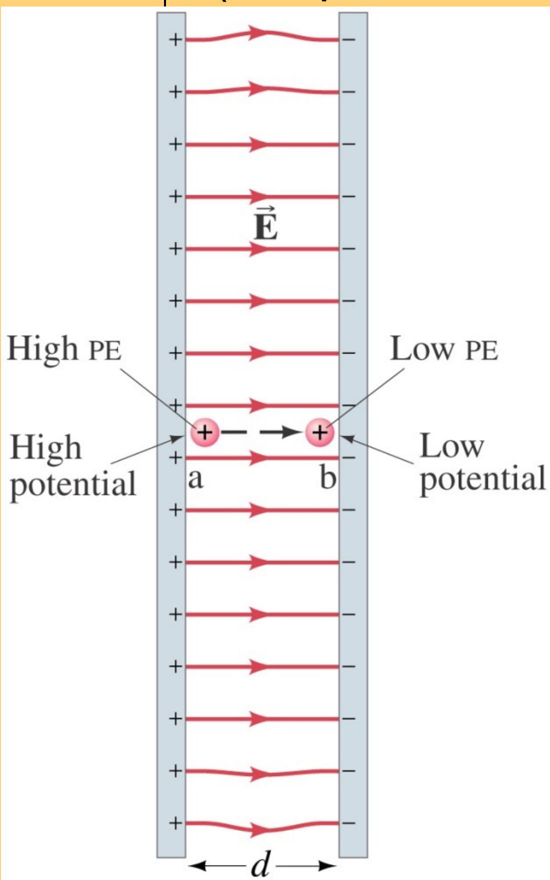
Chapter 24: Electric Potential

The electrostatic force is **conservative** – electric **potential** energy can be defined (just like for gravitational force)

Change in electric potential energy is negative of work done by electric force (independent of path):



$$W = \vec{F}\cdot\vec{d}.$$

θ Angle between the electric force and the displacement

$$W = q\vec{E}\cdot\vec{d}$$

ϕ Angle between the electric field and the displacement

$$\Delta U = U_f - U_i = -W. \quad = -qEd$$

Small positive charge q initially at a.
Electric force does work on it and accelerates it toward b.
The potential energy decreases and the particle’s kinetic energy increases.

Reverse is true for negative charge.

Electric Potential

Similarly to electric field, we define **electric potential** as the potential energy per unit charge:

$$V = \frac{U}{q}$$

V is a SCALAR

Unit of electric potential: the volt (V)   1 V = 1 J/C.

Only difference in potential is meaningful, so where to choose V=0 is arbitrary

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = -\frac{W}{q} \quad -\int_i^f \vec{E} \cdot d\vec{s}.$$

$$dW = \vec{F} \cdot d\vec{s}.$$

$$dW = q_0 \vec{E} \cdot d\vec{s}.$$

$$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}.$$

Unit: **N/C** or **V/m**

E is VECTOR
V is SCALAR

Positive vs Negative Charge

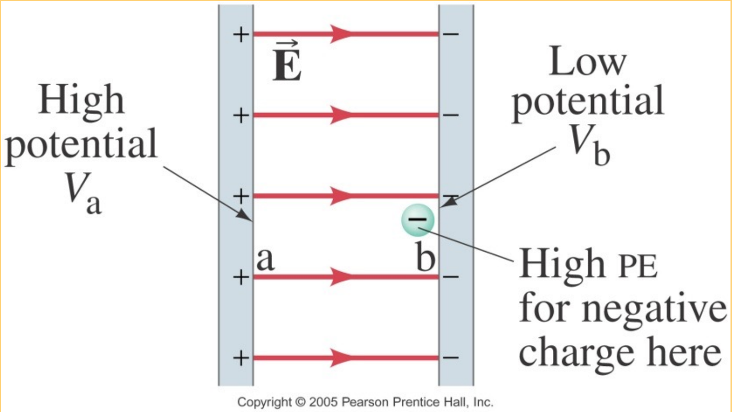
$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = -\frac{W}{q} \quad -\int_i^f \vec{E}\cdot d\vec{s}.$$

ELECTRIC POTENTIAL

V is due to charges on the plates

V is high on + side

V is low on - side



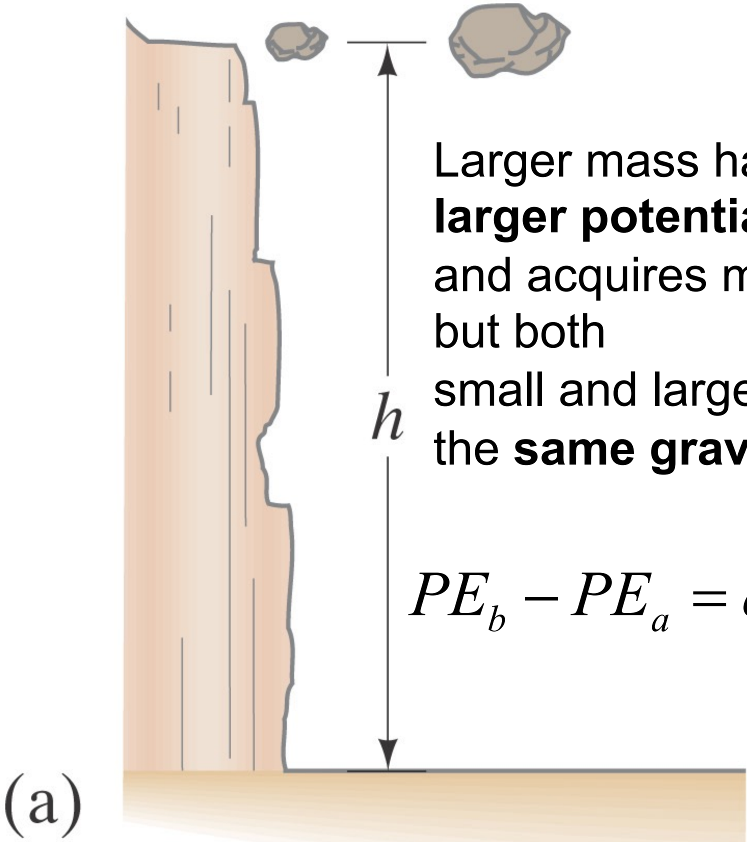
ELECTRIC POTENTIAL ENERGY

+q has high U in a

- q has high U in b

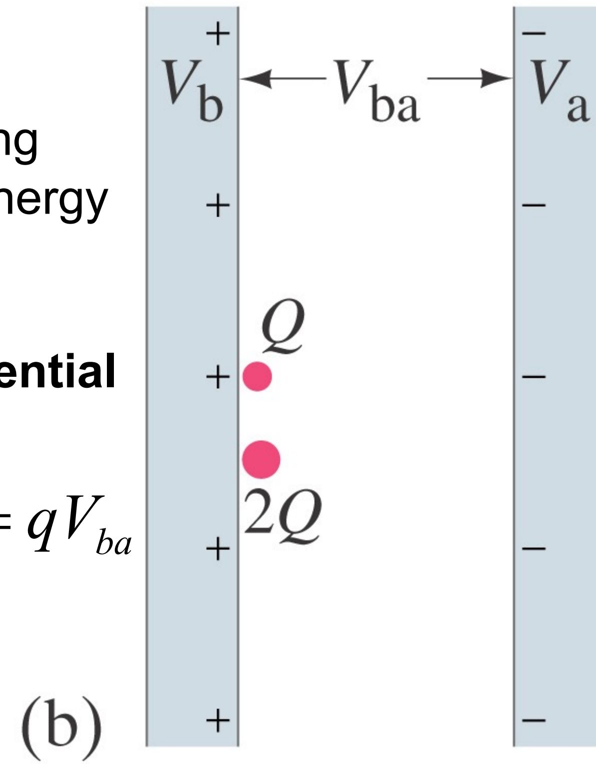
Electric Potential vs. Potential Energy

Analogy between gravitational and electrical potential energy:



