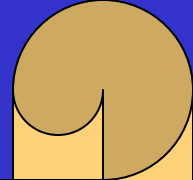
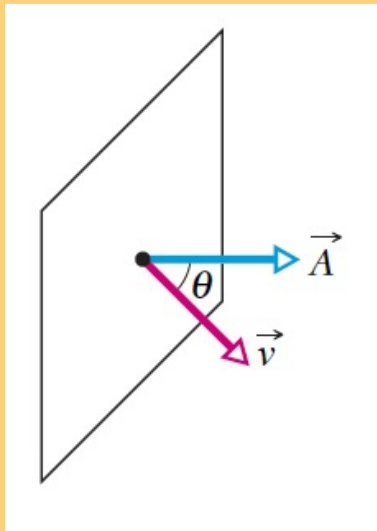


Gauss Law



Gauss Law: a way to find the electric field using the symmetries of the charge distribution involved.

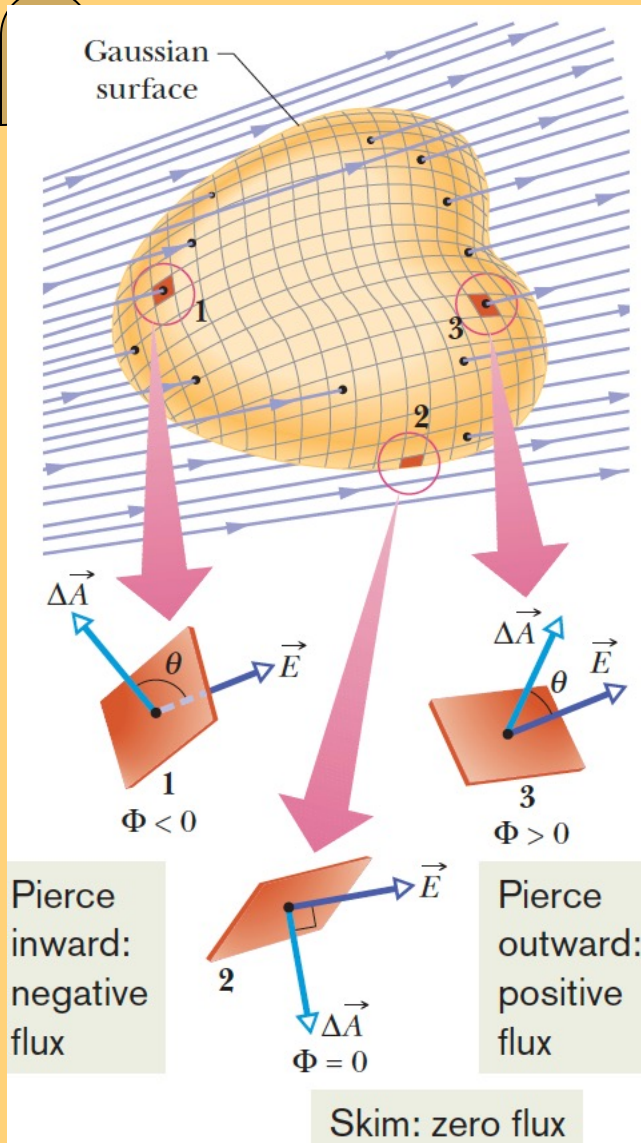
We consider a hypothetical (imaginary) closed surface (Gaussian surface) enclosing the charge distribution with the same symmetry as the charge distribution.



FLUX:

