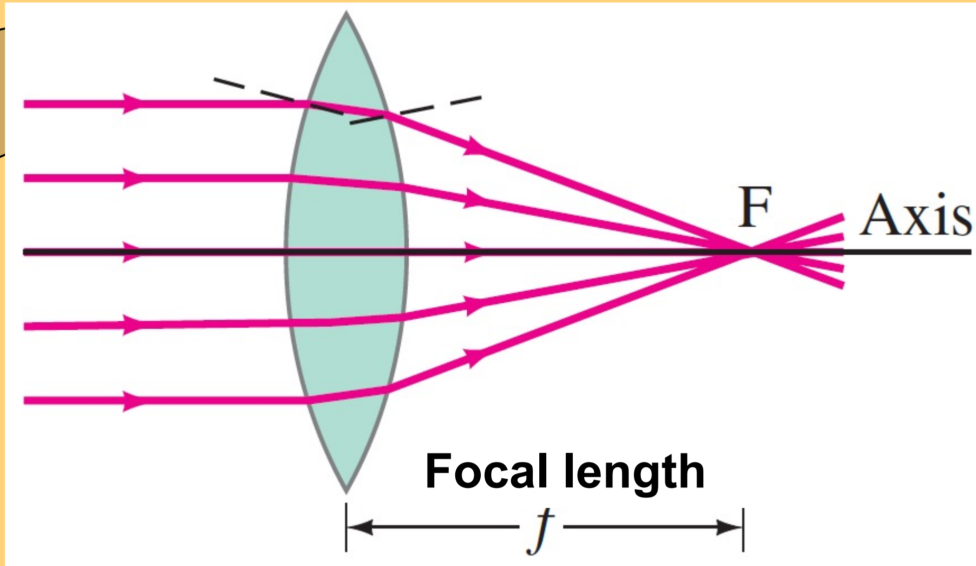
From Snell's law, the ray is bent toward the axis when the ray enters the lens and away when it leaves the lens at the back surface.

They are focused to the **focal point**, F.

Parallel rays are focused to a tiny region that is nearly a point—if the thickness of the lens is small compared to the radii of curvature of the two lens surfaces: **thin lens**.

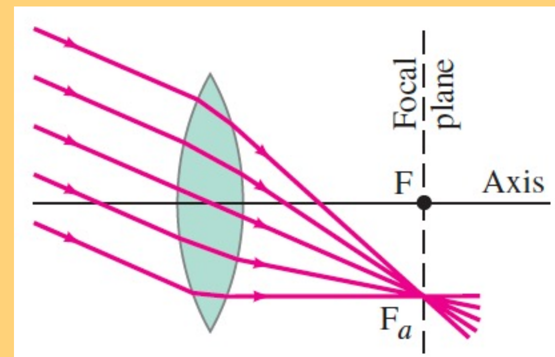


Converging vs Diverging Lenses

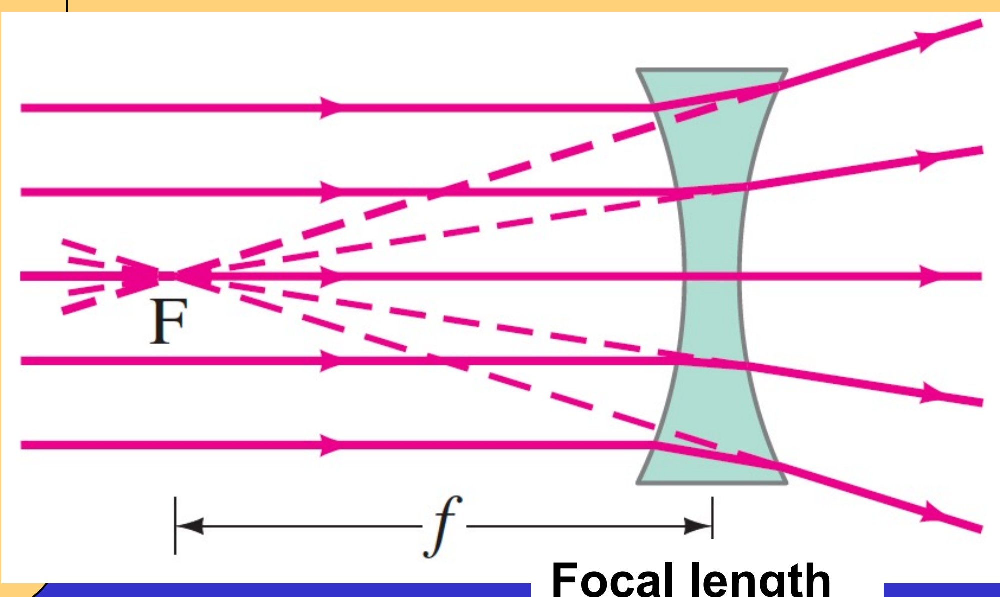


(thicker in the center than at the edges)

A lens can be turned around so that light can pass through it from the opposite side. The focal length is the same on both sides.