Larger mass has
larger potential energy: mhg
and acquires more kinetic energy
but both
small and large mass have
the **same gravitational potential**

$$PE_b - PE_a = q(V_b - V_a) = qV_{ba}$$



Gravitational PE depends on both m and h.g - effects of h.g depends on planet
Electrical PE deps on both Q and V_{ab} - effects of V_{ab} deps on charges on plates

Batteries, electric generators – keep potential difference

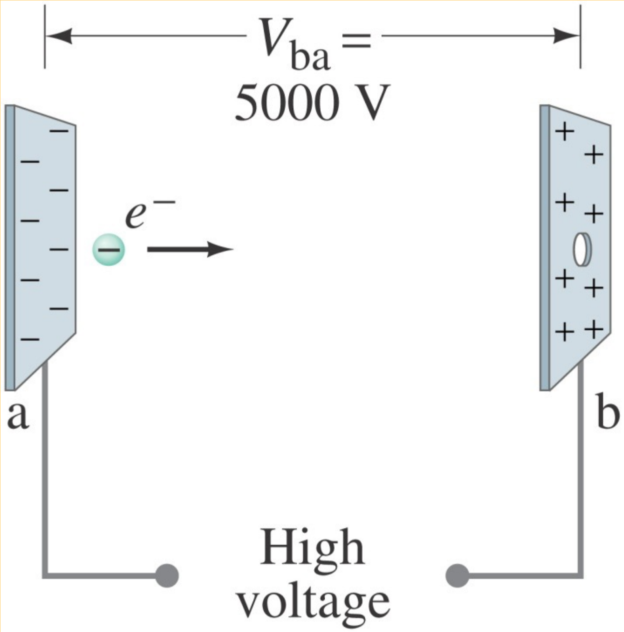
Exercise

Ex. Suppose an electron is accelerated from rest through a potential difference Vb – Va = Vba = +5000 V. (a) What is the change in electric potential energy of the electron? (b) What is the speed of the electron as a result of this acceleration?

$m_e = 9.1\times10^{-31}\,kg$

$$\Delta PE = qV_{ba} = -8.0\times10^{-16}\,J$$

$$\Delta KE = -\Delta PE \Rightarrow v = 4.2\times10^{7}\,m/s$$



Proton would be accelerated from rest by a potential difference -5000 V

$m_p = 1.67\times10^{-27}\,kg$

$$\boxed{1\ \text{eV} = 1.6\times10^{-19}\ \text{J}}$$

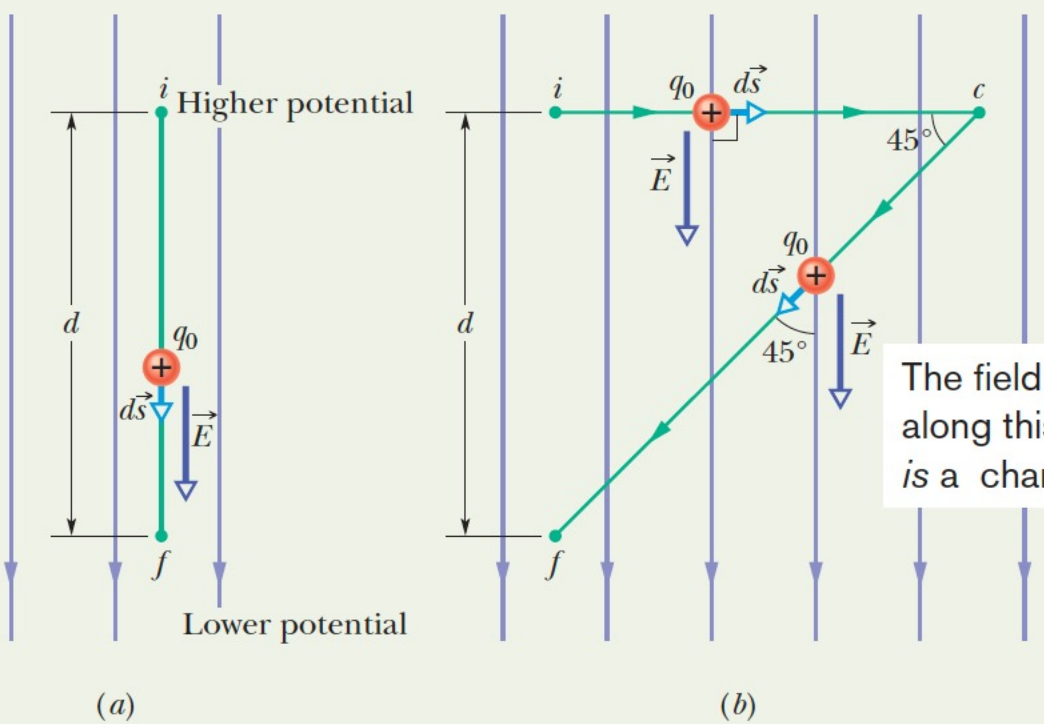
NOTE: Joule is a very large unit for dealing with energies of electrons, atoms or molecules, this is why eV was introduced. One electron volt (eV) is the energy gained by an electron moving through a potential difference of one volt.

Finding the potential change from the electric field

(a) Figure 24-5*a* shows two points *i* and *f* in a uniform electric field $\vec{E}$. The points lie on the same electric field line (not shown) and are separated by a distance *d*. Find the potential difference $V_f - V_i$ by moving a positive test charge q_0 from *i* to *f* along the path shown, which is parallel to the field direction.

The electric field points *from* higher potential *to* lower potential.

The field is perpendicular to this *ic* path, so there is no change in the potential.



The field has a component along this *cf* path, so there *is* a change in the potential.

