Electric Field

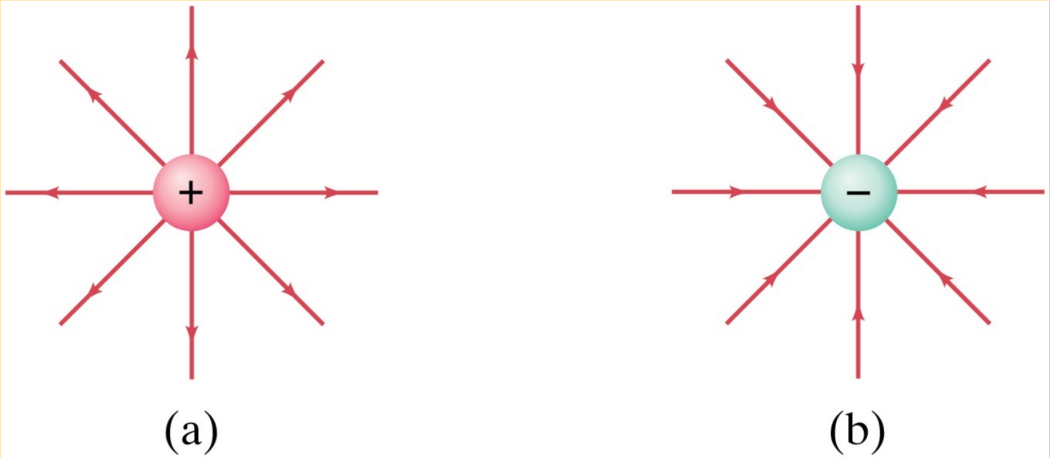
Many common forces are “contact forces”. But gravitational and electrical forces act over a distance, which was a difficult idea in the past. It helps to think in terms of **FIELD** as developed by Michael Faraday.

The electric field extends outward from a charge and permeates all space.

UNITS: N/C

$$E = k\frac{Q}{r^2}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$



The field concept can also be applied to the gravitational force.
A **gravitational field** exists for every object that has mass
It is the force per unit mass.

Electric Field

Many common forces are “contact forces”. But gravitational and electrical forces act over a distance, which was a difficult idea in the past. It helps to think in terms of **FIELD** as developed by Michael Faraday.

The electric field extends outward from a charge and permeates all space. A 2nd charge placed near it feels a force exerted by the electric field there.

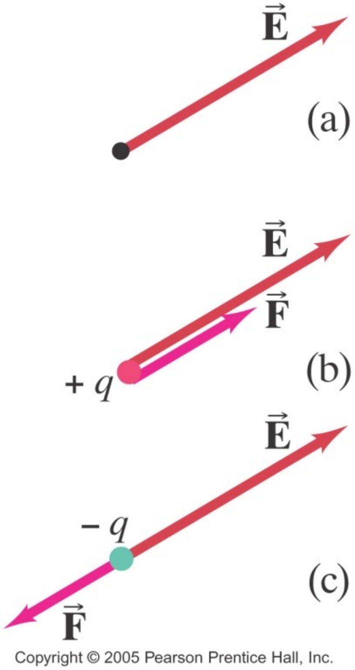
The electric field is the force on a small charge, divided by the charge: The charge (**test charge**) is so small that it does not affect the other particles which create the field UNITS: N/C

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}$$

$$F = k\frac{qQ}{r^2} \Rightarrow$$

$$E = k\frac{Q}{r^2}$$

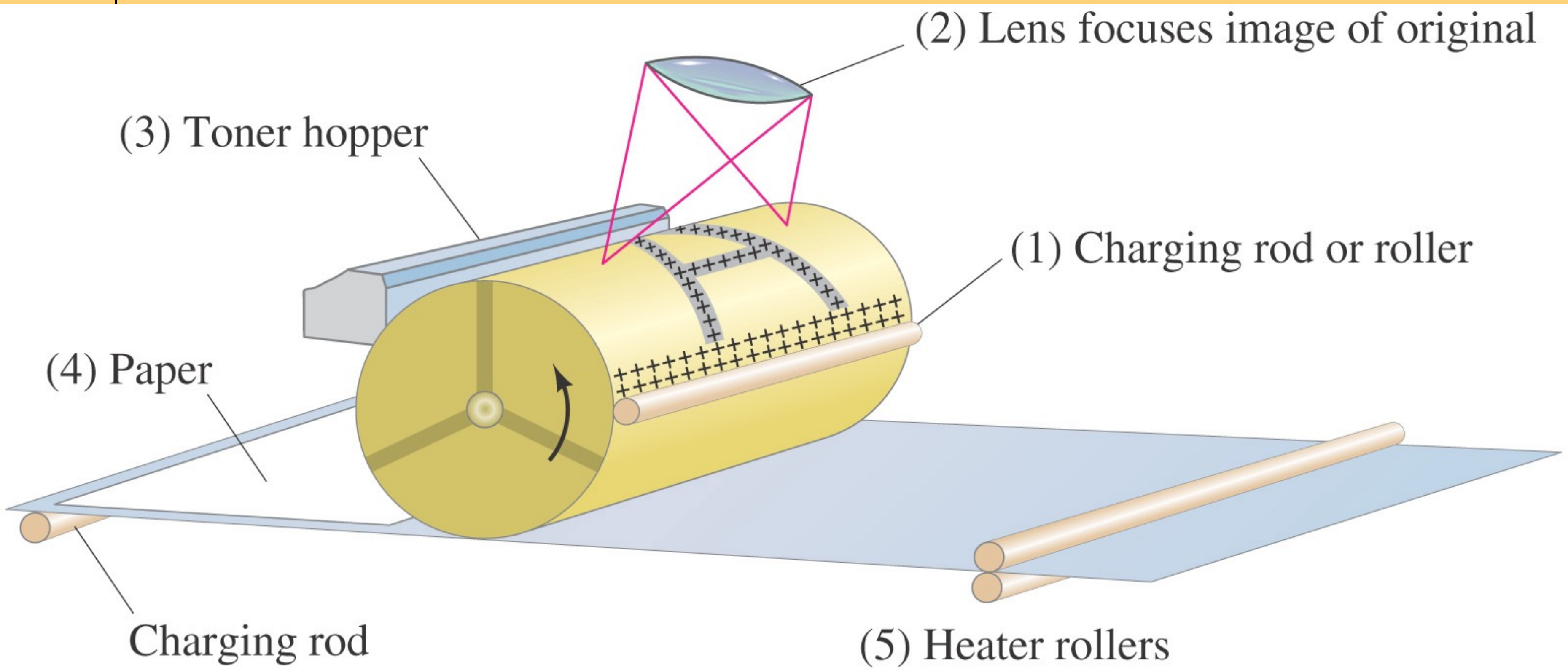
$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$



