Ch.28-29: Magnetism

N



Magnets have two ends – **poles** – called north and south.

Like poles repel; unlike poles attract

BUT magnetic poles ARE DIFFERENT from charges.

Charges can be isolated, but magnetic poles CANNOT. You cut a magnet and get two smaller magnets. Magnetic monopoles DO NOT exist.

compass needle = bar magnet, points North.

Iron and few other materials show strong magnetic effects. They are called **ferromagnetic** (Fe, Co, Ni)

We can think of a **magnetic field** surrounding a magnet, just like an electric field surrounds an electric charge

Magnetic Fields

Magnetic fields can be visualized using magnetic field lines, which are always closed loops. Force = interaction between one magnet and magnetic field of the other.



Direction =direction that the north pole of compass points

The Earth' s magnetic field is similar to that of a bar magnet.



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Note that the Earth's "North Pole" is really a south magnetic pole, as the north ends of the compass is attracted to it.

Earth's poles move in time and even reverse direction

https://www.youtube.com/watch?v=t2NqVJtNp6Y

//www.teachersdomain.org/resource/ess05.sci.ess.eiu.solarwind/

Earth Magnetic Field



Cause of Earth's magnetic field https://www.youtube.com/watch?v=t2NqVJtNp6Y

Auroras: https://www.youtube.com/watch?v=nXxwZVbDt1c

SOLAR WIND: http://www.teachersdomain.org/resource/ess05.sci.ess.eiu.solarwind/

Electric Current-Magnetic Fields

Experiment shows that, just like magnets, electric current produces a magnetic field

The direction of the field is given by a right-hand rule.





Compass around wire.

Therefore, electric current (by creating a magnetic field around it) exerts force on magnet.



Electric Current-Magnetic Fields

If electric current exerts force on magnet. From 3rd law: magnet must exert force on current F is perpendicular to I,B



Force on Electric Current

The force on the wire depends on the current, the length of the wire, the magnetic field, and its orientation.

 $F_B = iLB\sin\phi$

This equation defines the magnetic field B.

Maximum and minimum force depend on angle

Unit of B: the tesla, T. (SI)

 $1 T = 1 N/A \cdot m$.

the gauss (G). In CGS

 $1 \text{ G} = 10^{-4} \text{ T}$

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

Ex. A wire carrying a 30-A current has length 12 cm between the pole faces of a magnet at an angle of 60°. The magnetic field is approximately uniform at 0.90 T. We ignore the field beyond the pole pieces. What is the magnitude of the force on the wire?

2.8N

Force on Electric Current

The force on the wire depends on the current, the length of the wire, the magnetic field, and its orientation. Unit of B: the tesla, T. (SI)

 $F_B = iLB\sin\phi$

 $1 T = 1 N/A \cdot m$.

Convention for magnetic field pointing out of page and into the page.



Force on Electric Current

Ex. 20-2 A rectangular loop of wire hangs vertically as in the figure. A magnetic field is directed horizontally, perpendicular to the wire, and points out of the page at all points. The magnetic field is almost uniform. The top portion of the wire loop is free of the field. The loop hangs from a balance which measures a downward force (besides the gravitational force) of 3.48x10⁻² N when the wire carries I=0.245 A. What is the magnitude of the magnetic field B?

1.42 T



Magnetic force on a wire carrying current

A straight, horizontal length of copper wire has a current i = 28 A through it. What are the magnitude and direction of the minimum magnetic field \vec{B} needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

KEY IDEAS

(1) Because the wire carries a current, a magnetic force \vec{F}_B can act on the wire if we place it in a magnetic field \vec{B} . To balance the downward gravitational force \vec{F}_g on the wire, we want \vec{F}_B to be directed upward (Fig. 28-17). (2) The direction of \vec{F}_B is related to the directions of \vec{B} and the wire's length vector \vec{L} by Eq. 28-26 ($\vec{F}_B = i\vec{L} \times \vec{B}$).

Calculations: Because \vec{L} is directed horizontally (and the current is taken to be positive), Eq. 28-26 and the right-hand rule for cross products tell us that \vec{B} must be horizontal and rightward (in Fig. 28-17) to give the required upward \vec{F}_B .

The magnitude of \vec{F}_B is $F_B = iLB \sin \phi$ (Eq. 28-27). Because we want \vec{F}_B to balance \vec{F}_g , we want

$$iLB\sin\phi = mg, \qquad (28-29)$$

where mg is the magnitude of \vec{F}_{g} and m is the mass of the wire.