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

Finding the potential change from the electric field

Calculations: We begin by mentally moving a test charge q_0 along that path, from initial point i to final point f. As we move such a test charge along the path in Fig. 24-5a, its differential displacement $d\vec{s}$ always has the same direction as $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is zero and the dot product in Eq. 24-18 is

$$\vec{E}\cdot d\vec{s} = E\,ds\cos\theta = E\,ds. \qquad (24\text{-}20)$$

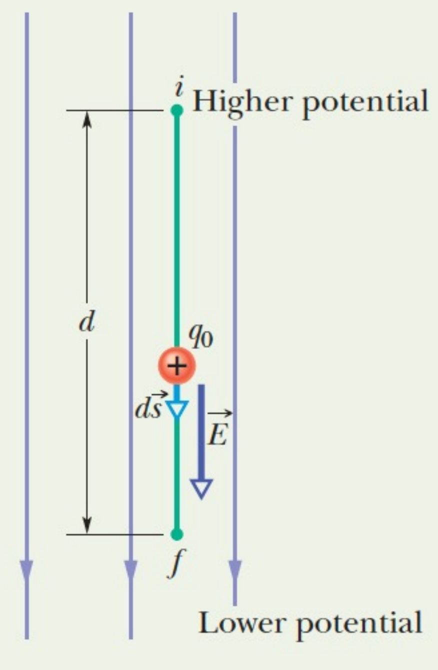
Equations 24-18 and 24-20 then give us

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} = -\int_i^f E\,ds. \qquad (24\text{-}21)$$

Since the field is uniform, E is constant over the path and can be moved outside the integral, giving us

$$V_f - V_i = -E\int_i^f ds = -Ed, \qquad \text{(Answer)}$$

in which the integral is simply the length d of the path. The minus sign in the result shows that the potential at point f in Fig. 24-5a is lower than the potential at point i. This is a general result: The potential always decreases along a path that extends in the direction of the electric field lines.



(a)

(b) Now find the potential difference $V_f - V_i$ by moving the positive test charge q_0 from i to f along the path icf shown in Fig. 24-5b.

Calculations: The Key Idea of (a) applies here too, except now we move the test charge along a path that consists of two lines: ic and cf. At all points along line ic, the displacement $d\vec{s}$ of the test charge is perpendicular to $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is 90°, and the dot product $\vec{E}\cdot d\vec{s}$ is 0. Equation 24-18 then tells us that points i and c are at the same potential: $V_c - V_i = 0$.

For line cf we have $\theta = 45°$ and, from Eq. 24-18,

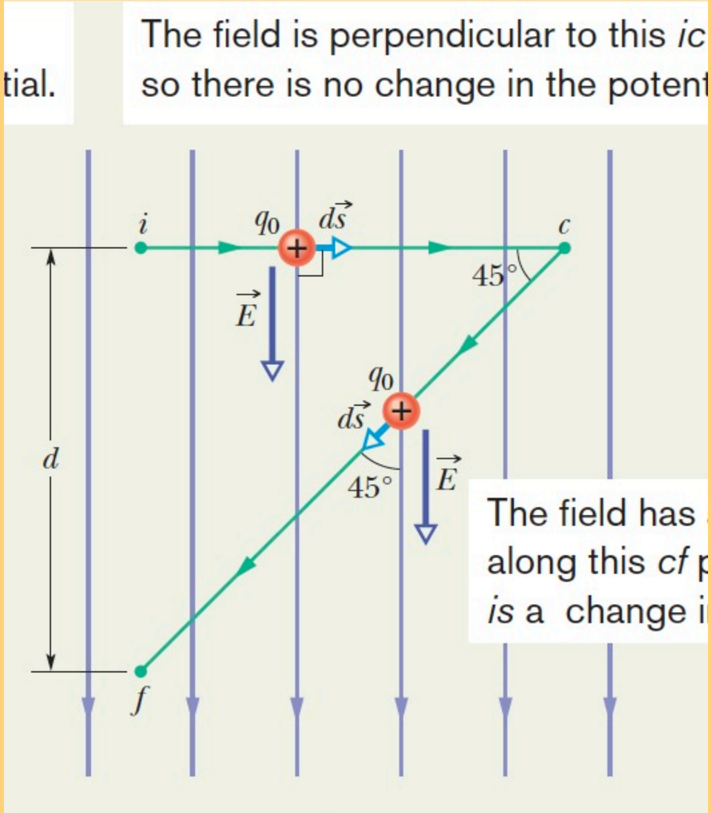
$$V_f - V_i = -\int_c^f \vec{E}\cdot d\vec{s} = -\int_c^f E(\cos 45°)\, ds$$

$$= -E(\cos 45°)\int_c^f ds.$$

The integral in this equation is just the length of line cf; from Fig. 24-5b, that length is $d/\cos 45°$. Thus,

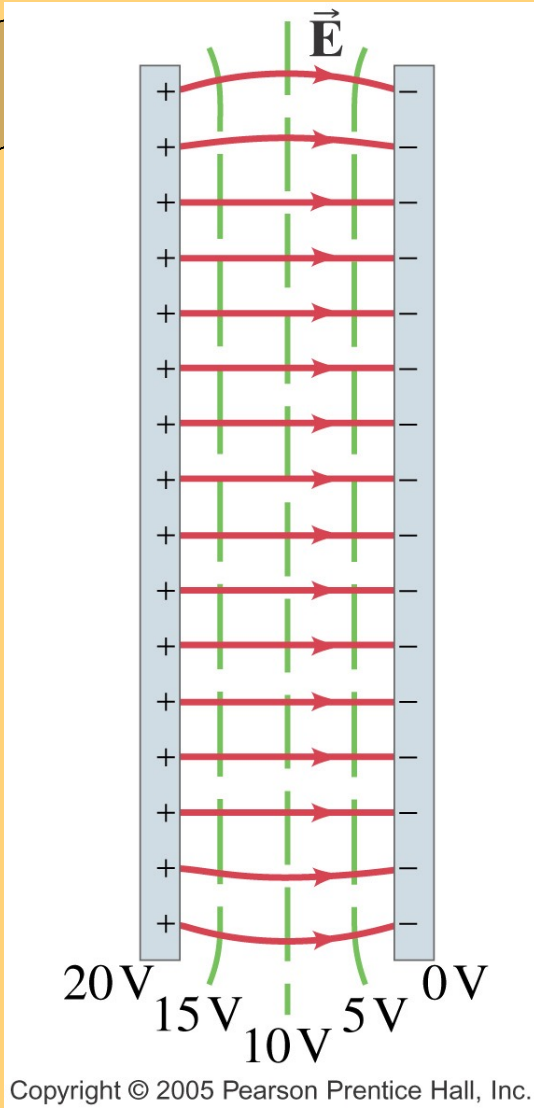
$$V_f - V_i = -E(\cos 45°)\frac{d}{\cos 45°} = -Ed. \quad \text{(Answer)}$$

This is the same result we obtained in (a), as it must be; the potential difference between two points does not depend on the path connecting them. Moral: When you want to find the potential difference between two points by moving a test charge between them, you can save time and work by choosing a path that simplifies the use of Eq. 24-18.