Above: parallel rays fall on a lens at an angle. The plane containing all focus points, such as F and F_a, is called the focal plane of the lens.



(thinner in the center than at the edges)

Power and Diopter

Optometrists and ophthalmologists, use the reciprocal of the focal length to specify the strength of eyeglass (or contact) lenses. This is called the **power**, P, of a lens.

$$P = \frac{1}{f}$$

The unit for lens power is the diopter (D), which is an inverse meter:

$$\mathbf{1\ D = 1\ m^{-1}}$$

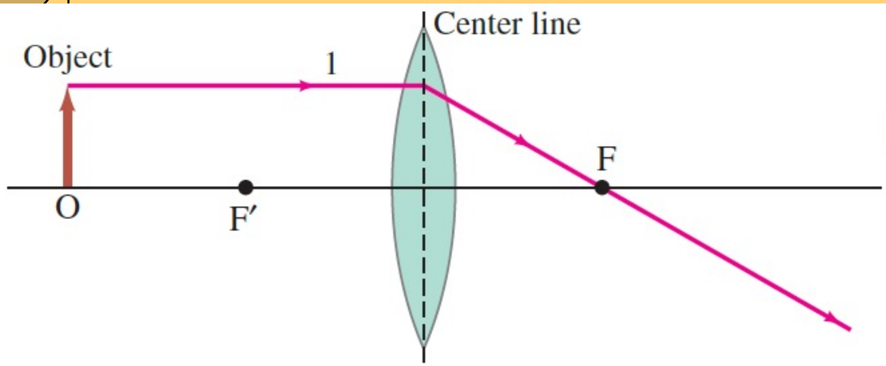
For example, a 20-cm-focal-length lens has a power

$$P = 1/(0.20\ \mathrm{m}) = 5.0\ \mathrm{D}$$

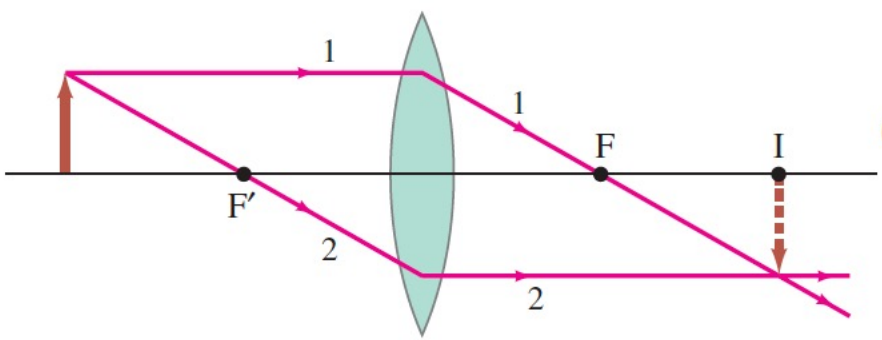
The focal point of a lens and thus the focal length can be found by locating the point where the Sun's rays (or those from some other distant object) are brought to a sharp image.

Real Image: Converging Lens

The rays, emanating are drawn as if the lens were infinitely thin, and we show only a single sharp bend at the center line of the lens instead of the refractions at each surface.



