Chapter 26: Current and Resistance

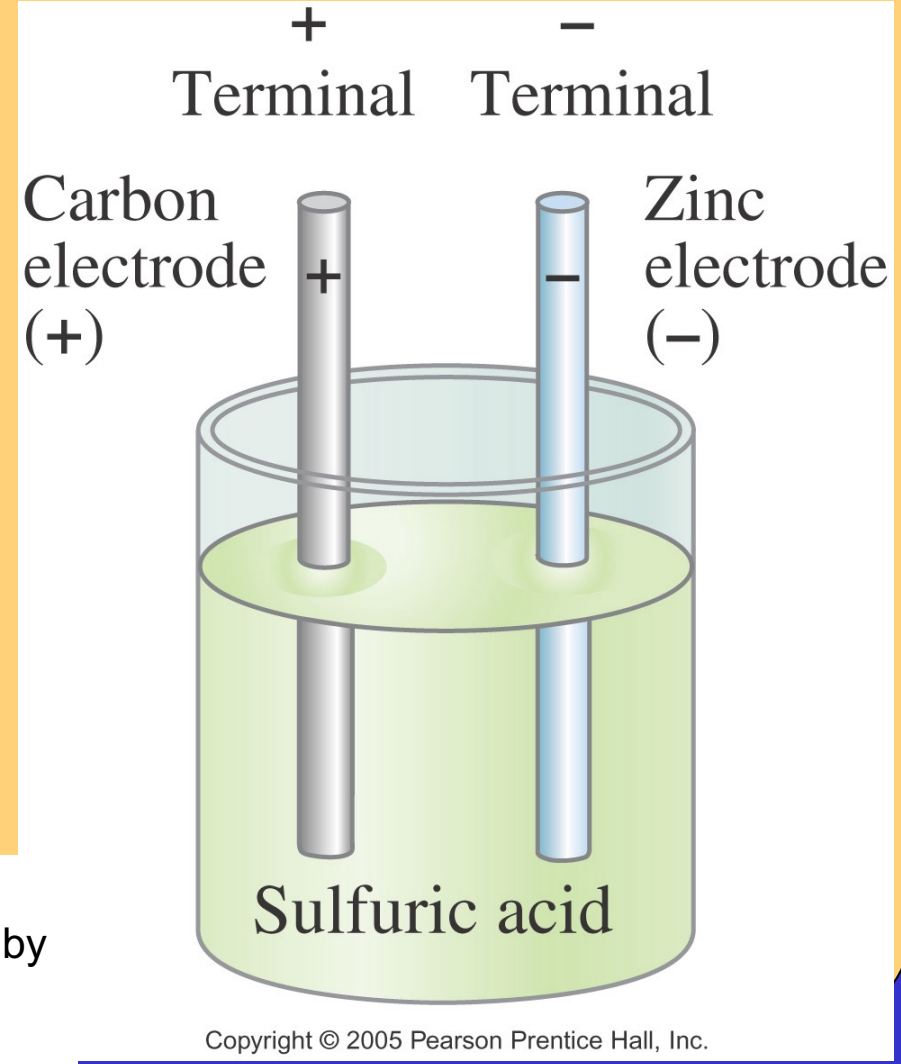
Here we study charges in motion

Flow of charge = **electric current**

To set charges in motion we need electric fields/ potential difference/ a battery
Alessandro Volta invented the electric battery – produced the 1st steady flow of electric charge.

A **battery** transforms chemical energy into electrical energy.