(b)

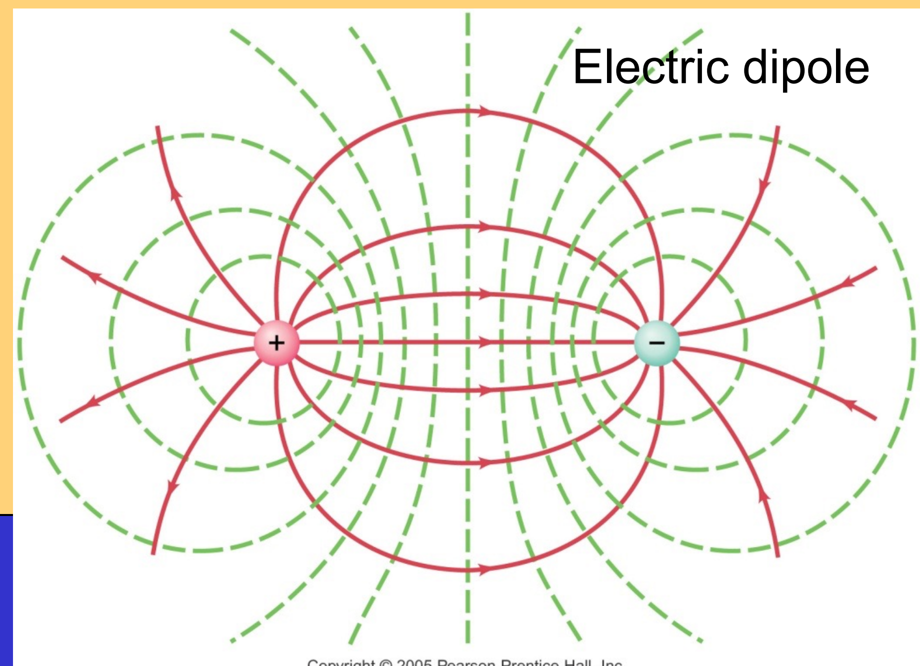
Equipotential Lines



An **equipotential** is a line or surface over which the **potential is constant**.

Electric field lines are **perpendicular** to equipotentials.

A conductor is entirely at the same potential in the static cases; the surface of a conductor is an equipotential.



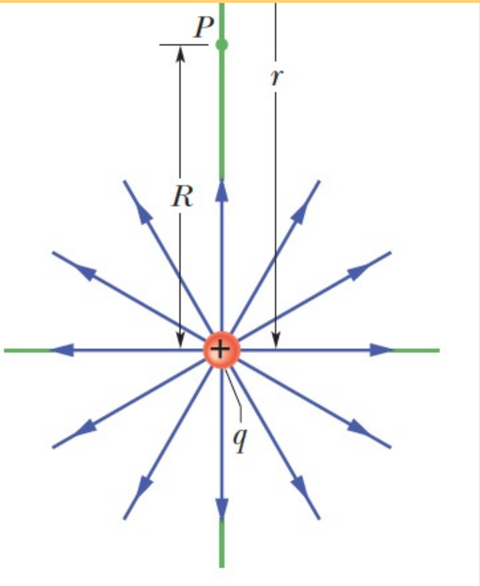
Similar to topographic maps

Electric Potential: Point Charge

The **electric potential** due to a *point charge* can be derived using calculus.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Potential is zero at infinity



Deriving the equation above

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}.$$

$$V_f - V_i = -\int_R^\infty E\,dr.$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

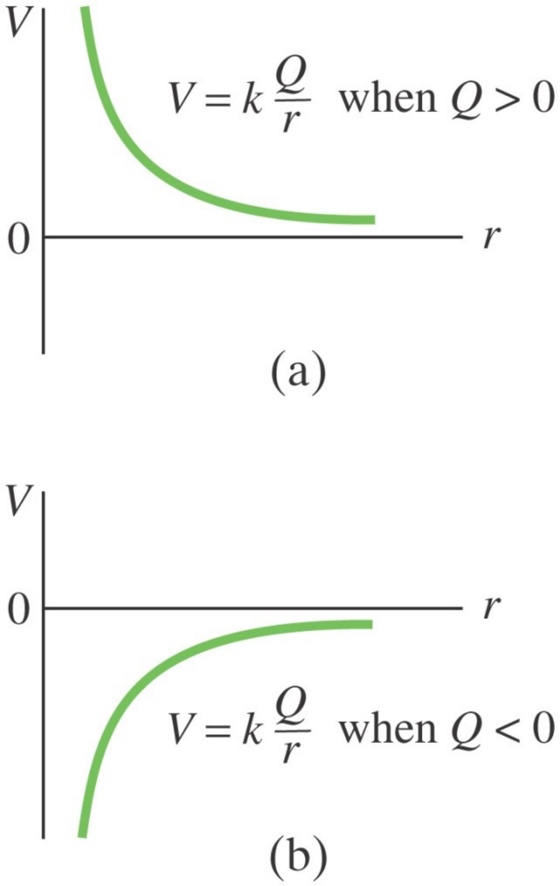
$$0 - V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Electric Potential: Point Charge

The **electric potential** due to a *point charge* can be derived using calculus.

$$V = k\frac{Q}{r} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$



Potential is zero at infinity

Exercises

$$\boxed{V = k\frac{Q}{r}}$$

Work by electric force:

W = **-** q (V_f – V_i)

Work by external force:

W = **+** q (V_f – V_i)

Ex. ---- Determine the potential at a point 0.50 m (a) from a + 20 μC point charge, (b) from a - 20 μ C point

$$(a)3.6\times10^{5}V \quad (b)-3.6\times10^{5}V$$

Ex. ---- What minimum work must be done by an external force to bring a charge q = 3.00 μ C from a great distance away (r: infinity) to a point 0.500 m from a charge Q = 20.0 μC ?