Fig. 28-17 A wire (shown in cross section) carrying current out of the page.

We also want the minimal field magnitude *B* for \vec{F}_B to balance \vec{F}_g . Thus, we need to maximize sin ϕ in Eq. 28-29. To do so, we set $\phi = 90^\circ$, thereby arranging for \vec{B} to be perpendicular to the wire. We then have sin $\phi = 1$, so Eq. 28-29 yields

$$B = \frac{mg}{iL\sin\phi} = \frac{(m/L)g}{i}.$$
 (28-30)

We write the result this way because we know m/L, the linear density of the wire. Substituting known data then gives us

$$B = \frac{(46.6 \times 10^{-3} \text{ kg/m})(9.8 \text{ m/s}^2)}{28 \text{ A}}$$

= 1.6 × 10⁻² T. (Answer)

This is about 160 times the strength of Earth's magnetic field.

Force on Electric Charge



Force on Electric Charge



X

×

×

×

X

B is into the page

× \

×

X

×

X

X

Path of electron

×

×

×

×

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Ex. A proton having a speed of 5.0×10^6 m/s in a magnetic field feels a force of 8.0×10^{-14} N toward the west when it moves vertically upward. When moving horizontally in a northerly direction, it feels zero force. Determine the magnitude and direction of the magnetic field in this region (q=+e=1.6 \times 10^{-19} C)

0.10 T

If a charged particle is moving perpendicular to a uniform magnetic field, its path will be a circle. $(a=v^2/r)$

Ex. An electron travels at $2.0x10^7$ m/s in a plane perpendicular to a uniform 0.010-T magnetic field. What is the radius of the electron motion? m= $9.1x10^{(-31)}kg$

1.1 cm

Uniform circular motion of a charged particle in a magnetic field

Figure 28-12 shows the essentials of a mass spectrometer, which can be used to measure the mass of an ion; an ion of mass *m* (to be measured) and charge *q* is produced in source *S*. The initially stationary ion is accelerated by the electric field due to a potential difference *V*. The ion leaves *S* and enters a separator chamber in which a uniform magnetic field \vec{B} is perpendicular to the path of the ion. A wide detector lines the bottom wall of the chamber, and the \vec{B} causes the ion to move in a semicircle and thus strike the detector. Suppose that B = 80.000 mT, V = 1000.0 V, and ions of charge $q = +1.6022 \times 10^{-19}$ C strike the detector at a point that lies at x = 1.6254 m. What is the mass *m* of the individual ions, in atomic mass units (Eq. 1-7: 1 u = 1.6605×10^{-27} kg)?

KEY IDEAS

(1) Because the (uniform) magnetic field causes the (charged) ion to follow a circular path, we can relate the ion's mass *m* to the path's radius *r* with Eq. 28-16 (r = mv/|q|B). From Fig. 28-12 we see that r = x/2 (the radius is half the diameter). From the problem statement, we know the magnitude *B* of the magnetic field. However, we lack the ion's speed *v* in the magnetic field after the ion has been accelerated due to the potential difference *V*. (2) To relate *v* and *V*, we use the fact that mechanical energy ($E_{mec} = K + U$) is conserved during the acceleration.

Finding speed: When the ion emerges from the source, its kinetic energy is approximately zero. At the end of the acceleration, its kinetic energy is $\frac{1}{2}mv^2$. Also, during the acceleration, the positive ion moves through a change in potential of -V. Thus, because the ion has positive charge q, its potential energy changes by -qV. If we now write the conservation of mechanical energy as

$$\Delta K + \Delta U = 0$$

we get

$$\frac{1}{2}mv^2 - qV = 0$$



Fig. 28-12 Essentials of a mass spectrometer. A positive ion, after being accelerated from its source *S* by a potential difference *V*, enters a chamber of uniform magnetic field \vec{B} . There it travels through a semicircle of radius *r* and strikes a detector at a distance *x* from where it entered the chamber.

$$v = \sqrt{\frac{2qV}{m}}.$$
 (28-22)

Finding mass: Substituting this value for v into Eq. 28-16 gives us

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$
$$x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}.$$

Thus,

or

Solving this for m and substituting the given data yield

$$m = \frac{B^2 q x^2}{8V}$$

= $\frac{(0.080000 \text{ T})^2 (1.6022 \times 10^{-19} \text{ C}) (1.6254 \text{ m})^2}{8(1000.0 \text{ V})}$
= $3.3863 \times 10^{-25} \text{ kg} = 203.93 \text{ u}.$ (Answer)