Electrodes – plates of dissimilar metals ---- terminal
Electrolyte – solution

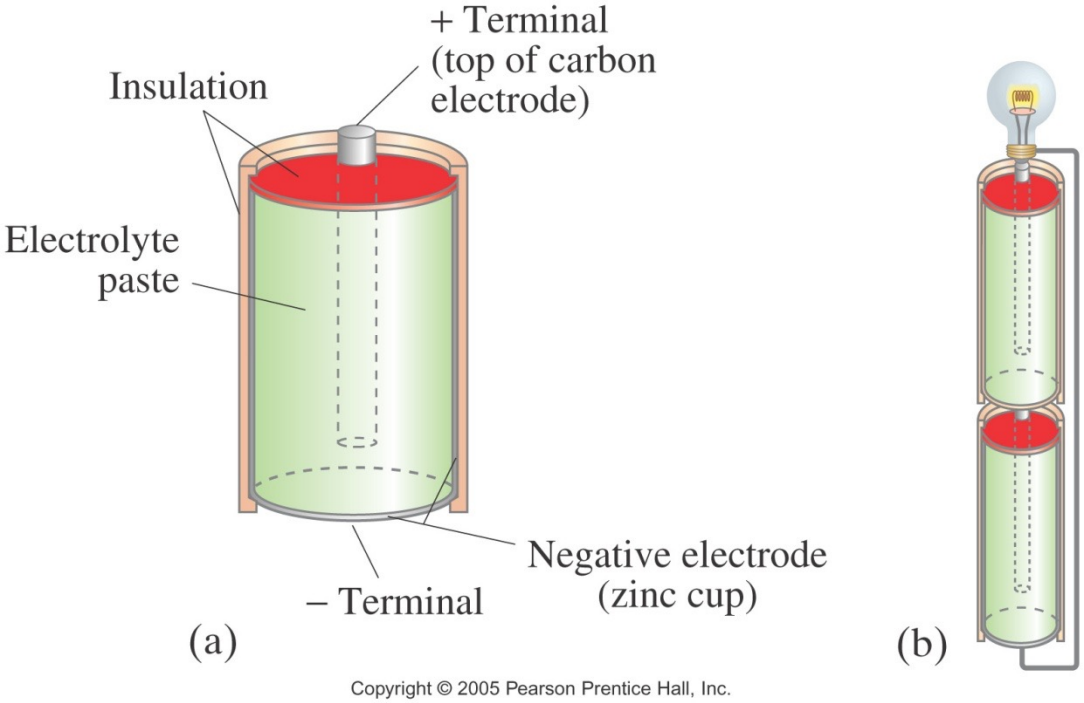


Chemical reactions within the cell create a potential difference between the terminals by slowly dissolving them. This potential difference can be maintained even if a current is kept flowing, until one or the other terminal is completely dissolved.

Unit cell/battery

Electric Battery

Several cells connected together make a battery, although now we refer to a single cell as a battery as well.



Two or more cells connected in **series** – voltages add up

Lightbulb is a thin coiled wire (filament) inside glass. Filament gets hot and glows when charges pass.

Electric Current

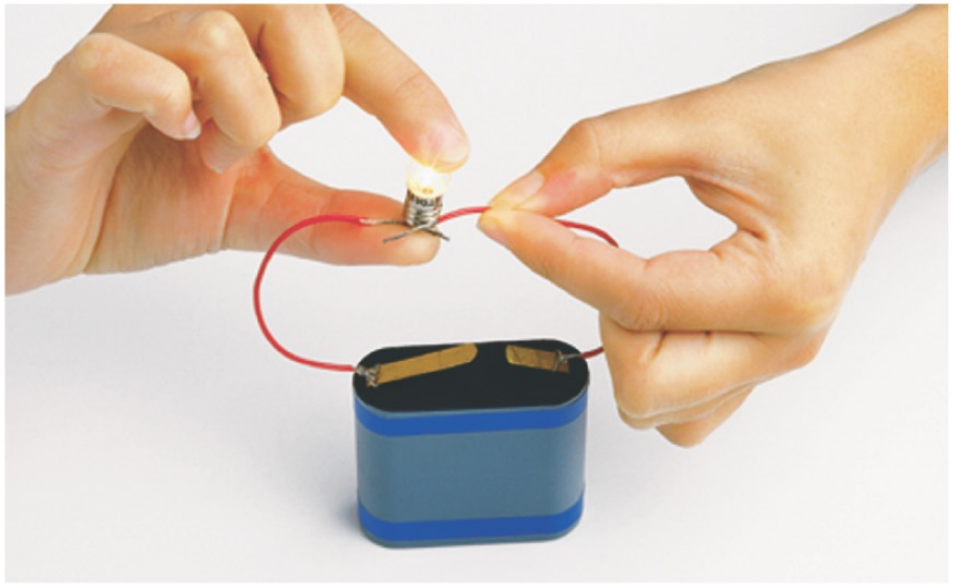
Battery produces potential difference and then charges can move.
Electric current is the **rate of flow** of charge through a conductor:

$$i = \frac{dq}{dt}$$

Unit of electric current: the ampere, A.

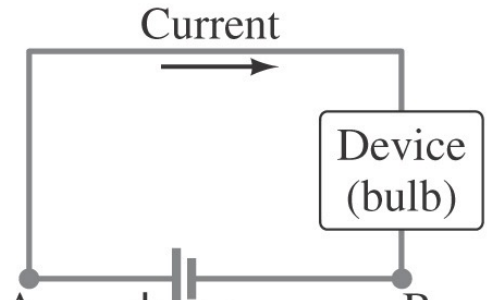
1 A = 1 C/s.