F and E have the same direction if q is +, but opposite if q is -

Photocopy machine

- drum is charged positively (drum Al with a layer of selenium – photoconductivity)
- image is focused on drum
- only black areas stay charged and therefore attract toner particles
- image is transferred to paper and sealed by heat



Exercises

Ex. A photocopy machine works by arranging positive charges (in the pattern to be copied) on the surface of a drum, then gently sprinkling negatively charged toner particles onto the drum. The toner particles temporarily stick to the pattern on the drum and are later transferred to paper and melted to produce the copy. Suppose each toner particle has a mass of 9.0x10^(-16)kg and carries an average of 20 extra electrons to provide an electric charge. Assuming that the electric force on a toner particle must exceed twice its weight in order to ensure sufficient attraction, compute the required electric field strength near the surface of the drum.

$$qE = 2mg \Rightarrow E = 5.5 \times 10^3 \, N / C$$

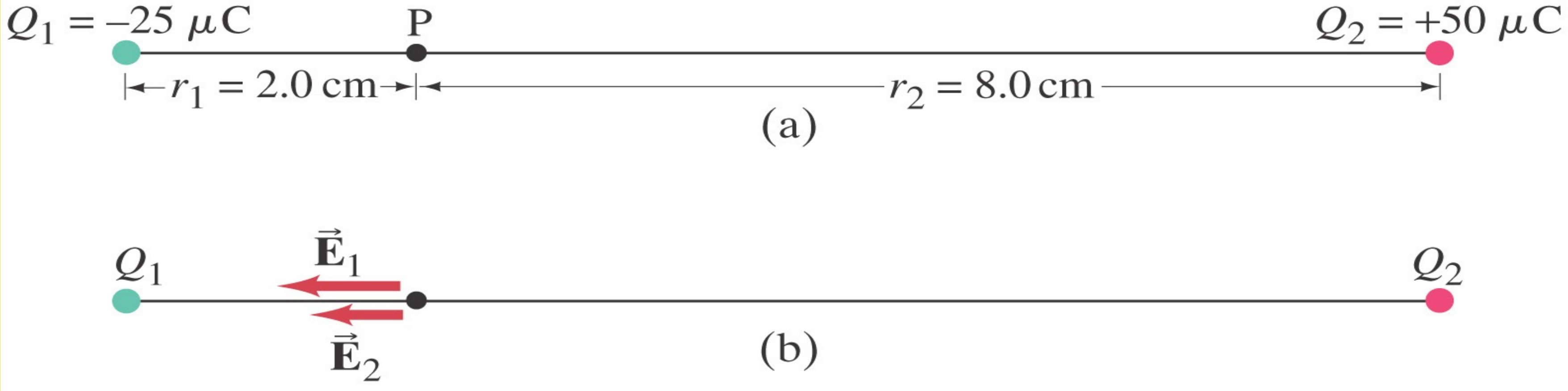
Ex. Calculate the magnitude and direction of the electric field at a point P which is 30 cm to the right of a point charge Q=-3.0x10^(-6) C.

$$E = k\frac{Q}{r^2} \Rightarrow E = 3.0 \times 10^5 \, N / C$$

Exercises

Superposition principle for electric fields: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \cdots$

Ex. Two point charges are separated by a distance of 10.0 cm. One has a charge of $-25\mu C$ and the other $+50\mu C$. (a) Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0 cm from the negative charge. (b) If an electron is placed at rest at P and then released, what will be its initial acceleration (direction and magnitude)?



Field due to **negative** charge points **TO** it
Field due to **positive** charge points **AWAY** from it