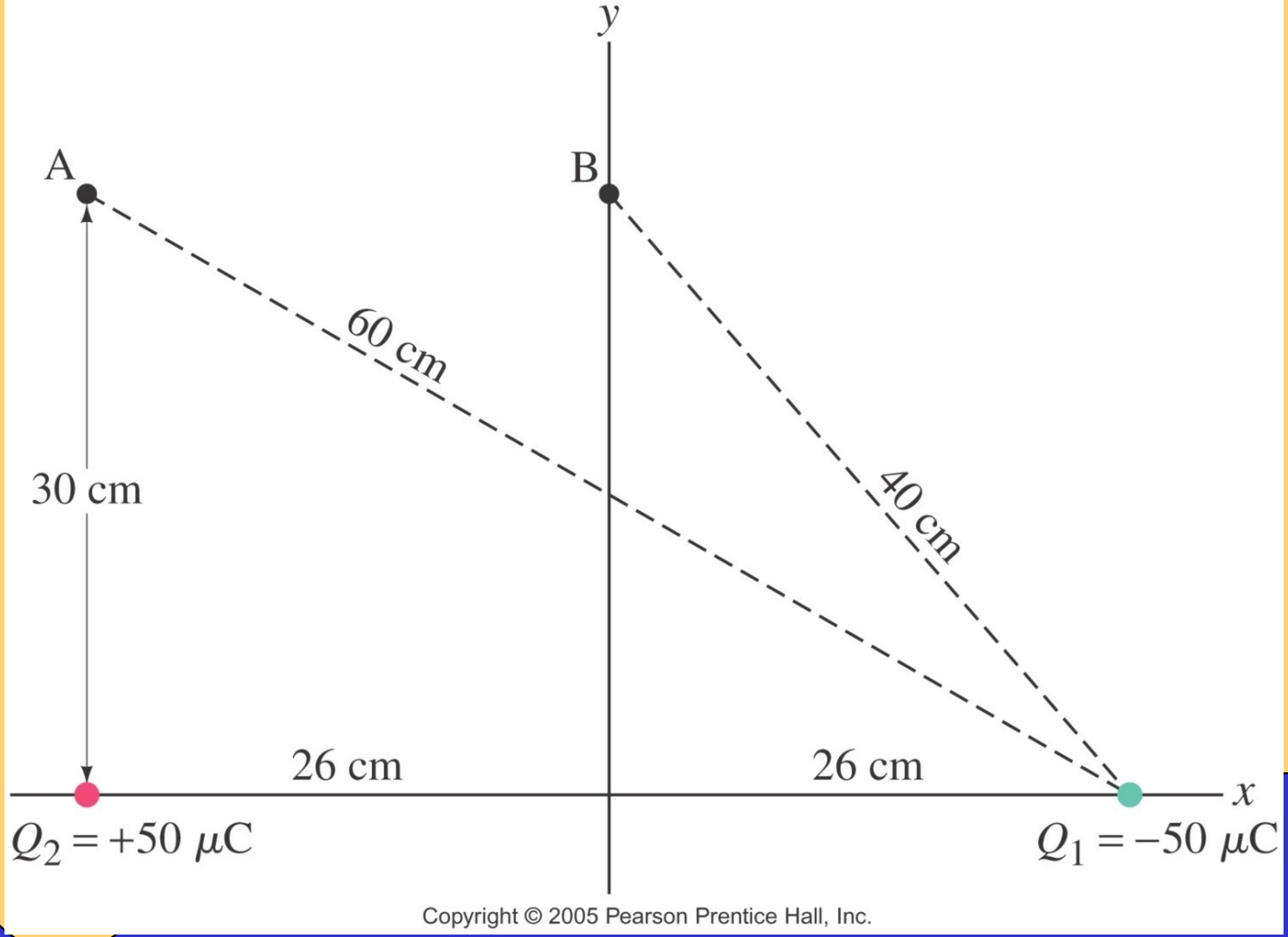
1.08 J

NOTE:
To find the electric field near a collection of two or more point charges requires adding VECTORS
To find the electric potential near a collection of two or more point charges is EASIER, it only requires adding NUMBERS
NOTE:
Take the **sign** of the charge into account when calculating electric potential

Exercise

$$\boxed{V = k\frac{Q}{r}}$$

Ex. Calculate the electric potential (a) at point A and (b) at point B



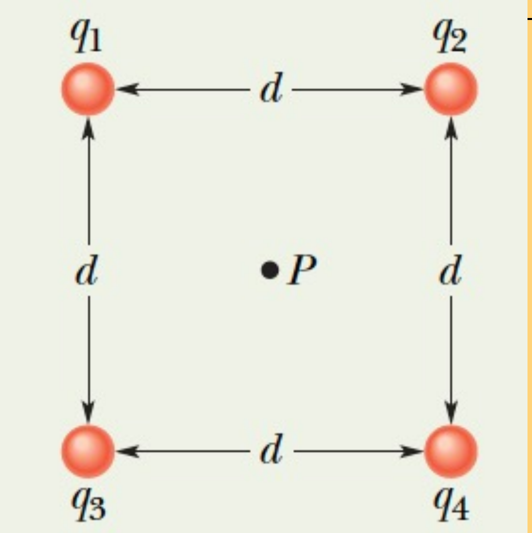
$$(a) V_a = 7.5 \times 10^5 V$$

$$(b) V_b = 0$$

Net potential of several charged particles

What is the electric potential at point P, located at the center of the square of point charges shown in Fig. 24-8a? The distance d is 1.3 m, and the charges are

$$q_1 = +12 \text{ nC}, \qquad q_3 = +31 \text{ nC},$$
$$q_2 = -24 \text{ nC}, \qquad q_4 = +17 \text{ nC}.$$



$$V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right).$$

The distance r is $d/\sqrt{2}$, which is 0.919 m, and the sum of the charges is

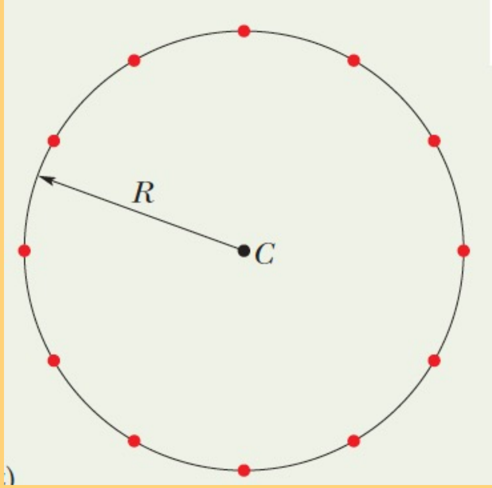
$$q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9} \text{ C}$$
$$= 36 \times 10^{-9} \text{ C}.$$

Thus, $$V = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(36 \times 10^{-9} \text{ C})}{0.919 \text{ m}}$$

$$\approx 350 \text{ V}. \qquad \text{(Answer)}$$

Potential is not a vector, orientation is irrelevant

(a) In Fig. 24-9*a*, 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?



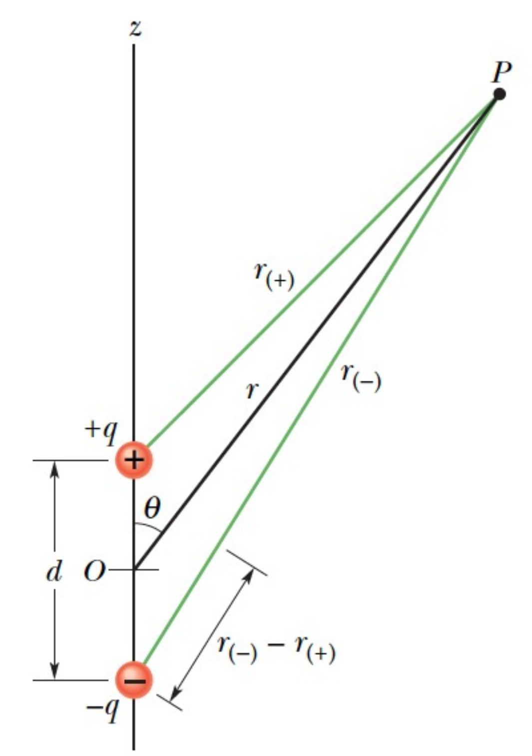
Calculations: Because the electrons all have the same negative charge $-e$ and are all the same distance R from C, Eq. 24-27 gives us

$$V = -12 \frac{1}{4\pi\varepsilon_0} \frac{e}{R}. \qquad \text{(Answer)} \qquad (24\text{-}28)$$

Because of the symmetry of the arrangement in Fig. 24-9*a*, the electric field vector at C due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at C,