A **complete circuit** is one where current can flow all the way around.
If there is a break in the circuit -- no current -- **open circuit**



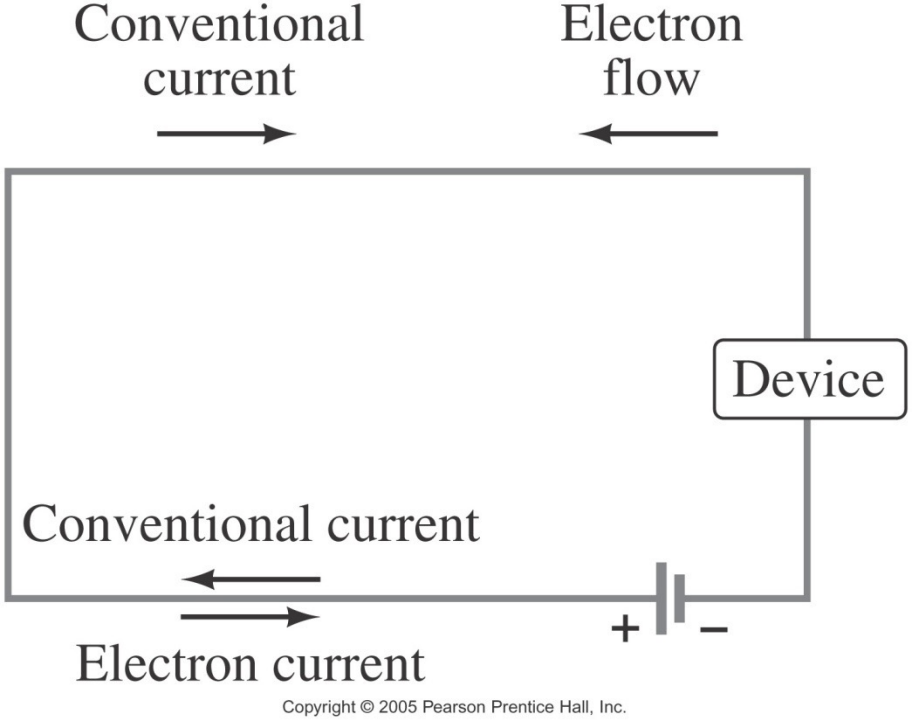
(a)

Steady current: current is the same at A or B



(b)

Electric Current



Conducting wire has free electrons. *Potential difference* between the terminals of a battery sets up an *electric field* in the wire (parallel to it). Free electrons are attracted into the positive terminal. There is a continuous *flow* of electrons.

By *convention*, current is defined as flowing from **+ to -.**

Electrons actually flow in the opposite direction, but not all currents consist of electrons.

Ex. A steady current of 2.5 A exists in a wire for 4.0 min. (a) How much total charge passed by a given point in the circuit during those 4.0 min? (b) How many electrons would this be?:

$$(a)\Delta Q = 600C \quad (b)3.8\times10^{21}\,electrons$$

Ohm's Law

Experimentally, it is found that the current in a wire is proportional to the potential difference between its ends: $I \propto V$

Compare flow of electric charge with flow of water in a river/pipe acted on by gravity
Height of a cliff vs. electric potential

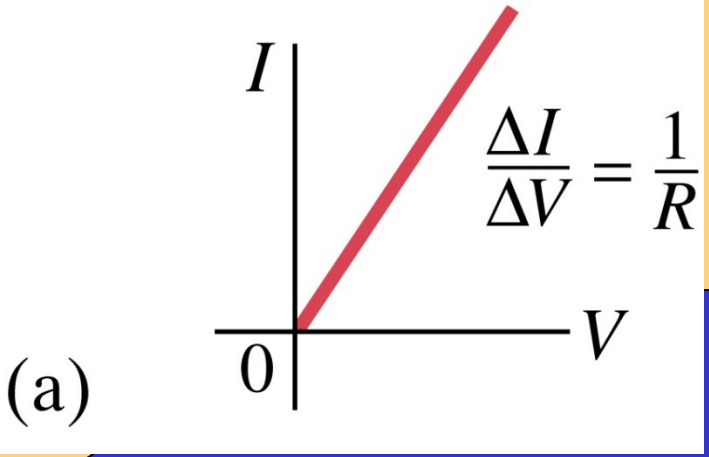
How large the current is depends on voltage and also **resistance**:
Electron flow is impeded because of interaction with atoms of the wire, like rocks, walls in river/pipe