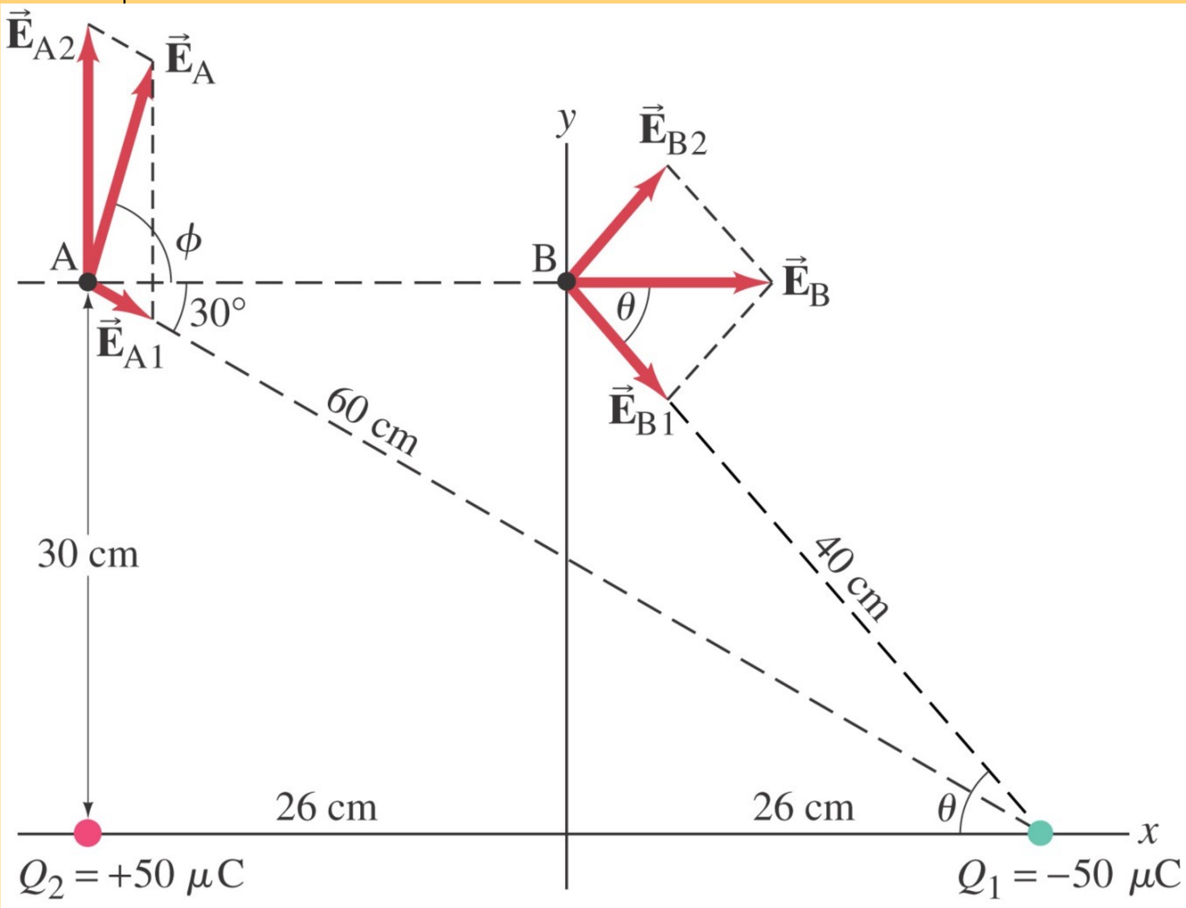
$m_e = 9.11 \times 10^{-31} kg$

$E = 6.3 \times 10^{8} N/C$

$a = 1.1 \times 10^{20} m/s^2$

Exercise

Ex. Calculate the total electric field (a) at point A and (b) point B due to both charges.



$$E_{A1}, E_{A2}, E_{Ax}, E_{Ay}$$

$$E_A = 4.5\times10^6\ N/C$$

$$\phi = 76^o$$

$$E_{B1} = E_{B2}$$

$$E_B = 2E_{B1}\cos\theta$$

$$E_B = 3.6\times10^6\ N/C$$

Net electric field due to three charged particles

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

KEY IDEA

Charges q_1, q_2, and q_3 produce electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, respectively, at the origin, and the net electric field is the vector sum $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. To find this sum, we first must find the magnitudes and orientations of the three field vectors.

Magnitudes and directions: To find the magnitude of $\vec{E}_1$, which is due to q_1, we use Eq. 22-3, substituting d for r and $2Q$ for q and obtaining

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

Similarly, we find the magnitudes of $\vec{E}_2$ and $\vec{E}_3$ to be

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} \quad \text{and} \quad E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}.$$

We next must find the orientations of the three electric field vectors at the origin. Because q_1 is a positive charge, the field vector it produces points directly *away* from it, and because q_2 and q_3 are both negative, the field vectors they produce point directly *toward* each of them. Thus, the three electric fields produced at the origin by the three charged particles are oriented as in Fig. 22-7*b*. (*Caution:* Note that we have placed the tails of the vectors at the point where the fields are to be evaluated; doing so decreases the chance of error. Error becomes very probable if the tails of the field vectors are placed on the particles creating the fields.)

Adding the fields: We can now add the fields vectorially just as we added force vectors in Chapter 21. However, here we can use symmetry to simplify the procedure. From Fig. 22-7*b*, we see that electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same direction. Hence, their vector sum has that direction and has the magnitude

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

which happens to equal the magnitude of field $\vec{E}_3$.

We must now combine two vectors, $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$, that have the same magnitude and that are oriented symmetrically about the x axis, as shown in Fig. 22-7*c*. From the symmetry of Fig. 22-7*c*, we realize that the equal y components of our two vectors cancel (one is upward and the other is downward) and the equal x components add (both are rightward). Thus, the net electric field $\vec{E}$ at the origin is in the positive direction of the x axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30° = (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}. \qquad \text{(Answer)}$$