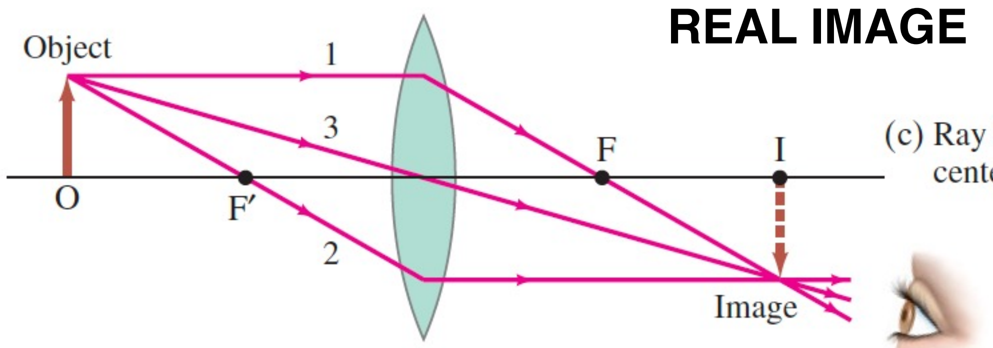
(a) Ray 1 leaves one point on object going parallel to the axis, then refracts through focal point behind the lens.



(b) Ray 2 passes through F′ in front of the lens; therefore it is parallel to the axis behind the lens.

We can see a sharp image only for rays diverging from each point on the image. Eye positioned between points F and I does not see a clear image.

REAL IMAGE



(c) Ray 3 passes straight through the center of the lens (assumed very thin).

The ray would be displaced slightly to one side, as we saw in an EXERCISE; but the lens is thin, so ray 3 is ~ straight.

Virtual Image: Diverging Lens

By drawing the same three rays emerging from a single object point, we can determine the image position formed by a diverging lens.

*) Ray 1 is drawn parallel to the axis, but does not pass through the focal point F’ behind the lens. It seems to come (dashed line) from the focal point F in front of the lens.

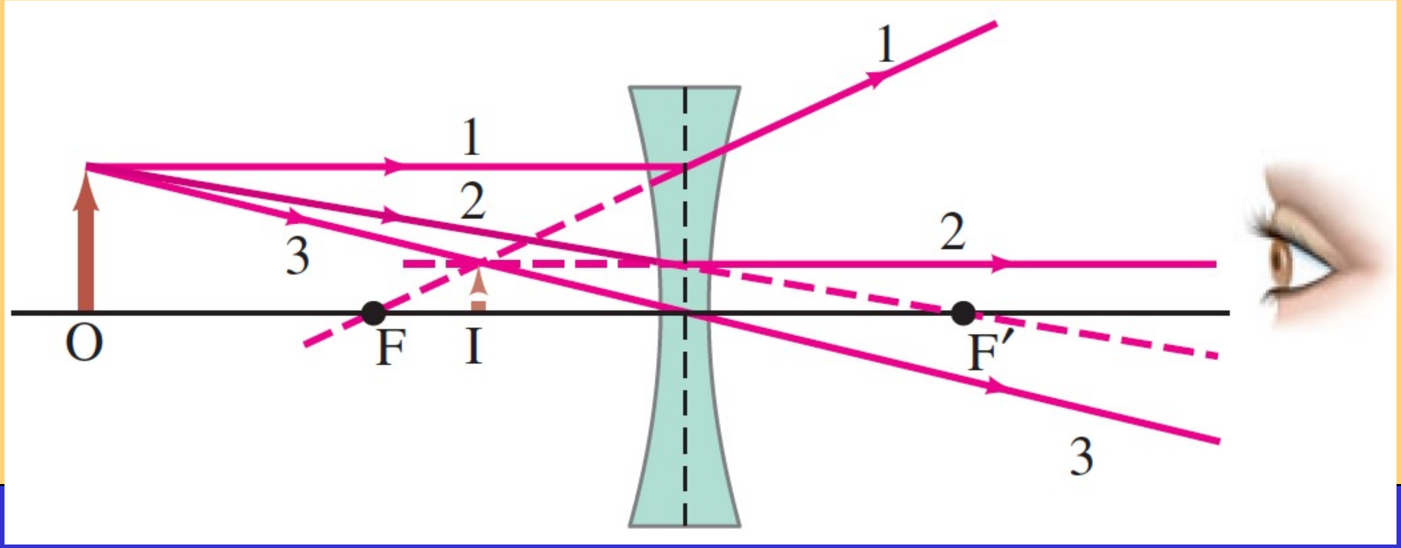
*) Ray 2 is directed toward and is refracted parallel to the lens axis by the lens.

*) Ray 3 passes directly through the center of the lens.