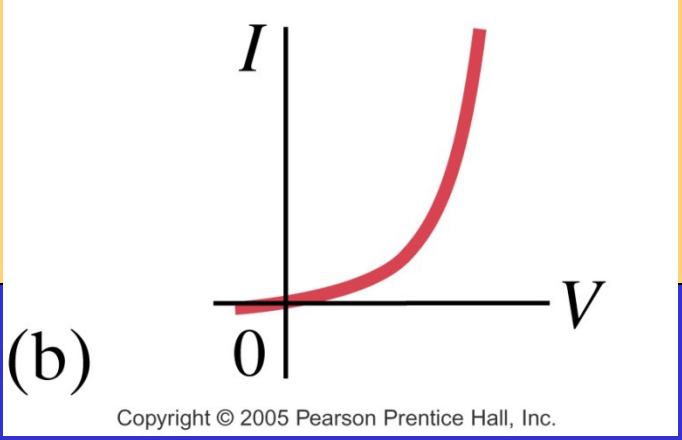
Resistance is defined as $R = \frac{V}{I}$

Ohm's law

$$V = IR$$

In many conductors, the resistance is independent of the voltage: Ohm's law.
Materials that do not follow Ohm's law are called nonohmic.





Unit of resistance: the ohm, Ω.

1 Ω = 1 V/A.

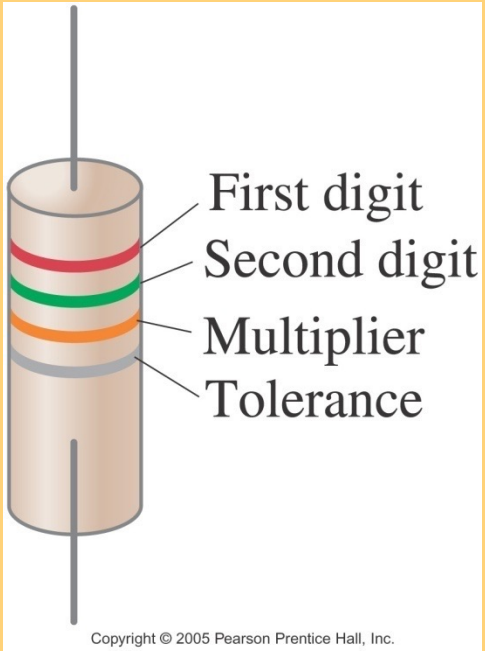
Resistors

Ex. A small flashlight bulb draws 300 mA from its 1.5-V battery. (a) What is the resistance of the bulb? (b) If the battery becomes weak and the voltage drops to 1.2 V, how would the current change?

$(a) R = 5.0\Omega \;\; (b) I = 240mA$

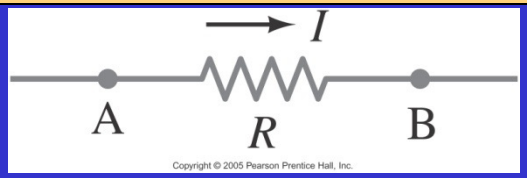
All electric devices offer resistance.

In many circuits, **resistors** are used to control the amount of current; they are color-coded to indicate their value and precision.



Ex. Current I enters a resistor R as shown in the Figure. (a) Is the potential higher at point A or at point B? (b) Is the current greater at point A or at point B?

(a) Positive charge flows from + to -, from high to low potential (like mass falling)
(b) Conservation of charge requires that whatever flows into R from A emerges at B, so the current is the same



Clarifications

Some clarifications:

• Batteries maintain a (nearly) constant potential difference; they are a source of voltage

• Electric current passes through a wire or device and its magnitude depends on the resistance

• Resistance is a property of a material or device. Voltage is external to wire/device

• Current is not a vector but it does have a direction.

• Current and charge do not get used up. Whatever charge goes in one end of a circuit comes out the other end.

Resistivity

The resistance of a wire is directly proportional to its length and inversely proportional to its cross-sectional area:

$$R = \rho \frac{L}{A}$$

The constant ρ, the **resistivity**, is characteristic of the material. Unit: $\Omega.m$

Ex. Suppose you want to connect your stereo to remote speakers. (a) If each wire must be 20 m long, what diameter copper wire should you use to keep the resistance less than 0.10Ω per wire? (b) If the current to each speaker is 4.0A, what is the potential difference, or voltage drop, across each wire?

$$\rho = 1.68 \times 10^{-8} \Omega.m$$

$$(a) A = 3.4 \times 10^{-6} m^2 \rightarrow r = 1.04 mm \rightarrow d = 2.1 mm$$

$$(b) V = 0.40 V$$

Silver has low resistivity (good conductor), but is expensive
Copper is close and less expensive, that is why most wires are made of copper

Current Density

Current Density: $\vec{J}$ The flow of charge through a cross section of the conductor at a particular point.