$$\Phi = vA \cos \theta = \vec{v} \cdot \vec{A} \quad (23-2)$$

Flux



$$\Phi = vA \cos \theta = \vec{v} \cdot \vec{A} \quad (23-2)$$

$$\Phi = \sum \vec{E} \cdot \Delta \vec{A}. \quad (23-3)$$

Electric flux through a Gaussian surface

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad (23-4)$$

Unit: $(\text{N} \cdot \text{m}^2/\text{C})$

Flux through a closed cylinder, uniform field

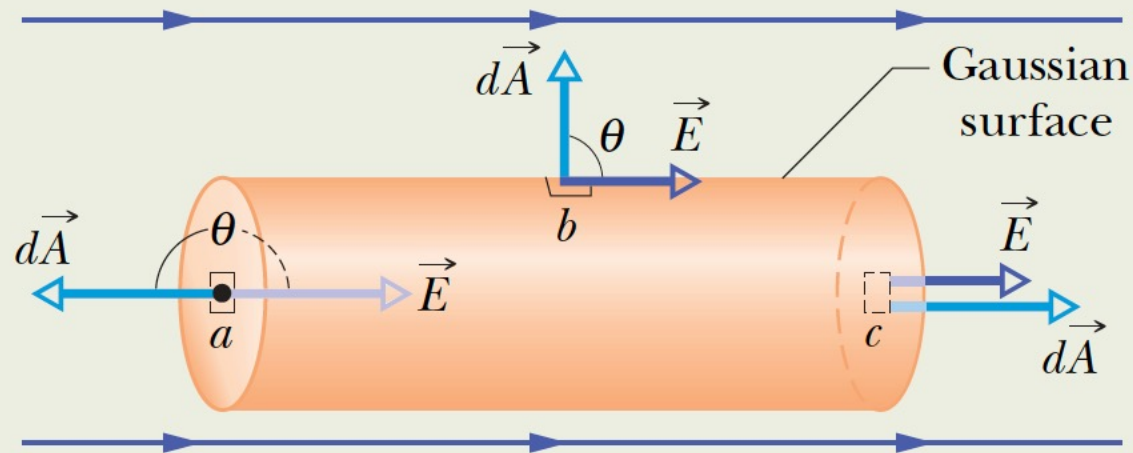
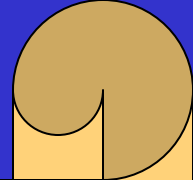


Fig. 23-4 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

Figure 23-4 shows a Gaussian surface in the form of a cylinder of radius R immersed in a uniform electric field \vec{E} , with the cylinder axis parallel to the field. What is the flux Φ of the electric field through this closed surface?



Calculations: We can do the integration by writing the flux as the sum of three terms: integrals over the left cylinder cap a , the cylindrical surface b , and the right cap c . Thus, from Eq. 23-4,

$$\begin{aligned}\Phi &= \oint \vec{E} \cdot d\vec{A} \\ &= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}.\end{aligned}\quad (23-5)$$

For all points on the left cap, the angle θ between \vec{E} and $d\vec{A}$ is 180° and the magnitude E of the field is uniform. Thus,

$$\int_a \vec{E} \cdot d\vec{A} = \int E(\cos 180^\circ) dA = -E \int dA = -EA,$$

right cap, where $\theta = 0$ for all points,

$$\int_c \vec{E} \cdot d\vec{A} = \int E(\cos 0) dA = EA.$$

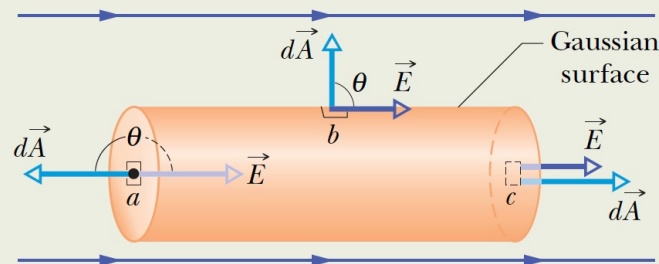
Finally, for the cylindrical surface, where the angle θ is 90° at all points,