Summary

TABLE 20–1 Summary of Right-hand Rules (= RHR)			
Physical Situation	Example	How to Orient Right Hand	Result
 Magnetic field produced by current (RHR-1) 	B Fig. 20–8c	Wrap fingers around wire with thumb pointing in direction of current <i>I</i>	Fingers point in direction of B
2. Force on electric current <i>I</i> due to magnetic field (RHR-2)	F F B F ig. 20–11c	Fingers point straight along current I , then bent along magnetic field $\mathbf{\vec{B}}$	Thumb points in direction of force
3. Force on electric charge +q due to magnetic field (RHR-3)	F Fig. 20–14	Fingers point along particle's velocity \vec{v} , then along \vec{B}	Thumb points in direction of force

Magnetic Field due to long Wire



The constant μ_0 is called the permeability of free space, and has the value: $\mu_0 = 4\pi \times 10^{-7} \,\mathrm{T} \cdot \mathrm{m/A}$

Ex. An electric wire in the wall of a building carries a dc current of 25 A vertically upward. What is the magnetic field due to this current at a point P 10 cm due north of the wire

B=5.0x10^(-5) T

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10 cm

 $\frac{\mu_0}{2\pi} \frac{I}{r}$

Magnetic Field due to long Wire

The field is inversely proportional to the distance from the wire:

 $B = \frac{\mu_0}{2\pi} \frac{I}{r}$

The constant μ_0 is called the permeability of free space, and has the value: $\mu_0 = 4\pi \times 10^{-7} \,\mathrm{T} \cdot \mathrm{m/A}$

Ex. Two parallel straight wires 10.0 cm apart carry currents in opposite directions. Current I1=5.0 A is out of the page and I2=7.0A is into the page. Determine the magnitude and direction of the magnetic field halfway between the two wires.

B=4.8x10^(-5) T up



Force between two parallel wires





Force between parallel two wires

Ex. The two wires of a 2.0-m –long appliance cord are 3.0 mm apart and carry a current of 8.0 A dc. Calculate the force one wire exerts on the other

F=8.5x10^(-3) N

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Ex. A horizontal wire carries a current I_1 =80A dc. A second parallel wire 20 cm below it must carry how much current I_2 so that it does not fall due to gravity? The lower wire has mass of 0.12 g per meter of length



Magnetic field off to the side of two long straight currents

Figure 29-8*a* shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point *P*? Assume the following values: $i_1 = 15$ A, $i_2 = 32$ A, and d = 5.3 cm.

KEY IDEAS

(1) The net magnetic field \vec{B} at point *P* is the vector sum of the magnetic fields due to the currents in the two wires. (2) We can find the magnetic field due to any current by applying the Biot-Savart law to the current. For points near the current in a long straight wire, that law leads to Eq. 29-4.

Finding the vectors: In Fig. 29-8*a*, point *P* is distance *R* from both currents i_1 and i_2 . Thus, Eq. 29-4 tells us that at point *P* those currents produce magnetic fields \vec{B}_1 and \vec{B}_2 with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R}$$
 and $B_2 = \frac{\mu_0 i_2}{2\pi R}$.

In the right triangle of Fig. 29-8*a*, note that the base angles (between sides *R* and *d*) are both 45°. This allows us to write $\cos 45^\circ = R/d$ and replace *R* with *d* $\cos 45^\circ$. Then the field magnitudes B_1 and B_2 become

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45^\circ}$$
 and $B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45^\circ}$



Fig. 29-8 (a) Two wires carry currents i_1 and i_2 in opposite directions (out of and into the page). Note the right angle at P.(b) The separate fields \vec{B}_1 and \vec{B}_2 are combined vectorially to yield the net field \vec{B} .

We want to combine \vec{B}_1 and \vec{B}_2 to find their vector sum, which is the net field \vec{B} at *P*. To find the directions of \vec{B}_1 and \vec{B}_2 , we apply the right-hand rule of Fig. 29-4 to each current in Fig. 29-8*a*. For wire 1, with current out of the page, we mentally grasp the wire with the right hand, with the thumb pointing out of the page. Then the curled fingers indicate that the field lines run counterclockwise. In particular, in the region of point *P*, they are directed upward to the left. Recall that the magnetic field at a point near a long, straight current-carrying wire must be directed perpendicular to a radial line between the point and the current. Thus, \vec{B}_1 must be directed upward to the left as drawn in Fig. 29-8*b*. (Note carefully the perpendicular symbol between vector \vec{B}_1 and the line connecting point *P* and wire 1.)