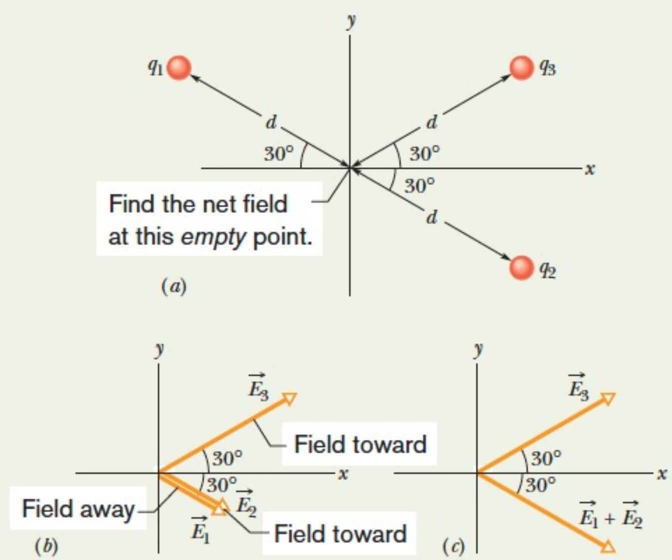


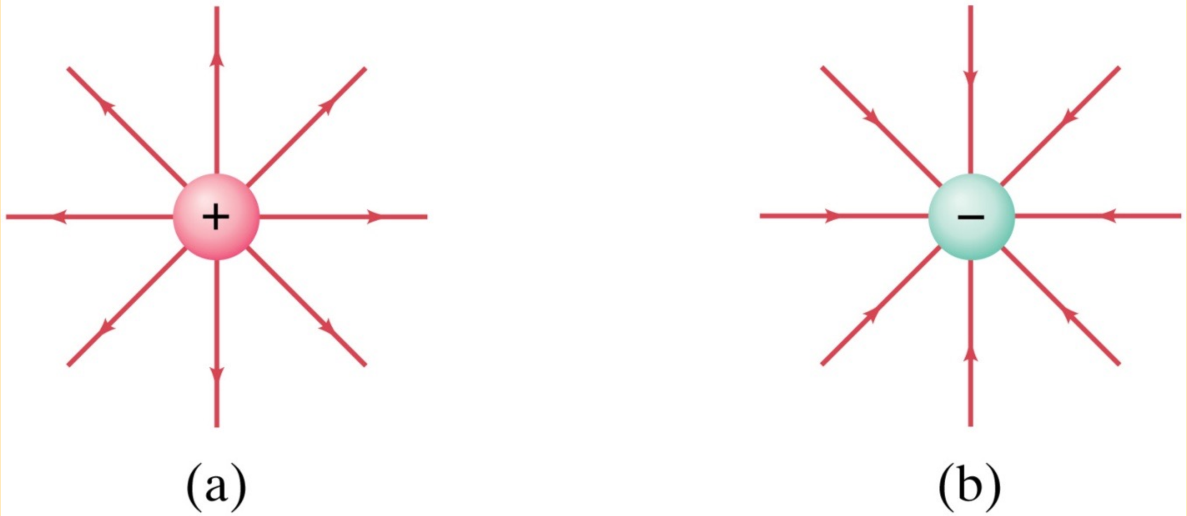
Fig. 22-7 (*a*) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (*b*) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (*c*) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

Field Lines

The electric field can be represented by field lines. These lines start on a positive charge and end on a negative charge.

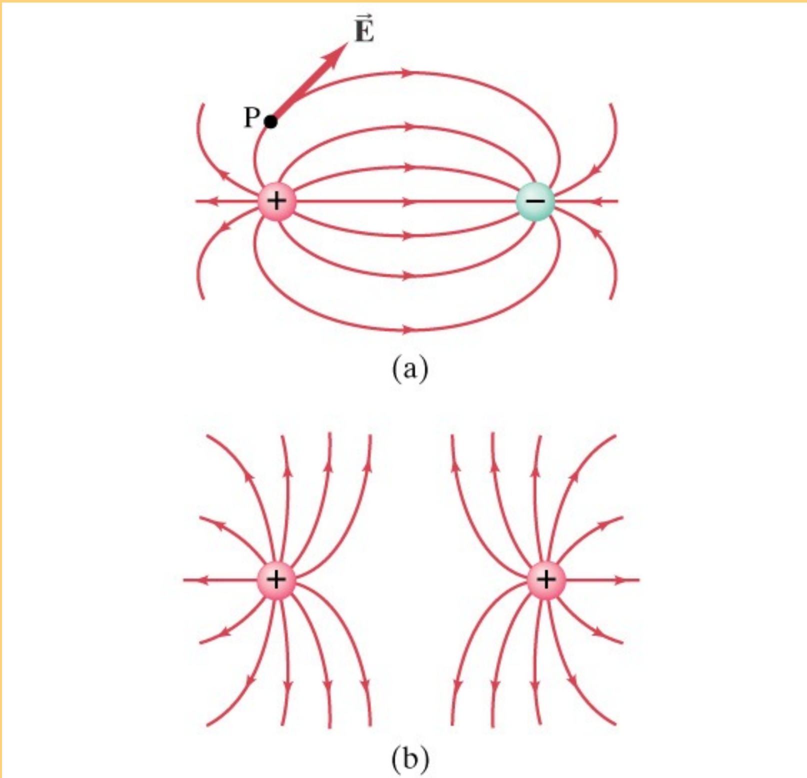
The number of field lines starting (ending) on a positive (negative) charge is proportional to the magnitude of the charge.

The electric field is stronger where the field lines are closer together.



Field Lines

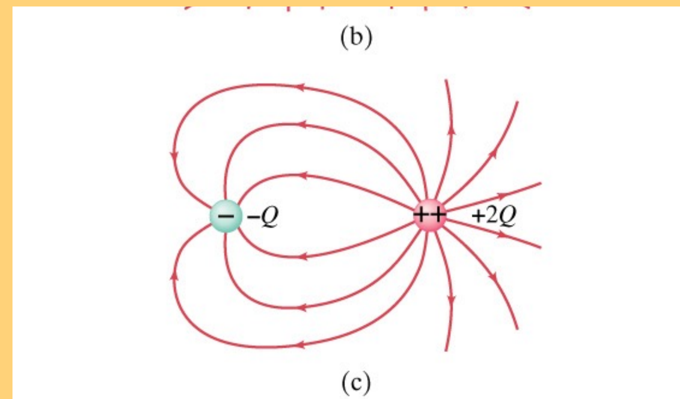
Electric dipole: two equal charges, opposite in sign:



(a)

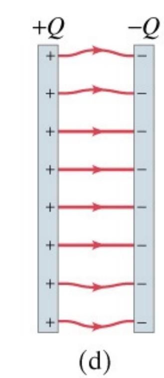
(b)

The direction of the electric field at any point is tangent to the field line.



(b)

(c)

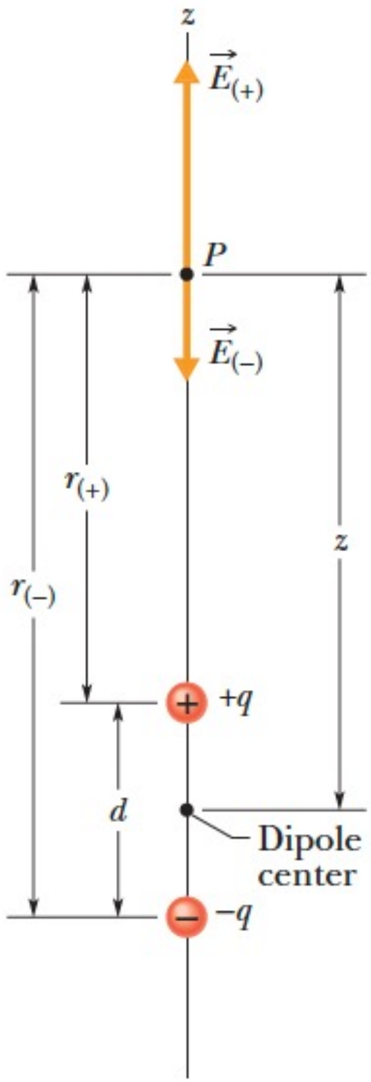


(d)

The electric field between two closely spaced, oppositely charged parallel plates is constant.