$$\int_b \vec{E} \cdot d\vec{A} = \int E(\cos 90^\circ) dA = 0.$$

Substituting these results into Eq. 23-5 leads us to

$$\Phi = -EA + 0 + EA = 0. \quad (\text{Answer})$$

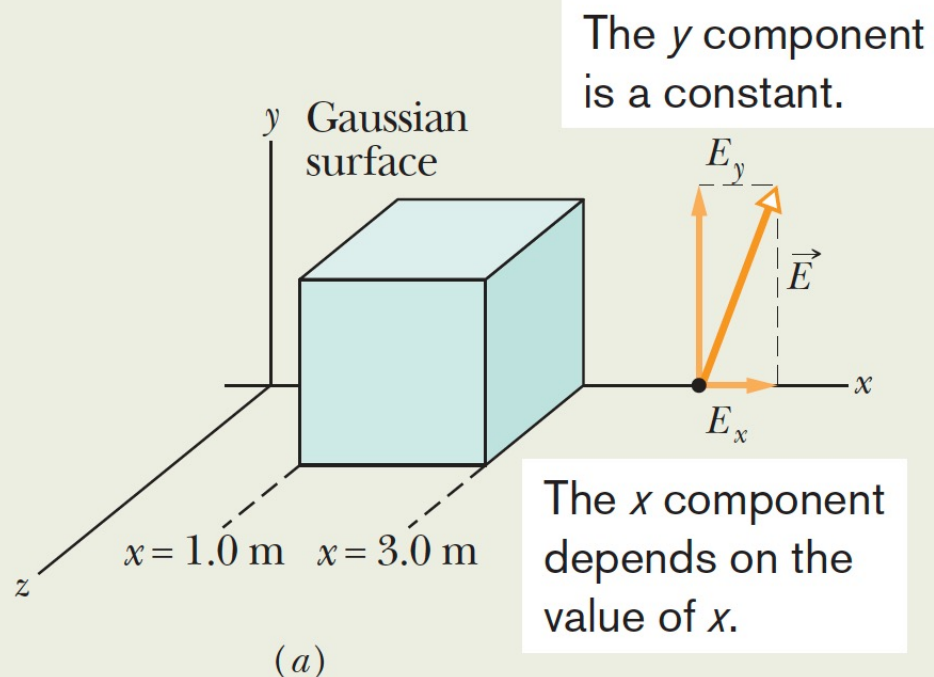
The net flux is zero because the field lines that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.



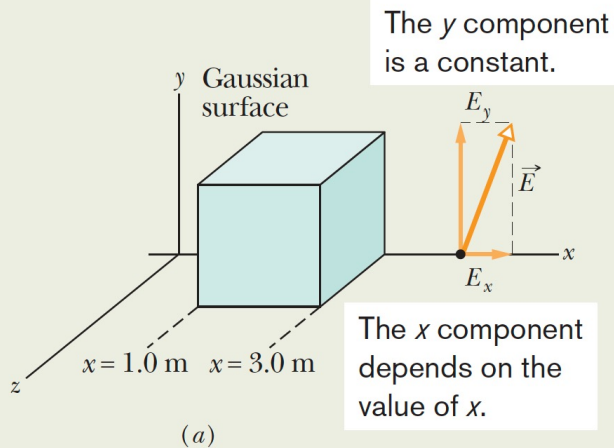
where $\int dA$ gives the cap's area $A (= \pi R^2)$. Similarly

Flux through a closed cube, nonuniform field

A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 23-5a. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)



Right Face



A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 23-5a. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)

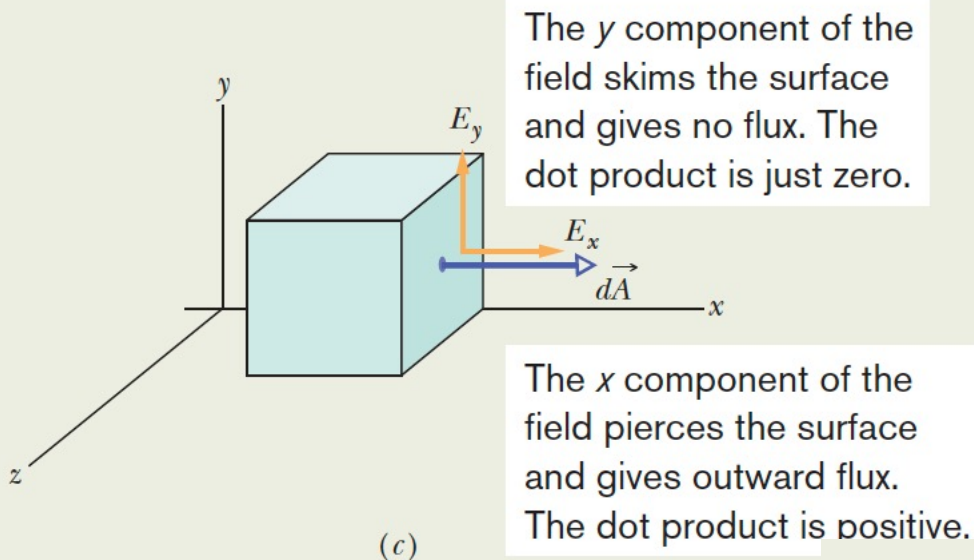
$$\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$$

$$d\vec{A} = dA\hat{i}$$

$$\begin{aligned}\Phi_r &= \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{i}) \\ &= \int [(3.0x)(dA)\hat{i} \cdot \hat{i} + (4.0)(dA)\hat{j} \cdot \hat{i}] \\ &= \int (3.0x dA + 0) = 3.0 \int x dA.\end{aligned}$$