Repeating this analysis for the current in wire 2, we find that \vec{B}_2 is directed upward to the right as drawn in Fig. 29-8b. (Note the perpendicular symbol between vector \vec{B}_2 and the line connecting point *P* and wire 2.)

Adding the vectors: We can now vectorially add \vec{B}_1 and \vec{B}_2 to find the net magnetic field \vec{B} at point *P*, either by using a vector-capable calculator or by resolving the vectors into components and then combining the components of \vec{B} . However, in Fig. 29-8*b*, there is a third method: Because \vec{B}_1 and \vec{B}_2 are perpendicular to each other, they form the legs of a right triangle, with \vec{B} as the hypotenuse. The Pythagorean theorem then gives us

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(\cos 45^\circ)} \sqrt{t_1^2 + t_2^2}$$
$$= \frac{(4\pi \times 10^{-7} \,\mathrm{T \cdot m/A})\sqrt{(15 \,\mathrm{A})^2 + (32 \,\mathrm{A})^2}}{(2\pi)(5.3 \times 10^{-2} \,\mathrm{m})(\cos 45^\circ)}$$
$$= 1.89 \times 10^{-4} \,\mathrm{T} \approx 190 \,\mu\mathrm{T}. \qquad (\mathrm{Answer})$$

The angle ϕ between the directions of \vec{B} and \vec{B}_2 in Fig. 29-8b follows from

$$b = \tan^{-1} \frac{B_1}{B_2},$$

which, with B_1 and B_2 as given above, yields

$$\phi = \tan^{-1} \frac{i_1}{i_2} = \tan^{-1} \frac{15 \text{ A}}{32 \text{ A}} = 25^\circ.$$

The angle between the direction of \vec{B} and the x axis shown in Fig. 29-8b is then

 $\phi + 45^\circ = 25^\circ + 45^\circ = 70^\circ$. (Answer)

Law of Biot and Savart



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,d\vec{s} \times \hat{r}}{r^2}$$

$$\mu_0 = 4\pi \times 10^{-7} \,\mathrm{T} \cdot \mathrm{m/A}$$

$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds\,\sin\,\theta}{r^2}$$

Magnetic Field Due to a Current in a Long Straight Wire

$$B = \frac{\mu_0 i}{2\pi R}$$

(29-3)

Magnetic Field Due to a Current in a Long Straight Wire

 $B = \frac{\mu_0 i}{2\pi R}$

θ

This element of current creates a magnetic field at *P*, into the page.

 $d\vec{B}$

$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds\,\sin\,\theta}{r^2}$$

The magnetic field produced at P by the current-length elements in the upper half of the infinitely long wire can be obtained by integrating dB from 0 to ∞

The magnetic field produced by the lower half of the wire is exactly the same as that produced by the upper half, so

$$B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi} \int_0^\infty \frac{\sin\theta \, ds}{r^2}$$

 $\sin \theta = \sin(\pi - \theta)$

 $=\frac{R}{\sqrt{s^2+}}$

$$r = \sqrt{s^2 + R^2}$$

Magnetic Field Due to a Current in a Long Straight Wire

This element of current creates a magnetic field at *P*, into the page.

 $d\vec{B}$

 $d\vec{s}$

S

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R

r

$$B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi} \int_0^\infty \frac{\sin\theta \, ds}{r^2}$$

$$r = \sqrt{s^2 + R^2}$$
 $\sin \theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}}$

$$B = \frac{\mu_0 i}{2\pi} \int_0^\infty \frac{R \, ds}{(s^2 + R^2)^{3/2}}$$
$$= \frac{\mu_0 i}{2\pi R} \left[\frac{s}{(s^2 + R^2)^{1/2}} \right]_0^\infty = \frac{\mu_0 i}{2\pi R}$$

$$B = \frac{\mu_0 i}{2\pi R}$$

Magnetic Field Due to a Current in a Circular Arc of Wire



$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds\,\sin\,\theta}{r^2}$$

No matter where the element \mathbf{ds} is located on the wire, the angle ϕ between the vectors \mathbf{ds} and \mathbf{r} is 90°

$$dB = \frac{\mu_0}{4\pi} \frac{i \, ds \, \sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi} \frac{i \, ds}{R^2}$$

 $C \bullet r$

The total field at C is simply the sum (via integration) of all the differential fields \mathbf{dB}