Positive test charge between plates feel repulsion from positive plate and attraction from negative plate

Electric Field due to Electric Dipole



$$E = E_{(+)} - E_{(-)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$

$$= \frac{q}{4\pi\varepsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z + \frac{1}{2}d)^2}.$$

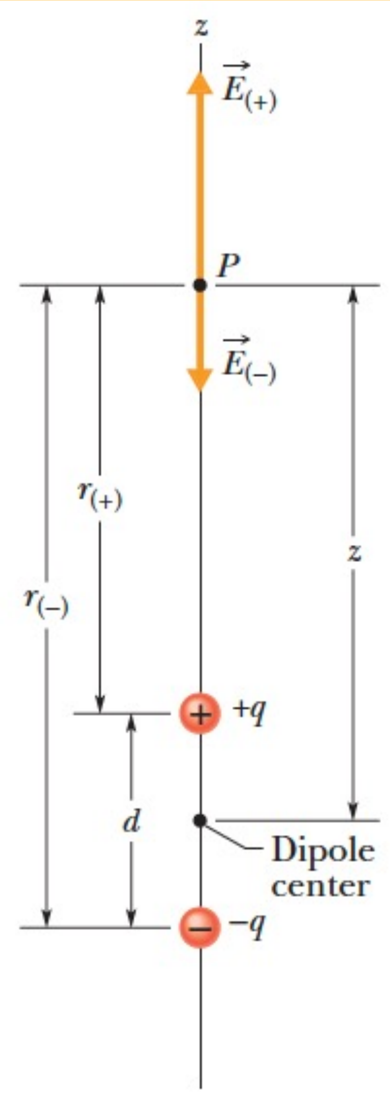
$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2}\right).$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}$$

We are usually interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole—that is, at distances such that $z \gg d$. At such large distances, we have $d/2z \ll 1$ in Eq. 22-7. Thus, in our approximation, we can neglect the $d/2z$ term in the denominator, which leaves us with

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}. \quad (22\text{-}8)$$

Electric Field due to Electric Dipole



$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}$$

The product qd, which involves the two intrinsic properties q and d of the dipole, is the magnitude p of a vector quantity known as the **electric dipole moment** $\vec{p}$ of the dipole. (The unit of $\vec{p}$ is the coulomb-meter.) Thus, we can write Eq. 22-8 as

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

E for a dipole varies as $1/r^3$ for all distant points, regardless of whether they lie on the dipole axis; here r is the distance between the point in question and the dipole center.

Electric dipole and atmospheric sprites

Sprites (Fig. 22-9*a*) are huge flashes that occur far above a large thunderstorm. They were seen for decades by pilots flying at night, but they were so brief and dim that most pilots figured they were just illusions. Then in the 1990s sprites were captured on video. They are still not well understood but are believed to be produced when especially powerful lightning occurs between the ground and storm clouds, particularly when the lightning transfers a huge amount of negative charge $-q$ from the ground to the base of the clouds (Fig. 22-9*b*).

Just after such a transfer, the ground has a complicated distribution of positive charge. However, we can model the electric field due to the charges in the clouds and the ground by assuming a vertical electric dipole that has charge $-q$ at cloud height h and charge $+q$ at below-ground depth h (Fig. 22-9*c*). If $q = 200$ C and $h = 6.0$ km, what is the magnitude of the dipole's electric field at altitude $z_1 = 30$ km somewhat above the clouds and altitude $z_2 = 60$ km somewhat above the stratosphere?

KEY IDEA

We can approximate the magnitude E of an electric dipole's electric field on the dipole axis with Eq. 22-8.

Calculations: We write that equation as

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q(2h)}{z^3},$$

where $2h$ is the separation between $-q$ and $+q$ in Fig. 22-9*c*. For the electric field at altitude $z_1 = 30$ km, we find

$$E = \frac{1}{2\pi\varepsilon_0}\frac{(200\text{ C})(2)(6.0\times10^3\text{ m})}{(30\times10^3\text{ m})^3}$$
$$= 1.6\times10^3\text{ N/C}. \qquad \text{(Answer)}$$

Similarly, for altitude $z_2 = 60$ km, we find

$$E = 2.0\times10^2\text{ N/C}. \qquad \text{(Answer)}$$

As we discuss in Section 22-8, when the magnitude of an electric field exceeds a certain critical value E_c, the field can pull electrons out of atoms (ionize the atoms), and then the freed electrons can run into other atoms, causing those atoms to emit light. The value of E_c depends on the density of the air in which the electric field exists. At altitude $z_2 = 60$ km the density of the air is so low that $E = 2.0\times10^2$ N/C exceeds E_c, and thus light is emitted by the atoms in the air. That light forms sprites. Lower down, just above the clouds at $z_1 = 30$ km, the density of the air is much higher, $E = 1.6\times10^3$ N/C does not exceed E_c, and no light is emitted. Hence, sprites occur only far above storm clouds.