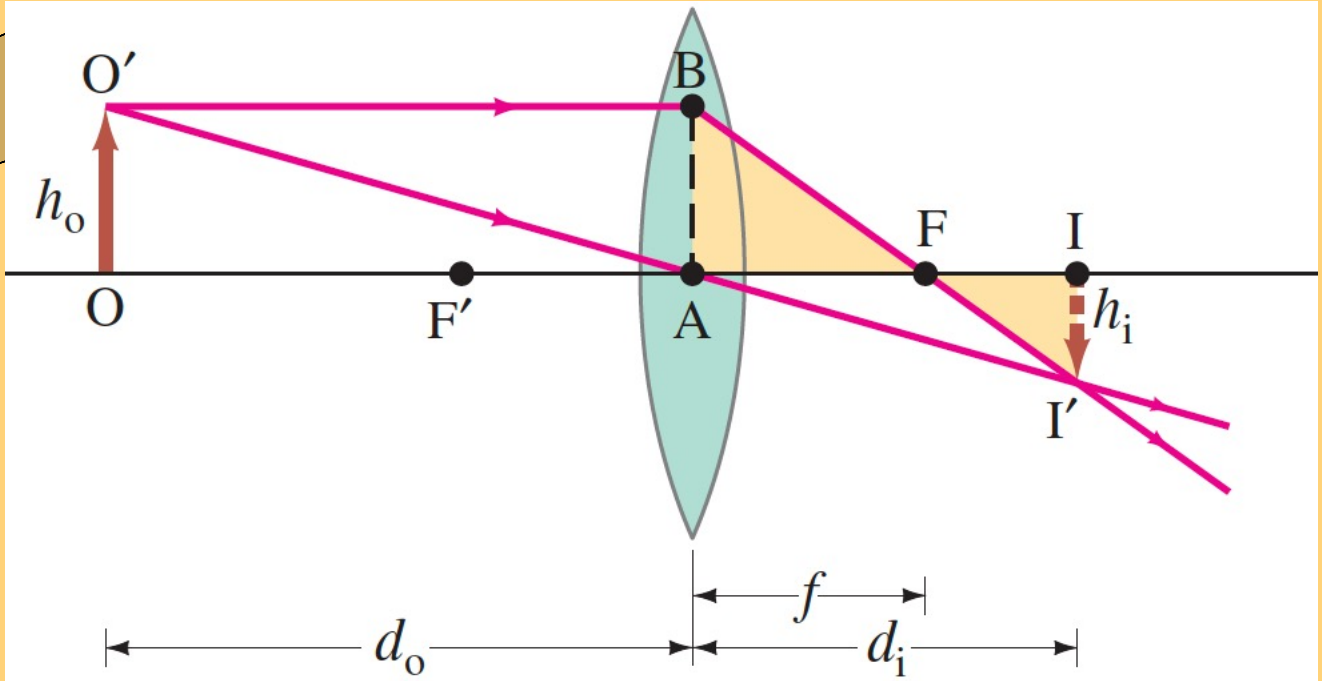
The three refracted rays seem to emerge from a point on the left of the lens. This is the image point, I.

Because the rays do not pass through the image, it is a **<u>VIRTUAL IMAGE</u>**.

The eye does not distinguish between real and virtual images—both are visible.



The Thin Lens Equation



The focal length is the image distance for an object at infinity, $1/d_o=0$

The right triangles FI'I and FBA (highlighted in yellow) are similar

$$\frac{h_i}{h_o} = \frac{d_i - f}{f}$$

Triangles OAO' and IAI' are similar as well.