It has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.

$$i = \int \vec{J} \cdot d\vec{A} \qquad (26\text{-}4)$$

If the current is uniform across the surface and parallel to $d\vec{A}$, then $\vec{J}$ is also uniform and parallel to $d\vec{A}$. Then Eq. 26-4 becomes

$$i = \int J\,dA = J \int dA = JA,$$

so

$$J = \frac{i}{A}, \qquad (26\text{-}5)$$

where A is the total area of the surface. From Eq. 26-4 or 26-5 we see that the SI unit for current density is the ampere per square meter (A/m^2).

Current density, uniform and nonuniform

(a) The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5$ A/m^2. What is the current through the outer portion of the wire between radial distances $R/2$ and R (Fig. 26-6a)?

KEY IDEA

Because the current density is uniform across the cross section, the current density J, the current i, and the cross-sectional area A are related by Eq. 26-5 ($J = i/A$).

Calculations: We want only the current through a reduced cross-sectional area A' of the wire (rather than the entire area), where

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \pi\left(\frac{3R^2}{4}\right)$$

$$= \frac{3\pi}{4}(0.0020\text{ m})^2 = 9.424 \times 10^{-6}\text{ m}^2.$$

So, we rewrite Eq. 26-5 as

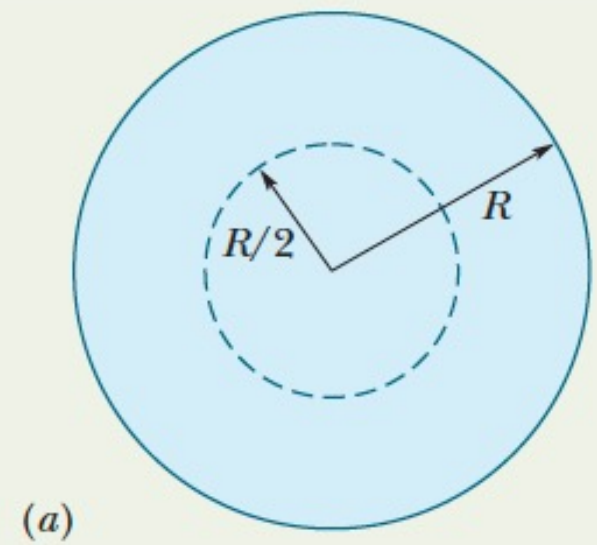
$$i = JA'$$

and then substitute the data to find

$$i = (2.0 \times 10^5\text{ A/m}^2)(9.424 \times 10^{-6}\text{ m}^2)$$

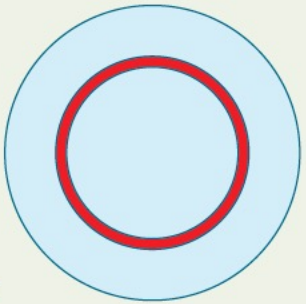
$$= 1.9\text{ A}. \qquad \text{(Answer)}$$

We want the current in the area between these two radii.



(b) Suppose, instead, that the current density through a cross section varies with radial distance r as $J = ar^2$, in which $a = 3.0 \times 10^{11}$ A/m^4 and r is in meters. What now is the current through the same outer portion of the wire?

If the current is nonuniform, we start with a ring that is so thin that we can approximate the current density as being uniform within it.

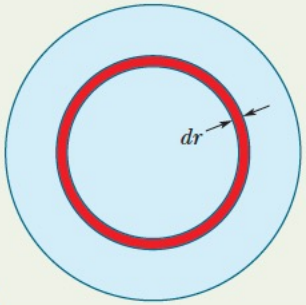


(b)

KEY IDEA

Because the current density is not uniform across a cross section of the wire, we must resort to Eq. 26-4 ($i = \int \vec{J} \cdot d\vec{A}$) and integrate the current density over the portion of the wire from $r = R/2$ to $r = R$.

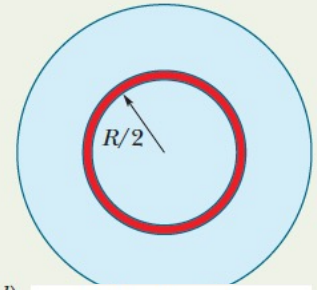
Its area is the product of the circumference and the width.



(c)

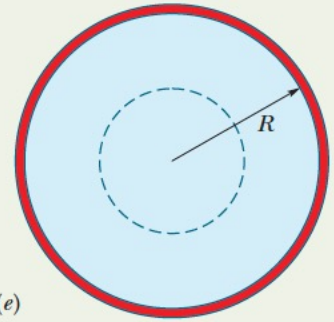
The current within the ring is the product of the current density and the ring's area.