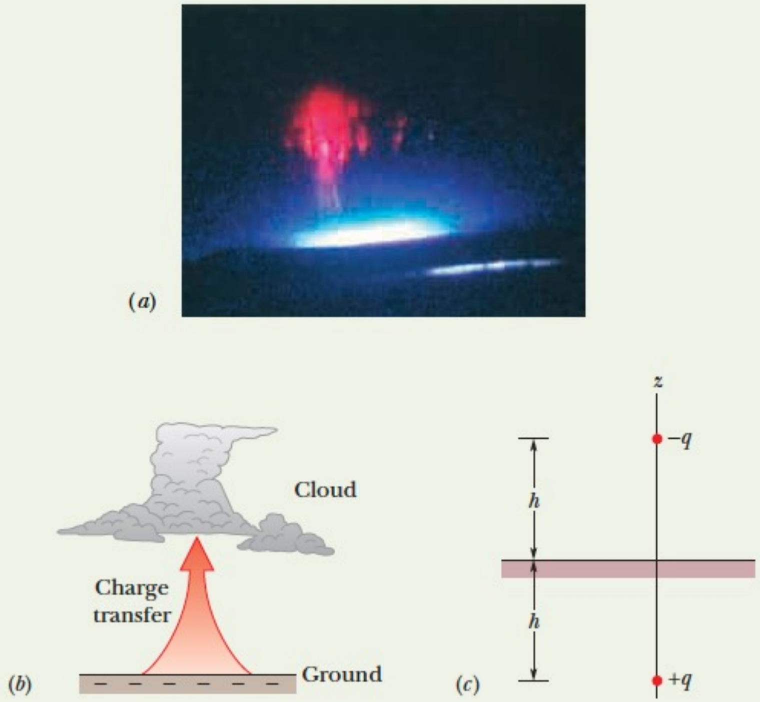
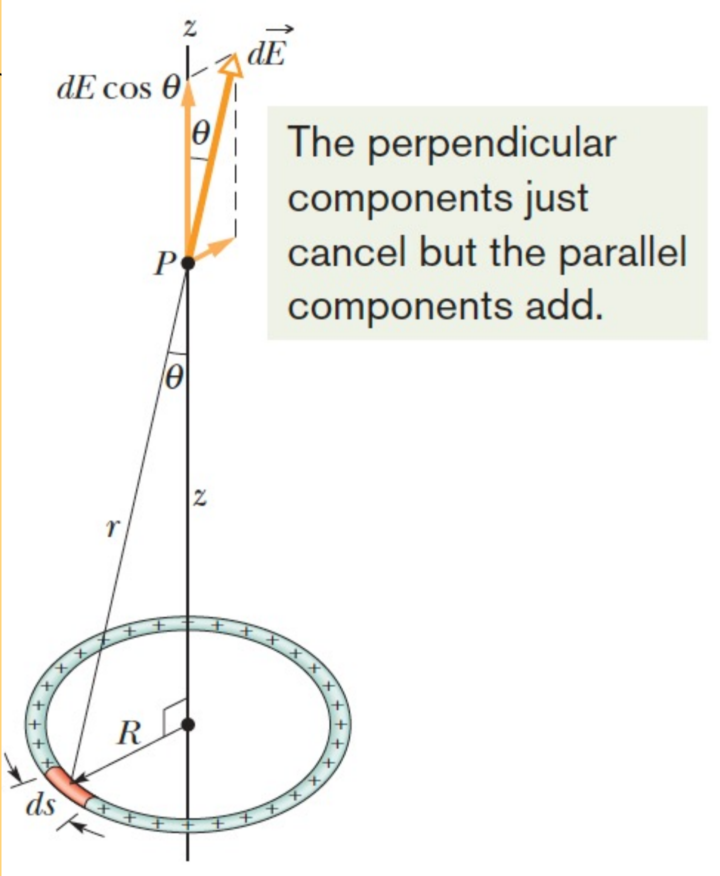


Fig. 22-9 (*a*) Photograph of a sprite. (*Courtesy NASA*) (*b*) Lightning in which a large amount of negative charge is transferred from ground to cloud base. (*c*) The cloud–ground system modeled as a vertical electric dipole.

Electric Field due to Line of Charge



Linear charge density λ

$$dq = \lambda\, ds$$

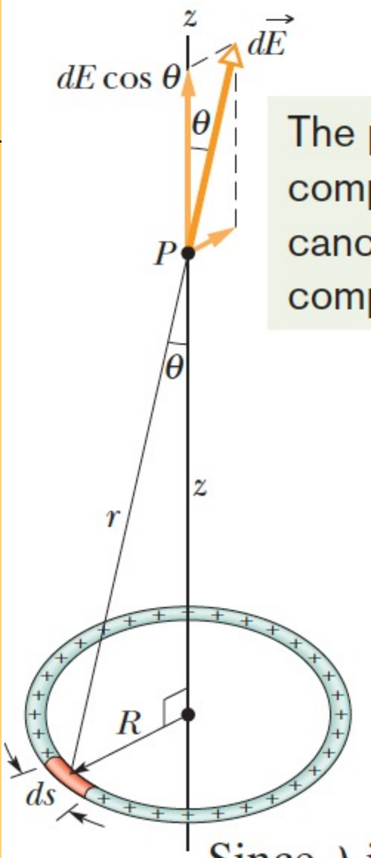
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}$$

All the components of dE that are perpendicular to the z-axis cancel! Only thos along the z-axis survive

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}$$

Electric Field due to Line of Charge



The perpendicular components just cancel but the parallel components add.

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}$$

$$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\,ds.$$

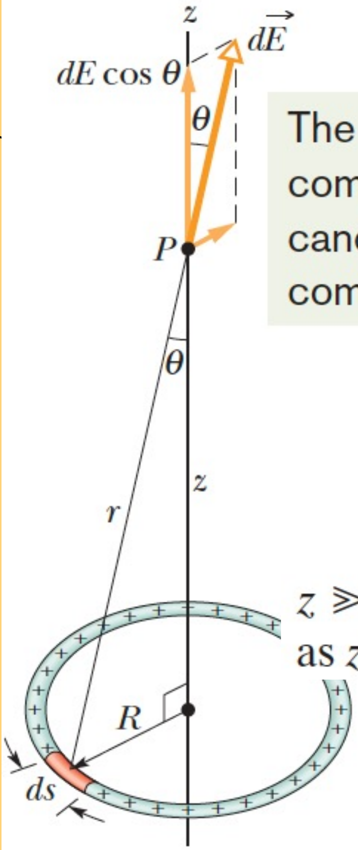
$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}.$$

Since λ is the charge per length of the ring, the term $\lambda(2\pi R)$ in Eq. 22-15 is q, the total charge on the ring. We then can rewrite Eq. 22-15 as

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \qquad \text{(22-16)}$$

Electric Field due to Line of Charge



The perpendicular components just cancel but the parallel components add.

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \quad (22\text{-}16)$$

If the charge on the ring is negative, instead of positive as we have assumed, the magnitude of the field at P is still given by Eq. 22-16. However, the electric field vector then points toward the ring instead of away from it.