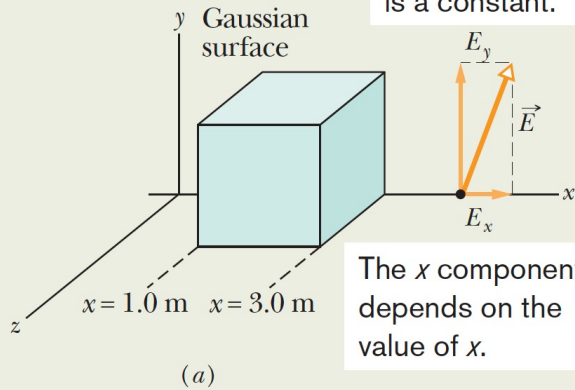
$$\Phi_r = 3.0 \int (3.0) dA = 9.0 \int dA.$$

$$A = 4.0 \text{ m}^2$$

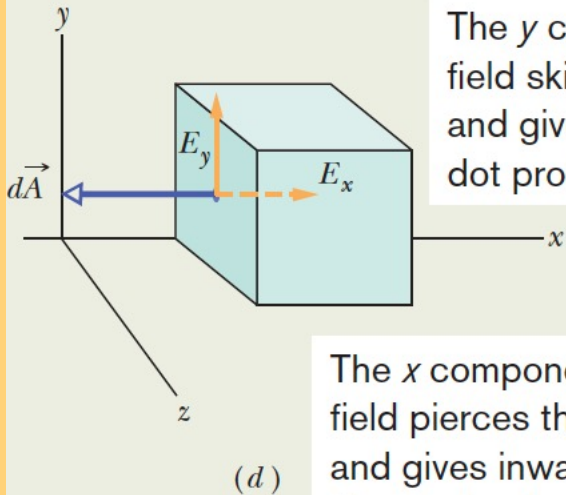


$$\Phi_r = (9.0 \text{ N/C})(4.0 \text{ m}^2) = 36 \text{ N} \cdot \text{m}^2/\text{C}$$

Left Face



The x component depends on the value of x .



The x component of the field pierces the surface and gives inward flux. The dot product is negative.

A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 23-5a. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)

$$\vec{E} = 3.0x\hat{i} + 4.0\hat{j} \quad d\vec{A} = -dA\hat{i}$$

$$\begin{aligned} \Phi_r &= \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (-dA\hat{i}) \\ &= \int \left[(3.0x)(dA)\hat{i} \cdot \hat{i} + (4.0)(dA)\hat{j} \cdot \hat{i} \right] \\ &= \int (3.0x dA + 0) = -3.0 \int x dA. \end{aligned}$$

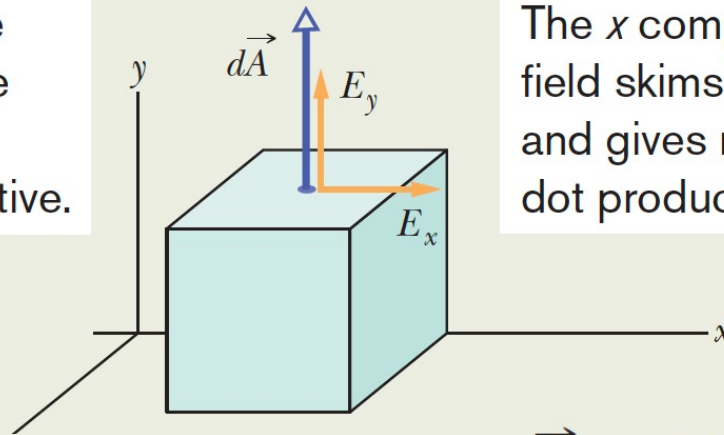
$$x = 1.0 \text{ m.}$$

$$A = 4.0 \text{ m}^2$$

$$\Phi_l = -12 \text{ N} \cdot \text{m}^2/\text{C.}$$

Top Face

The y component of the field pierces the surface and gives outward flux. The dot product is positive.

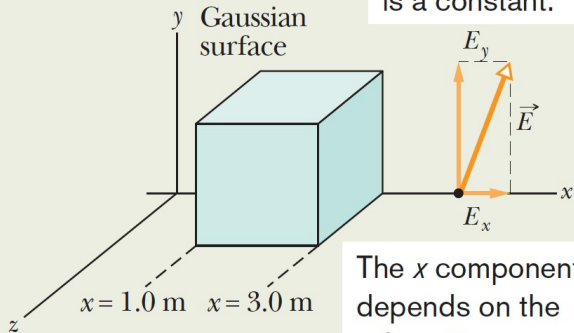


The x component of the field skims the surface and gives no flux. The dot product is just zero.