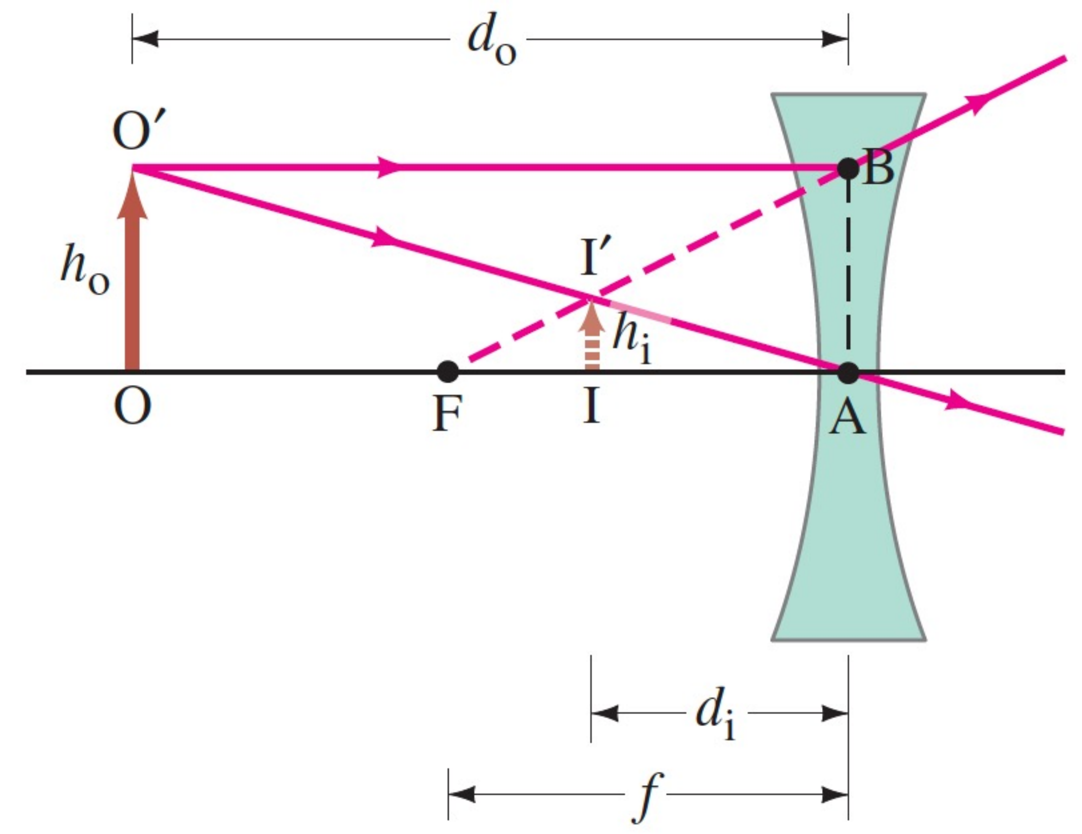
$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$

Equate the right sides and divide by d_i

$$\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$$

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$

The Thin Lens Equation



Triangles OAO’ and IAI’ are similar.

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$

Triangles IFI’ and AFB are similar.

$$\frac{h_i}{h_o} = \frac{f - d_i}{f}$$

This equation becomes the same as in the previous slide if we make *f* and d_i negative.

$$\boxed{\frac{1}{d_o} - \frac{1}{d_i} = -\frac{1}{f}}$$

The Thin Lens Equation

This equation

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$

is valid for both converging and diverging lenses, and for all situations, if we use the following sign conventions:

1. The focal length is <u>positive for converging</u> lenses and <u>negative for diverging</u> lenses.
2. The object distance is positive if the object is on the side of the lens from which the light is coming, otherwise, it is negative.
3. The <u>image distance is positive if the image is on the opposite side of the lens</u> from where the light is coming; if it is on the same side, it is negative. Equivalently, the <u>image distance is positive for a real image</u> and <u>negative for a virtual image</u>.
4. The height of the image, h_i <u>is positive if the image is upright</u>, and negative if the image is inverted relative to the object.

(h_o is always taken as upright and positive.)

Magnification

The magnification, *m*, of a lens is defined as the ratio of the image height to object height.

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

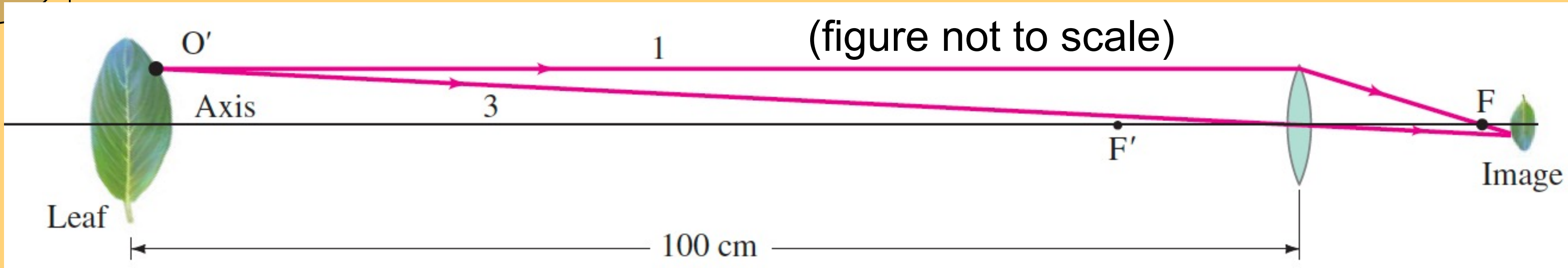
For an upright image the magnification is positive, and for an inverted image the magnification is negative.