Let us check Eq. 22-16 for a point on the central axis that is so far away that $z \gg R$. For such a point, the expression $z^2 + R^2$ in Eq. 22-16 can be approximated as z^2, and Eq. 22-16 becomes

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2} \quad \text{(charged ring at large distance)}$$

This is a reasonable result because from a large distance, the ring "looks like" a point charge.

Electric field of a charged circular rod

Figure 22-11*a* shows a plastic rod having a uniformly distributed charge $-Q$. The rod has been bent in a 120° circular arc of radius r. We place coordinate axes such that the axis of symmetry of the rod lies along the x axis and the origin is at the center of curvature P of the rod. In terms of Q and r, what is the electric field $\vec{E}$ due to the rod at point P?

KEY IDEA

Because the rod has a continuous charge distribution, we must find an expression for the electric fields due to differential elements of the rod and then sum those fields via calculus.

An element: Consider a differential element having arc length ds and located at an angle θ above the x axis (Figs. 22-11*b* and *c*). If we let λ represent the linear charge density of the rod, our element ds has a differential charge of magnitude

$$dq = \lambda\, ds. \tag{22-18}$$

The element's field: Our element produces a differential electric field $d\vec{E}$ at point P, which is a distance r from the element. Treating the element as a point charge, we can rewrite Eq. 22-3 to express the magnitude of $d\vec{E}$ as

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}. \tag{22-19}$$

The direction of $d\vec{E}$ is toward ds because charge dq is negative.

Symmetric partner: Our element has a symmetrically located (mirror image) element ds' in the bottom half of the rod. The electric field $d\vec{E}'$ set up at P by ds' also has the magnitude given by Eq. 22-19, but the field vector points toward ds' as shown in Fig. 22-11*d*. If we resolve the electric field vectors of ds and ds' into x and y components as shown in Figs. 22-11*e* and *f*, we see that their y components cancel (because they have equal magnitudes and are in opposite directions). We also see that their x components have equal magnitudes and are in the same direction.

Summing: Thus, to find the electric field set up by the rod, we need sum (via integration) only the x components of the differential electric fields set up by all the differential elements of the rod. From Fig. 22-11*f* and Eq. 22-19, we can write the component dE_x set up by ds as

$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, ds. \tag{22-20}$$

Equation 22-20 has two variables, θ and s. Before we can integrate it, we must eliminate one variable. We do so by replacing ds, using the relation

$$ds = r\, d\theta,$$

in which $d\theta$ is the angle at P that includes arc length ds (Fig. 22-11*g*). With this replacement, we can integrate Eq. 22-20 over the angle made by the rod at P, from $\theta = -60°$ to $\theta = 60°$; that will give us the magnitude of the electric field at P due to the rod:

$$\begin{aligned} E &= \int dE_x = \int_{-60°}^{60°}\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, r\, d\theta \\ &= \frac{\lambda}{4\pi\varepsilon_0 r}\int_{-60°}^{60°}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-60°}^{60°} \\ &= \frac{\lambda}{4\pi\varepsilon_0 r}[\sin 60° - \sin(-60°)] \\ &= \frac{1.73\lambda}{4\pi\varepsilon_0 r}. \end{aligned} \tag{22-21}$$

(If we had reversed the limits on the integration, we would have gotten the same result but with a minus sign. Since the integration gives only the magnitude of $\vec{E}$, we would then have discarded the minus sign.)

Charge density: To evaluate λ, we note that the rod subtends an angle of 120° and so is one-third of a full circle. Its arc length is then $2\pi r/3$, and its linear charge density must be

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477Q}{r}.$$

Substituting this into Eq. 22-21 and simplifying give us

$$\begin{aligned} E &= \frac{(1.73)(0.477Q)}{4\pi\varepsilon_0 r^2} \\ &= \frac{0.83Q}{4\pi\varepsilon_0 r^2}. \end{aligned} \qquad \text{(Answer)}$$

The direction of $\vec{E}$ is toward the rod, along the axis of symmetry of the charge distribution. We can write $\vec{E}$ in unit-vector notation as