Our job is to sum the current in all rings from this smallest one ...



(d)

... to this largest one.



(e)

$$i = \int \vec{J} \cdot d\vec{A} = \int J\, dA$$

$$= \int_{R/2}^{R} ar^2\, 2\pi r\, dr = 2\pi a \int_{R/2}^{R} r^3\, dr$$

$$= 2\pi a \left[\frac{r^4}{4}\right]_{R/2}^{R} = \frac{\pi a}{2}\left[R^4 - \frac{R^4}{16}\right] = \frac{15}{32}\pi a R^4$$

$$= \frac{15}{32}\pi(3.0 \times 10^{11}\ \text{A/m}^4)(0.0020\ \text{m})^4 = 7.1\ \text{A}.$$

Resistivity

$$i = \frac{V}{R}$$

$$J = \frac{i}{A},$$

$$V = \int_{-}^{+} E\,ds = E\int_{0}^{d} ds = Ed.$$

$$R = \rho\frac{L}{A}$$

$$\rho = \frac{E}{J}$$

Electric Power

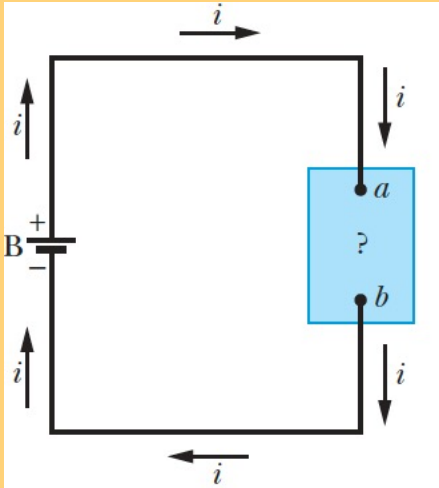
Power, as in kinematics, is the energy transformed by a device per unit time:

$$P = \frac{\text{energy transformed}}{\text{time}} = \frac{QV}{t}$$

dU/dt

$$dU = dq\,V = i\,dt\,V.$$

$$\boxed{P = IV}$$



As the charge moves from high potential Va to low potential Vb, its potential energy decreases. The decrease in electric potential energy from a to b is accompanied by a transfer of energy to some other form, so that the device functions.

The unit of power is the watt, W.

For ohmic devices, we can make the substitutions:

$$P = IV = I(IR) = I^2R$$

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$$

Exercises

Ex Calculate the resistance of a 40-W automobile headlight designed for 12V

$$R = \frac{V^2}{P} = 3.6\Omega$$

Ex. An electric heater draws a steady 15.0 A on a 120-V line. How much power does it require and how much does it cost per month (30 days) if it operates 3.0 h per day and the electric company charges 9.2 cents per kWh?

P=I.V=1800W (3.0h/d)(30d)=90h so (1.80kW)(90 h)($0.092/kWh)=$15

Ex. Lightning is a spectacular example of electric current in a natural phenomenon. A typical event can transfer 10^9 J of energy across a potential difference of perhaps 5x10^7 V during a time interval of about 0.2 s. Use this information to estimate (a) the total amount of charge transferred between cloud and ground, (b) the current in the lightning bolt, and (c) the average power delivered over the 0.2 s

(a) 20 C, (b) 100 A (c) 5x10^9W=5GW

What you pay for on your electric bill is not power, but energy – the power consumption multiplied by the time.

We have been measuring energy in joules, but the electric company measures it in kilowatt-hours, kWh.

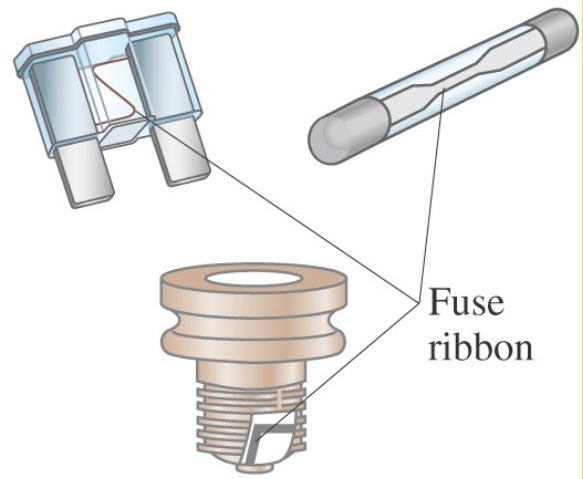
$$\text{One kWh} = (1000\ \text{W})(3600\ \text{s}) = 3.60 \times 10^6\ \text{J}$$