The y component is a constant.

(e)

$$\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$$



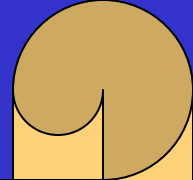
The x component depends on the value of x .

(a)

$$A = 4.0 \text{ m}^2$$

$$\begin{aligned} \Phi_t &= \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{j}) \\ &= \int [(3.0x)(dA)\hat{i} \cdot \hat{j} + (4.0)(dA)\hat{j} \cdot \hat{j}] \\ &= \int (0 + 4.0 dA) = 4.0 \int dA \\ &= 16 \text{ N} \cdot \text{m}^2/\text{C}. \end{aligned}$$

Gauss' Law



Gauss' law relates the net flux of an electric field through a closed surface (a Gaussian surface) to the net charge q_{enc} that is **enclosed** by that surface.

$$\epsilon_0 \Phi = q_{\text{enc}} \quad (23-6)$$

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \quad (23-7)$$

Equations 23-6 and 23-7 hold only when the net charge is located in a vacuum

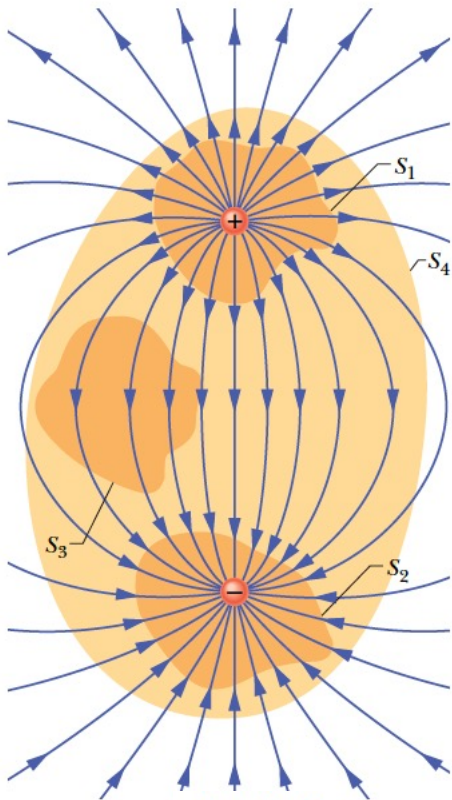


Fig. 23-6

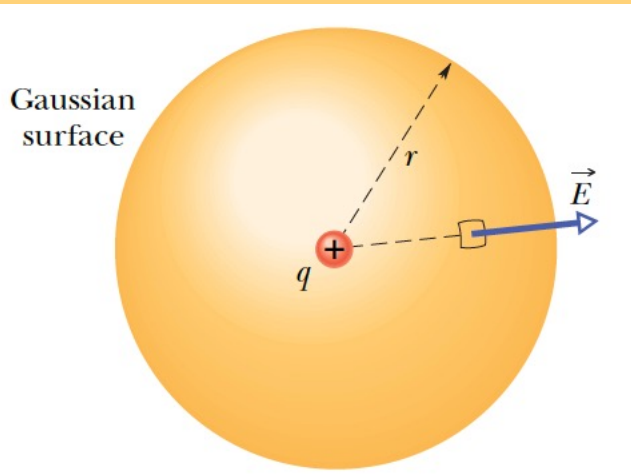
Surface S_1 . The electric field is outward for all points on this surface. Thus, the flux of the electric field through this surface is positive, and so is the net charge within the surface, as Gauss' law requires. (That is, in Eq. 23-6, if Φ is positive, q_{enc} must be also.)

Surface S_2 . The electric field is inward for all points on this surface. Thus, the flux of the electric field through this surface is negative and so is the enclosed charge, as Gauss' law requires.

Surface S_3 . This surface encloses no charge, and thus $q_{\text{enc}} = 0$. Gauss' law (Eq. 23-6) requires that the net flux of the electric field through this surface be zero. That is reasonable because all the field lines pass entirely through the surface, entering it at the top and leaving at the bottom.

Surface S_4 . This surface encloses no *net* charge, because the enclosed positive and negative charges have equal magnitudes. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because there are as many field lines leaving surface S_4 as entering it.