To solve the integral below, we use the definition of radians, that is $ds = R \ d\phi$



$$B = \int dB = \int_0^{\phi} \frac{\mu_0}{4\pi} \frac{iR \, d\phi}{R^2} = \frac{\mu_0 i}{4\pi R} \int_0^{\phi} d\phi$$
$$B = \frac{\mu_0 i\phi}{4\pi R}$$

Magnetic Field Due to a Current in a Circular Arc of Wire

$$B = \frac{\mu_0 i\phi}{4\pi R}$$

Note that this equation gives us the magnetic field only at the center of curvature of a circular arc of current. When you insert data into the equation, you must be careful to express ϕ in <u>radians</u> rather than degrees. For example, to find the magnitude of the magnetic field at the center of a full circle of current, you would substitute ϕ with 2π

$$B = \frac{\mu_0 i(2\pi)}{4\pi R} = \frac{\mu_0 i}{2R}$$

Magnetic field at the center of a circular arc of current

The wire in Fig. 29-7*a* carries a current *i* and consists of a circular arc of radius *R* and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center *C* of the arc. What magnetic field \vec{B} (magnitude and direction) does the current produce at *C*?



and carries current *i*. (*b*) For a current-length element in section 1, the angle between $d\vec{s}$ ar \hat{r} is zero. (*c*) Determining the direction of magnetic field \vec{B}_3 at *C* due to the current in the circular arc; the field is into the page there.

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$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad \text{(Ampere's law).} \quad (29-14)$

The loop on the integral sign means that the scalar (dot) product $\vec{B} \cdot d\vec{s}$ is to be integrated around a *closed* loop, called an *Amperian loop*. The current i_{enc} is the *net* current encircled by that closed loop.

We can now interpret the scalar product $\vec{B} \cdot d\vec{s}$ as being the product of a length ds of the Amperian loop and the field component $B \cos \theta$ tangent to the loop. Then we can interpret the integration as being the summation of all such products around the entire loop.



$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad \text{(Ampere's law).} \quad (29-14)$

The loop on the integral sign means that the scalar (dot) product $\vec{B} \cdot d\vec{s}$ is to be integrated around a *closed* loop, called an *Amperian loop*. The current i_{enc} is the *net* current encircled by that closed loop.

Fig. 29-13 Using Ampere's law to find the magnetic field that a current *i* produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire. All of the current is $\oint \vec{B} \cdot d\vec{s} = \oint B \cos \theta \, ds = B \oint ds = B(2\pi r)$ encircled and thus all is used in Ampere's law. Amperian $B(2\pi r) = \mu_0 i$ Wire loop surface **O**i $\mu_0 l$ (outside straight wire) 28

Problems: Ch.28

•8 An electric field of 1.50 kV/m and a perpendicular magnetic field of 0.400 T act on a moving electron to produce no net force. What is the electron's speed?

3.75×10^3 m/s

•9 ILW In Fig. 28-31, an electron accelerated from rest through potential difference $V_1 = 1.00$ kV enters the gap between two par-

allel plates having separation d = 20.0 mm and potential difference $V_2 = 100$ V. The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates. In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap?



Fig. 28-31 Problem 9.

Note:

From the figure, the x-direction is to the right, the y-direction is up and the z-direction is out of the page.

$$\vec{B} = -(2.67 \times 10^{-4} \text{ T})\hat{k}$$

Problems: Ch.29

•8 In Fig. 29-39, two semicircular arcs have radii $R_2 = 7.80$ cm and $R_1 = 3.15$ cm, carry current i = 0.281 A, and share the same center of curvature C. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at C?



•10 In Fig. 29-40, a wire forms a semicircle of radius R = 9.26 cm and two (radial) straight segments each of length L = 13.1 cm. The wire carries current i = 34.8 mA. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the semicircle's center of curvature C?



Fig. 29-40 Problem 10.

Wine 1 @



 $1.67 \times 10^{-6} \text{ T}$

into the page

into the page

••29 **SSM** In Fig. 29-56, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length a = 20 cm. The currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3, and each wire carries 20 A. In unit-vector notation, what is the net magnetic field at the square's center?



 $\vec{B}_{\rm net} = (8.0 \times 10^{-5} \,{\rm T})\hat{j}$

••21 Figure 29-48 shows two very long straight wires (in cross section) that each carry a current of 4.00 A directly out of the page. Distance $d_1 = 6.00$ m and distance $d_2 = 4.00$ m. What is the magnitude of the net magnetic field at point *P*, which lies on a perpendicular bisector to the wires?



RECITATION: Ch.28, Problem 9 Ch.29, Problems 21, 29