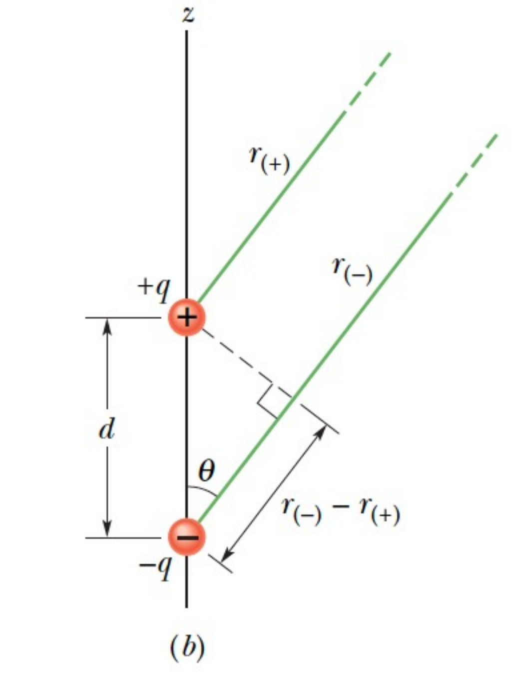
$$\vec{E} = 0. \qquad \text{(Answer)}$$

Potential Due to an Electric Dipole



$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

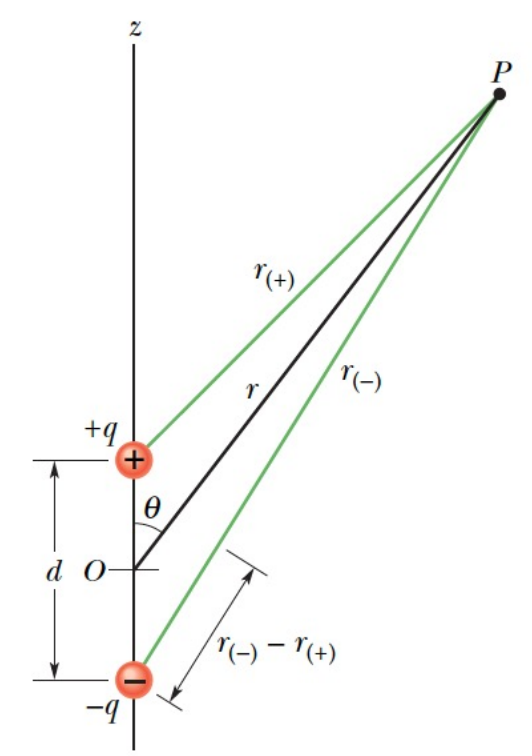


Naturally occurring dipoles—such as those possessed by many molecules—are quite small; so we are usually interested only in points that are relatively far from the dipole, such that r>>d, where d is the distance between the charges.

$$r_{(-)} - r_{(+)} \approx d\cos\theta$$

$$r_{(-)}r_{(+)} \approx r^2$$

Potential Due to an Electric Dipole



$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

$$r_{(-)} - r_{(+)} \approx d\cos\theta \qquad r_{(-)}r_{(+)} \approx r^2$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

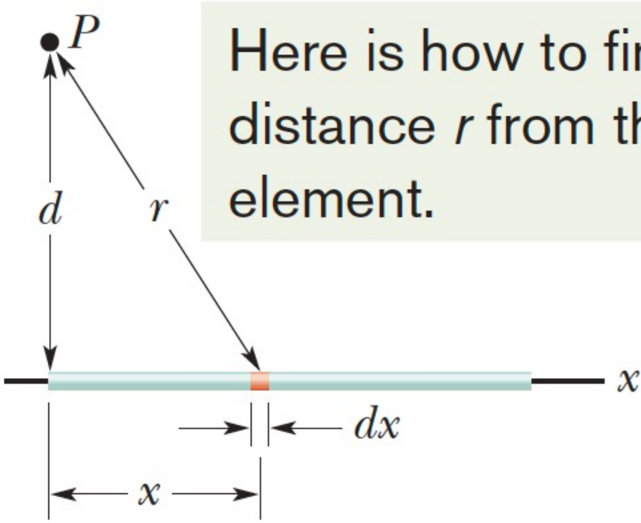
in which p $(= qd)$ is the magnitude of the electric dipole moment

Potential Due to a Continuous Charge Distribution

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

Line of Charge



Here is how to find distance r from the element.

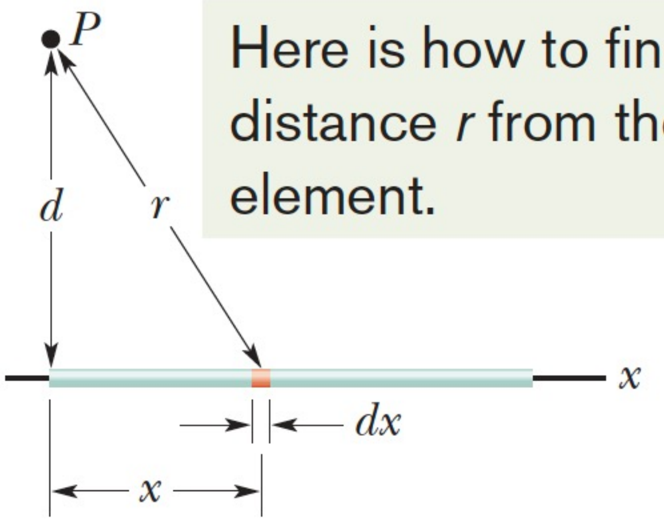
$$dq = \lambda\, dx$$

$$r = (x^2 + d^2)^{1/2}$$

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{(x^2+d^2)^{1/2}}\,dx$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right)$$

Line of Charge



Here is how to find distance r from the element.

$$dq = \lambda\, dx$$

$$r = (x^2 + d^2)^{1/2}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

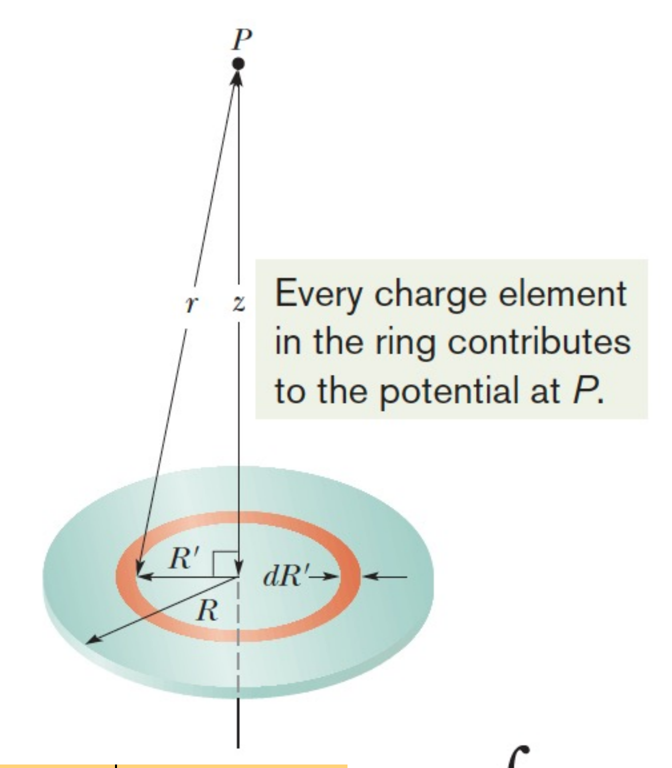
$$V = \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \Big[\ln\Big(x + (x^2 + d^2)^{1/2}\Big) \Big]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \Big[\ln\Big(L + (L^2 + d^2)^{1/2}\Big) - \ln d \Big]$$

$$V = \frac{\lambda}{4\pi\varepsilon_0} \ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right].$$