Power in Household Circuits

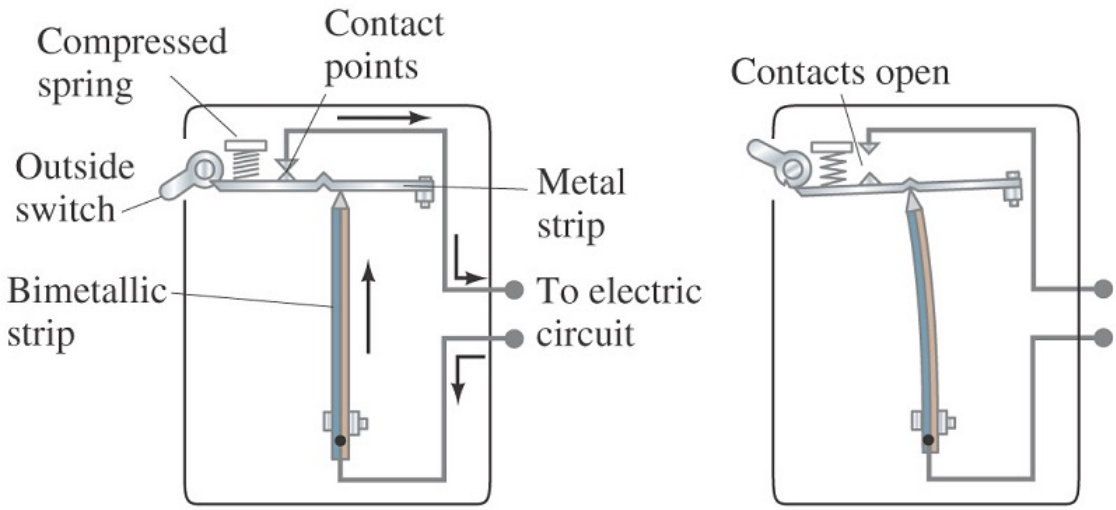
The wires used in homes to carry electricity have very low resistance. However, if the current is high enough, the power will increase and the wires can become hot enough to start a fire.

To avoid this, we use fuses or circuit breakers, which disconnect when the current goes above a predetermined value.

$$P = I^2 R$$



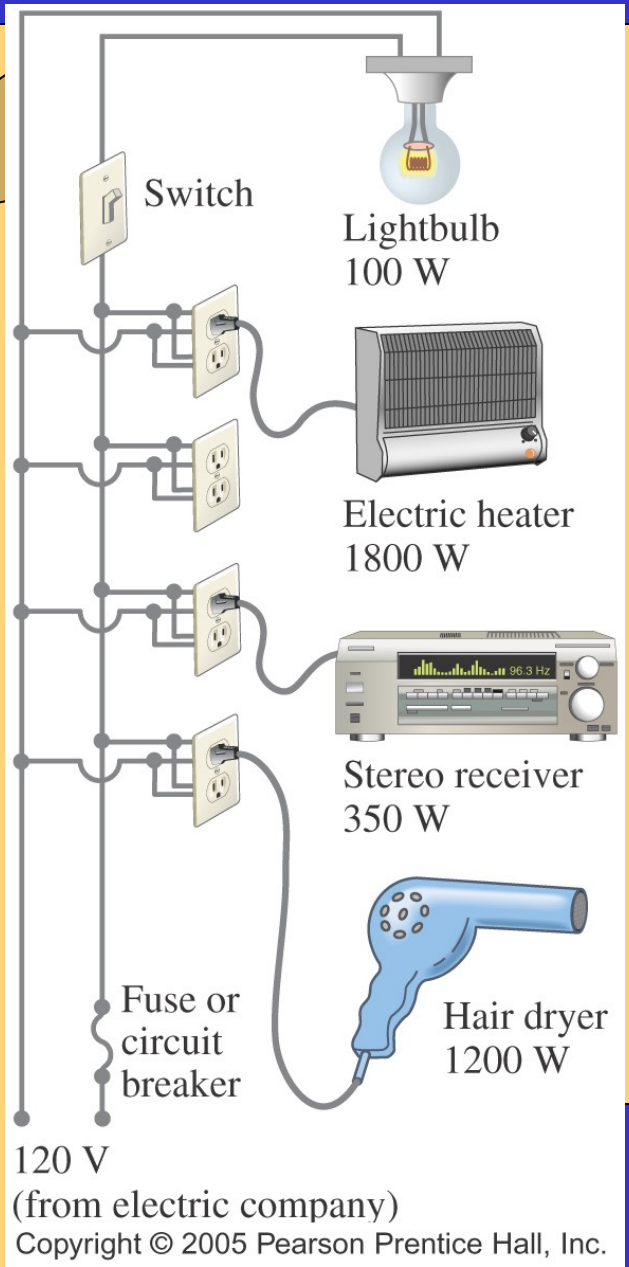
(a) Types of fuses



(b) Circuit breaker (closed)

(c) Circuit breaker (open)

Power in Household Circuits



$$P = IV$$

Ex. Determine the total current drawn by all the devices in the circuit of the figure. If the circuit is designed for a 20-A fuse, will the fuse blow?

$$0.8A + 15.0A + 2.9A + 10.0A = 28.7A$$

Alternating Current

Current from a battery flows steadily in one direction (direct current, DC). Current from a power plant varies sinusoidally (alternating current, AC).