From sign convention, it follows that the power of a converging lens, in diopters, is positive, whereas the power of a diverging lens is negative.
A **converging lens** is sometimes referred to as a **positive lens**, and a **diverging lens** as a **negative lens**.

Diverging lenses always produce an **upright virtual image** for any real object, no matter where that object is.
Converging lenses can produce real (inverted) images or virtual (upright) images, depending on the object position.

Exercise: Image formed by converging lens

What is (a) the position, and (b) the size, of the image of a 7.6-cm-high leaf placed 1.00 m from a +50.0-mm-focal-length camera lens?



The camera lens is converging, so *f* is positive.
The object is on the side of the lens from which the light is coming, so d_o is positive.

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{5.00\text{ cm}} - \frac{1}{100\text{ cm}} = \frac{20.0 - 1.0}{100\text{ cm}} = \frac{19.0}{100\text{ cm}}$$

$$d_i = \frac{100\text{ cm}}{19.0} = 5.26\text{ cm}$$

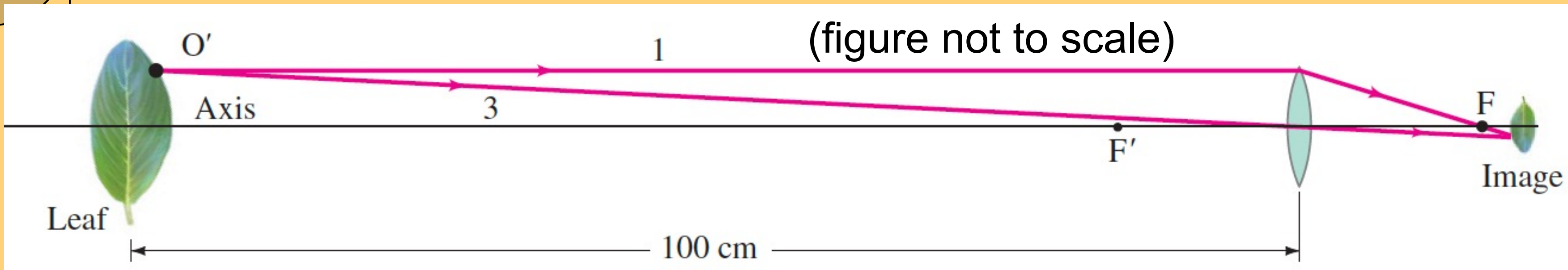
$$m = -\frac{d_i}{d_o} = -\frac{5.26\text{ cm}}{100\text{ cm}} = -0.0526$$

$$h_i = mh_o = (-0.0526)(7.6\text{ cm}) = -0.40\text{ cm}$$

The image distance came out positive, so the image is behind the lens.
The image height has the minus sign, so the image is inverted.

Exercise: Image formed by converging lens

What is (a) the position, and (b) the size, of the image of a 7.6-cm-high leaf placed 1.00 m from a +50.0-mm-focal-length camera lens?



The camera lens is converging, so *f* is positive.
The object is on the side of the lens from which the light is coming, so d_o is positive.

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{5.00\text{ cm}} - \frac{1}{100\text{ cm}} = \frac{20.0 - 1.0}{100\text{ cm}} = \frac{19.0}{100\text{ cm}}$$

$$d_i = \frac{100\text{ cm}}{19.0} = 5.26\text{ cm}$$

$$m = -\frac{d_i}{d_o} = -\frac{5.26\text{ cm}}{100\text{ cm}} = -0.0526$$

$$h_i = mh_o = (-0.0526)(7.6\text{ cm}) = -0.40\text{ cm}$$

The image distance came out positive, so the image is behind the lens.
The image height has the minus sign, so the image is inverted.

Part (a) tells us that the image is 2.6 mm farther from the lens than the image for an object at infinity, which would equal the focal length, 50.0 mm.