Gauss' Law and Coulomb's Law



$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 \oint E dA = q_{\text{enc.}}$$

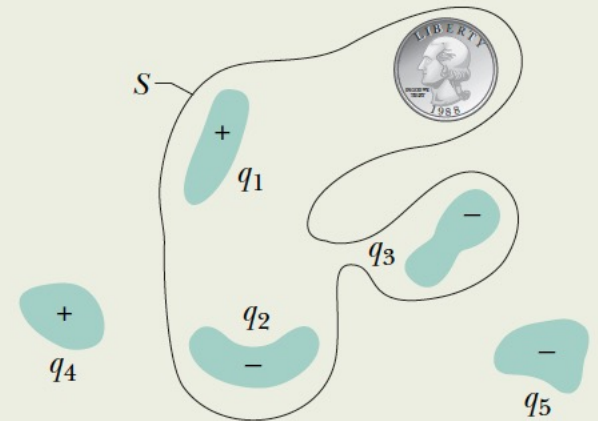
$$\epsilon_0 E \oint dA = q$$

$$\epsilon_0 E (4\pi r^2) = q$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Relating the net enclosed charge and the net flux

Figure 23-7 shows five charged lumps of plastic and an electrically neutral coin. The cross section of a Gaussian surface S is indicated. What is the net electric flux through the surface if $q_1 = q_4 = +3.1 \text{ nC}$, $q_2 = q_5 = -5.9 \text{ nC}$, and $q_3 = -3.1 \text{ nC}$?



$$\begin{aligned}\Phi &= \frac{q_{\text{enc}}}{\epsilon_0} = \frac{q_1 + q_2 + q_3}{\epsilon_0} \\ &= \frac{+3.1 \times 10^{-9} \text{ C} - 5.9 \times 10^{-9} \text{ C} - 3.1 \times 10^{-9} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} \\ &= -670 \text{ N} \cdot \text{m}^2/\text{C}. \quad (\text{Answer})\end{aligned}$$

The minus sign shows that the net flux through the surface is inward and thus that the net charge within the surface is negative.

Charges q_4 and q_5 do not contribute because they are outside surface S . They certainly send electric field lines

through the surface, but as much enters as leaves and no net flux is contributed.

Enclosed charge in a nonuniform field

What is the net charge enclosed by the Gaussian cube of Fig. 23-5, which lies in the electric field $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$? (E is in newtons per coulomb and x is in meters.)

KEY IDEA

The net charge enclosed by a (real or mathematical) closed surface is related to the total electric flux through the surface by Gauss' law as given by Eq. 23-6 ($\epsilon_0\Phi = q_{\text{enc}}$).

Flux: To use Eq. 23-6, we need to know the flux through all six faces of the cube. We already know the flux through the right face ($\Phi_r = 36 \text{ N}\cdot\text{m}^2/\text{C}$), the left face ($\Phi_l = -12 \text{ N}\cdot\text{m}^2/\text{C}$), and the top face ($\Phi_t = 16 \text{ N}\cdot\text{m}^2/\text{C}$).

For the bottom face, our calculation is just like that for the top face *except* that the differential area vector $d\vec{A}$ is now directed downward along the y axis (recall, it must be *outward* from the Gaussian enclosure). Thus, we have

$d\vec{A} = -dA\hat{j}$, and we find

$$\Phi_b = -16 \text{ N}\cdot\text{m}^2/\text{C}.$$

For the front face we have $d\vec{A} = dA\hat{k}$, and for the back face, $d\vec{A} = -dA\hat{k}$. When we take the dot product of the given electric field $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ with either of these expressions for $d\vec{A}$, we get 0 and thus there is no flux through those faces. We can now find the total flux through the six sides of the cube:

$$\begin{aligned}\Phi &= (36 - 12 + 16 - 16 + 0 + 0) \text{ N}\cdot\text{m}^2/\text{C} \\ &= 24 \text{ N}\cdot\text{m}^2/\text{C}.\end{aligned}$$

Enclosed charge: Next, we use Gauss' law to find the charge q_{enc} enclosed by the cube:

$$\begin{aligned}q_{\text{enc}} &= \epsilon_0\Phi = (8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(24 \text{ N}\cdot\text{m}^2/\text{C}) \\ &= 2.1 \times 10^{-10} \text{ C}.\end{aligned}\quad (\text{Answer})$$

Thus, the cube encloses a *net* positive charge.

Problems to Solve

Chapter 23: 3, 10