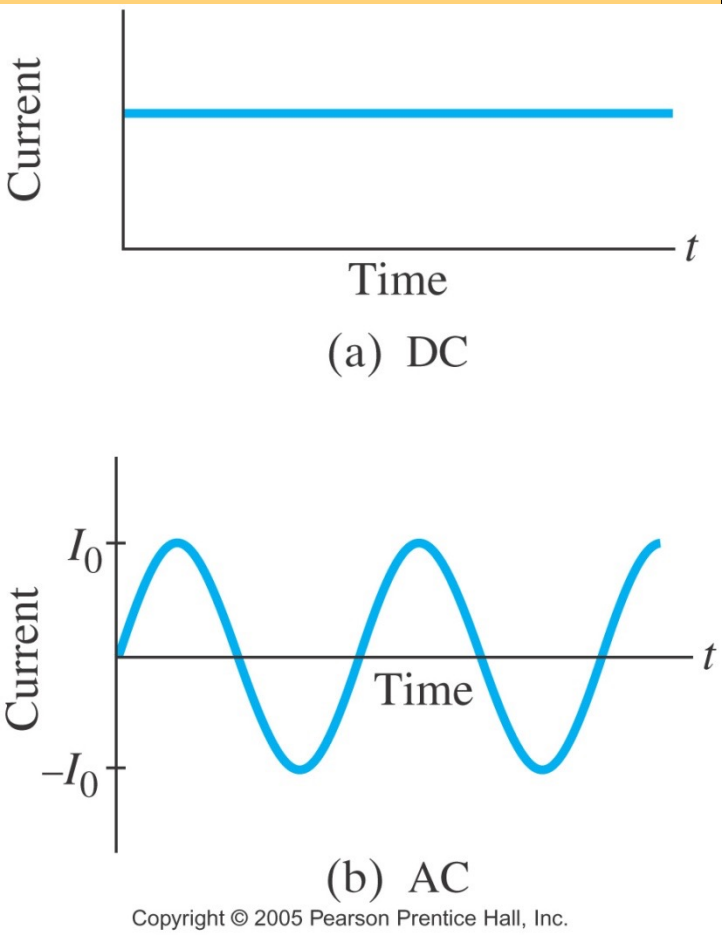
The voltage varies sinusoidally with time:

$$V = V_0 \sin 2\pi f t = V_0 \sin \omega t$$

as does the current:

$$I = \frac{V}{R} = \frac{V_0}{R} \sin \omega t = I_0 \sin \omega t$$

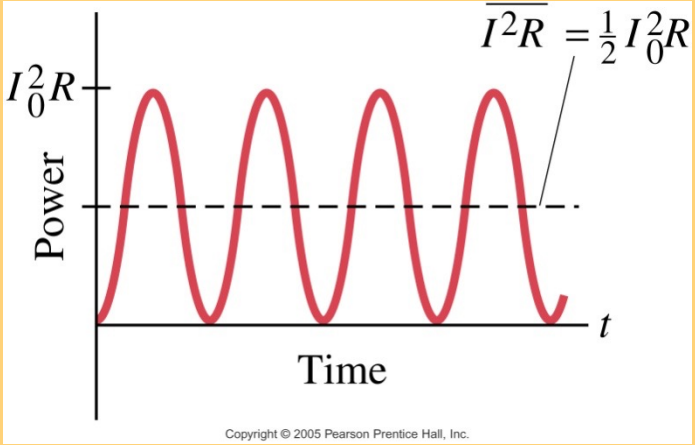
In most areas of the US and Canada f=60 Hz



Alternating Current

Multiplying the current and the voltage gives the power:

$$P = I^2 R = I_0^2 R \sin^2 \omega t$$



$$\bar{I} = 0 \;\; but \;\; \bar{P} = I_0^2 R / 2$$

$$P = V^2 / R = (V_0^2 / R) \sin^2(\omega t)$$

$$\bar{P} = V_0^2 / (2R)$$