Exercise: Object close to converging lens

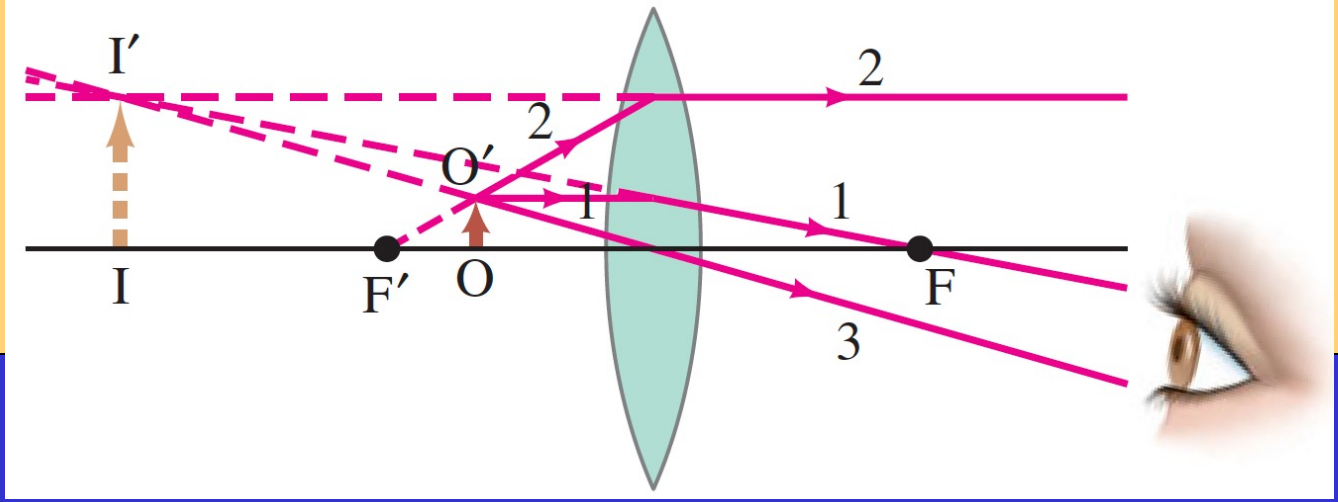
An object is placed 10 cm from a 15-cm-focal-length converging lens. Determine the image position and size (a) analytically, and (b) using a ray diagram.

The object is within the focal point—closer to the lens than the focal point F

$$\frac{1}{d_i} = \frac{1}{15\text{ cm}} - \frac{1}{10\text{ cm}} = \frac{2-3}{30\text{ cm}} = -\frac{1}{30\text{ cm}} \Rightarrow d_i = -30\text{ cm}$$

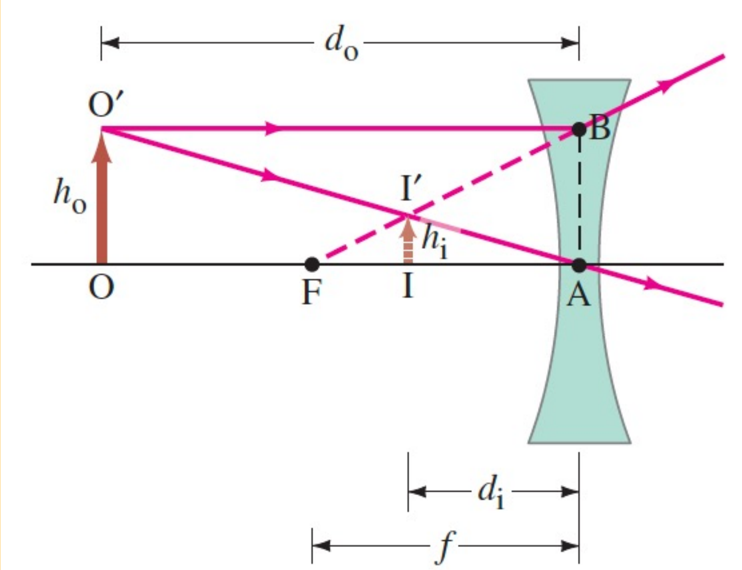
$$m = -\frac{d_i}{d_o} = -\frac{-30\text{ cm}}{10\text{ cm}} = 3.0$$

This lens is being used as a magnifying glass.



Exercise: Diverging lens

Where must a small insect be placed if a 25-cm-focal-length diverging lens is to form a virtual image 20 cm from the lens, on the same side as the object?



The lens is diverging, so f is negative.

The image distance must be negative too because the image is in front of the lens

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = -\frac{1}{25\text{ cm}} + \frac{1}{20\text{ cm}} = \frac{-4 + 5}{100\text{ cm}} = \frac{1}{100\text{ cm}}$$

So the object must be 100 cm in front of the lens.