Charged Disk



$$dq = \sigma(2\pi R')(dR')$$

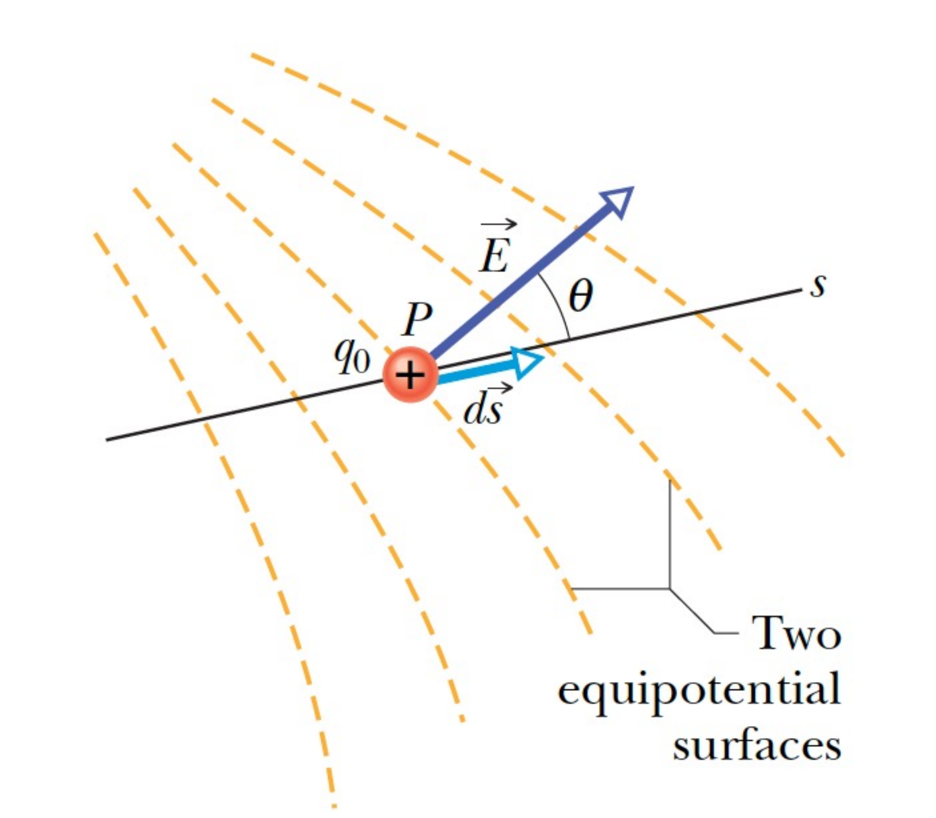
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}$$

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{R'\,dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right)$$

Electric Potential and Electric Field

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

$$E_s = -\frac{\partial V}{\partial s}.$$



$$E_x = -\frac{\partial V}{\partial x};$$

$$E_y = -\frac{\partial V}{\partial y};$$

$$E_z = -\frac{\partial V}{\partial z}.$$

Finding the field from the potential

The electric potential at any point on the central axis of a uniformly charged disk is given by Eq. 24-37,

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right).$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.

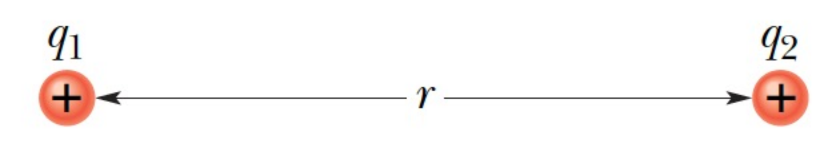
We want the electric field $\vec{E}$ as a function of distance z along the axis of the disk. For any value of z, the direction of $\vec{E}$ must be along that axis because the disk has circular symmetry about that axis.

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\frac{d}{dz}\left(\sqrt{z^2 + R^2} - z\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right). \qquad \text{(Answer)}$$

This is the same expression that we derived in Section 22-7 by integration, using Coulomb's law.

Electric Potential Energy of a System of Point Charges



Potential created by q_1 at a distance r

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

Potential ENERGY of q_2

$$U = W = q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

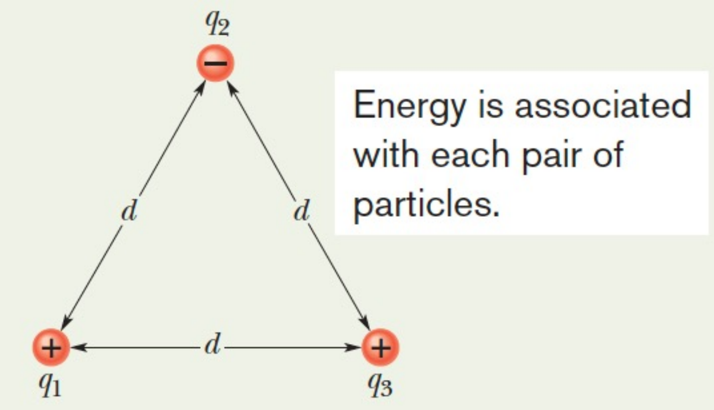
This is also the work done by an EXTERNAL force to bring q_2 from an infinity to a distance r from particle q_1

Potential energy of a system of three charged particles

Figure 24-16 shows three point charges held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.



The potential energy U of the system is equal to the work we must do to assemble the system, bringing in each charge from an infinite distance.

Figure 24-16 shows three point charges held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

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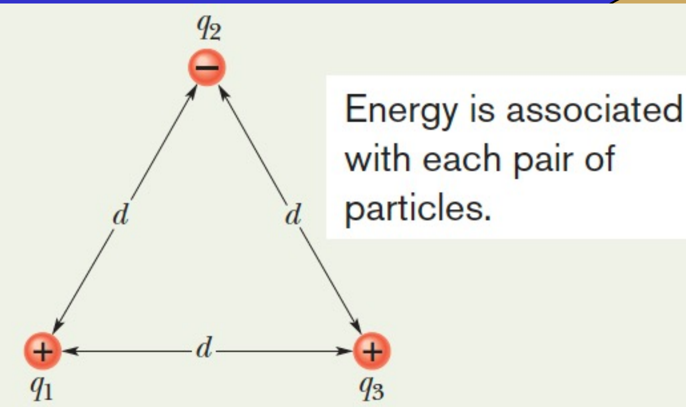


Calculations: Let's mentally build the system of Fig. 24-16, starting with one of the point charges, say q_1, in place and the others at infinity. Then we bring another one, say q_2, in from infinity and put it in place. From Eq. 24-43 with d substituted for r, the potential energy U_{12} associated with the pair of point charges q_1 and q_2 is

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}.$$

We then bring the last point charge q_3 in from infinity and put it in place. The work that we must do in this last step is equal to the sum of the work we must do to bring q_3 near q_1 and the work we must do to bring it near q_2. From Eq. 24-43, with d substituted for r, that sum is

$$W_{13} + W_{23} = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{d} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{d}.$$

$$\begin{aligned} U &= U_{12} + U_{13} + U_{23} \\ &= \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right) \\ &= -\frac{10q^2}{4\pi\varepsilon_0 d} \\ &= -\frac{(8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2})(10)(150 \times 10^{-9}\ \mathrm{C})^2}{0.12\ \mathrm{m}} \\ &= -1.7 \times 10^{-2}\ \mathrm{J} = -17\ \mathrm{mJ}. \qquad \text{(Answer)} \end{aligned}$$

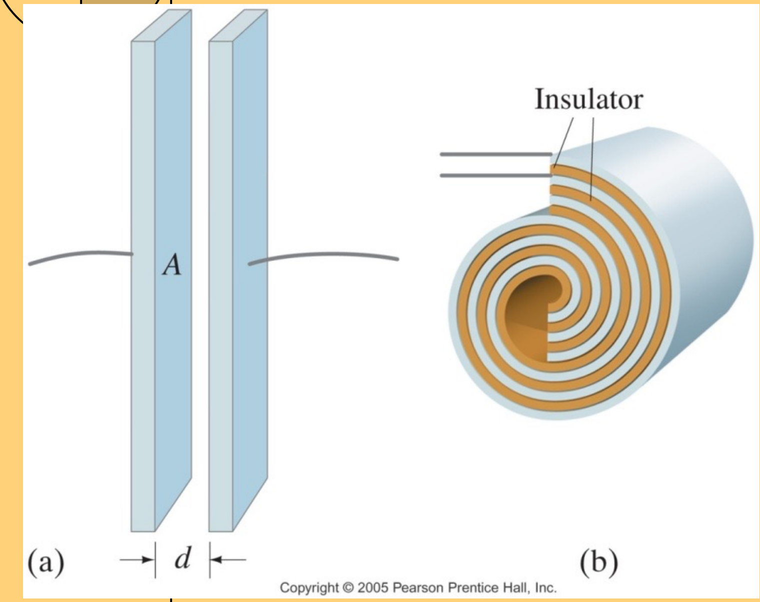
The negative potential energy means that negative work would have to be done to assemble this structure, starting with the three charges infinitely separated and at rest. Put another way, an external agent would have to do 17 mJ of work to disassemble the structure completely, ending with the three charges infinitely far apart.

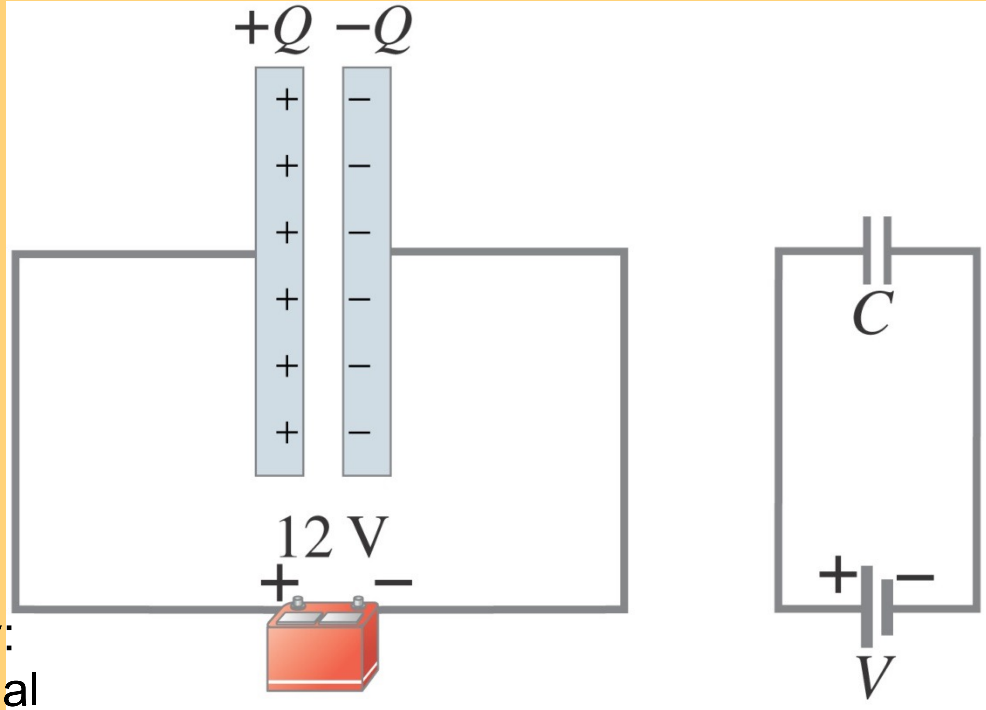
Problems

Chapter 24:
Problems: 16, 17, 19, 25, 37, 43, 44

Chapter 25: Capacitance

A capacitor is a device that can **store** electric charge.
It consists of two conductors that are close but not touching.





Capacitor connected to a battery: charge on its plates is proportional to the voltage:

$$Q = CV$$

The quantity C is called the capacitance.

Unit of capacitance: the farad (F)

1 F = 1 C/V

Examples of capacitors: power backups, camera flash

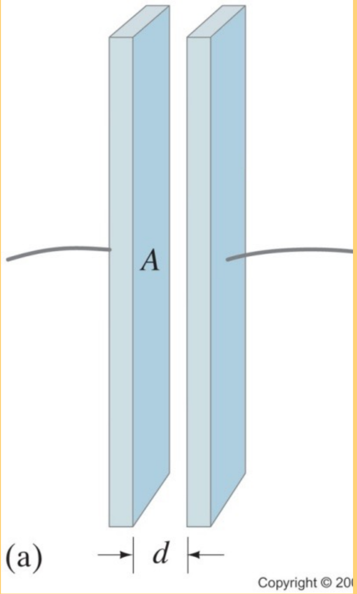
Capacitance

$$Q = CV$$

Common capacitors have capacitance in the range of 1 pF to $10^3\,\mu F$

$$1pF = 10^{-12}F$$

From here on V indicates potential difference



In general, C does not depend on Q or V
For a parallel-plate capacitor:

$$C = \epsilon_0 \frac{A}{d}$$

Ex. (a) Calculate the capacitance of a parallel-plate capacitor whose plates are 20 cm x 3.0 cm and are separated by a 1.0-mm air gap. (b) What is the charge on each plate if a 12-V battery is connected across the two plates? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1 F, given that the same air gap d.

$$(a) 53\,pF \quad (b) 6.4\times10^{-10}C \quad (c) 1.2\times10^{4}V/m \quad (d) A \approx 10^{8}m^{2}$$

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2}$$

Dielectrics

A dielectric is an insulator, and is characterized by a dielectric constant K.

Capacitance of a parallel-plate capacitor filled with dielectric:

$$C = K\epsilon_0 \frac{A}{d}$$

Purpose of dielectrics:

(i) They do not break (=charge flow) as easily as in air; higher voltage can be applied

(ii) Plates can be together without touching; C increases as d decreases

(iii) They increase C by K

Permittivity of the material $\varepsilon = K\varepsilon_0$

Ex. An airfilled capacitor consisting of 2 parallel plates separated by a distance d is connected to a battery of voltage V and acquires a charge Q. While it is still connected to the battery, a slab of dielectric material with K=3 is inserted between the plates of the capacitor. Will Q increases, decrease, or stay the same?

V stays constant, C increases, so Q increases as well

Ex. Suppose the capacitor above is instead disconnected from the battery and then a dielectric is inserted between the plates. Will Q, C, or V change?

Q stays constant, C increases, so V decreases

Storage of Electric Energy

A charged capacitor stores electric energy; the energy stored is equal to the work done to charge the capacitor.

$$dW = V'\,dq' = \frac{q'}{C}\,dq'$$

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}$$

$$U = \frac{q^2}{2C}$$

$$= \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\frac{Q^2}{C}$$