$$\vec{E} = \frac{0.83Q}{4\pi\varepsilon_0 r^2}\,\hat{\mathrm{i}}.$$

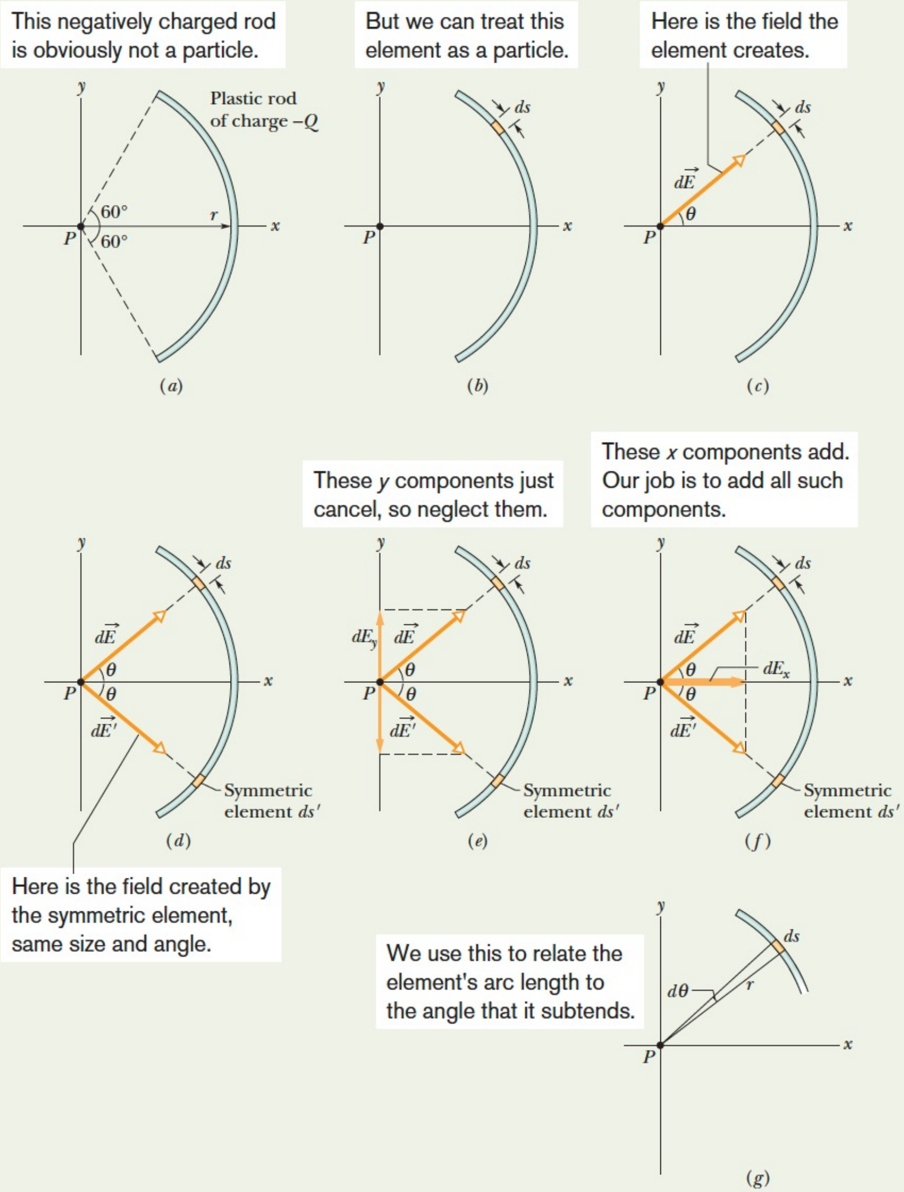


Fig. 22-11 (*a*) A plastic rod of charge $-Q$ is a circular section of radius r and central angle 120°; point P is the center of curvature of the rod. (*b*)–(*c*) A differential element in the top half of the rod, at an angle θ to the x axis and of arc length ds, sets up a differential electric field $d\vec{E}$ at P. (*d*) An element ds', symmetric to ds about the x axis, sets up a field $d\vec{E}'$ at P with the same magnitude. (*e*)–(*f*) The field components. (*g*) Arc length ds makes an angle $d\theta$ about point P.

Electric Field due to Charged Disk

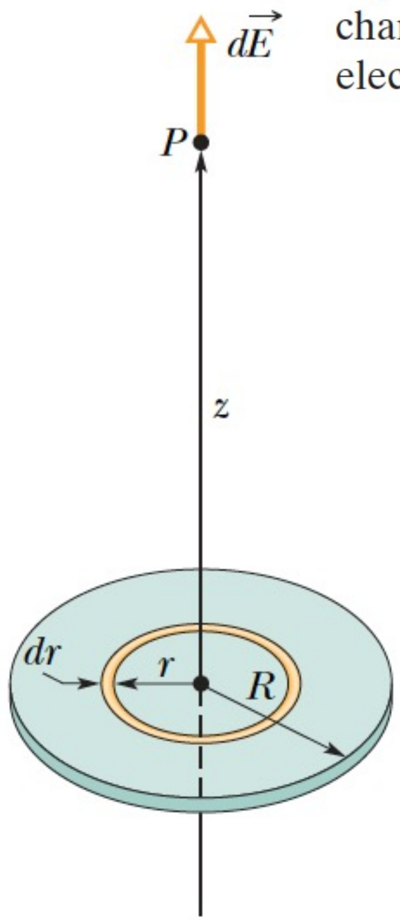


Fig. 22-13

Figure 22-13 shows a circular plastic disk of radius R that has a positive surface charge of uniform density σ on its upper surface (see Table 22-2). What is the electric field at point P, a distance z from the disk along its central axis?

This is what we found for the ring

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \qquad (22\text{-}16)$$

The disk is equivalent to a ring where r goes from 0 to R, so we can substitute "q" above by

$$dq = \sigma\, dA = \sigma(2\pi r\, dr)$$

$$dE = \frac{z\sigma 2\pi r\, dr}{4\pi\varepsilon_0(z^2 + r^2)^{3/2}}$$

$$dE = \frac{\sigma z}{4\varepsilon_0}\frac{2r\, dr}{(z^2 + r^2)^{3/2}}$$

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0}\int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr$$

Electric Field due to Charged Disk

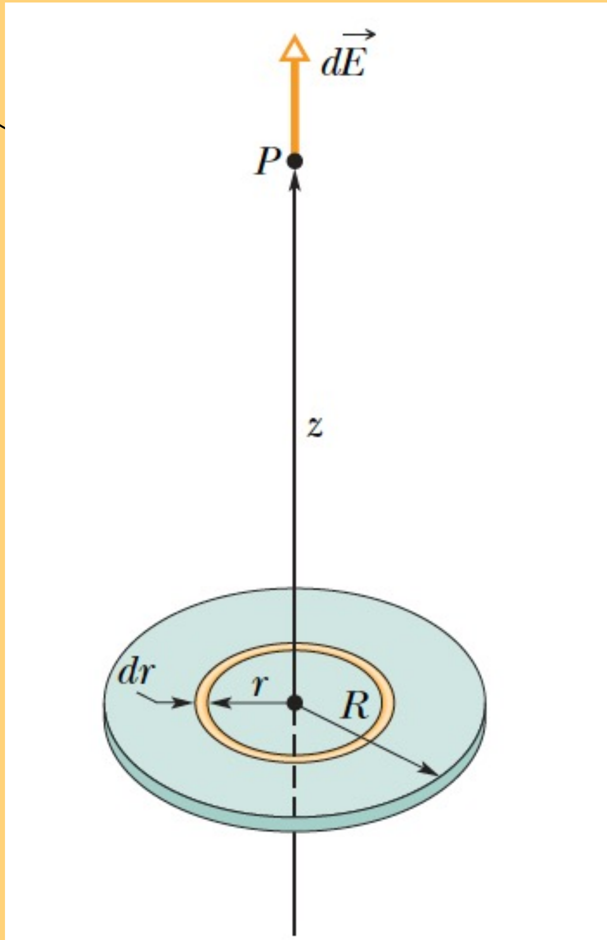


Fig. 22-13

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0} \int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr$$

To solve, we change variables as

$$X = (z^2 + r^2) \qquad dX = (2r)\, dr$$

$$\int X^m\, dX = \frac{X^{m+1}}{m+1}$$

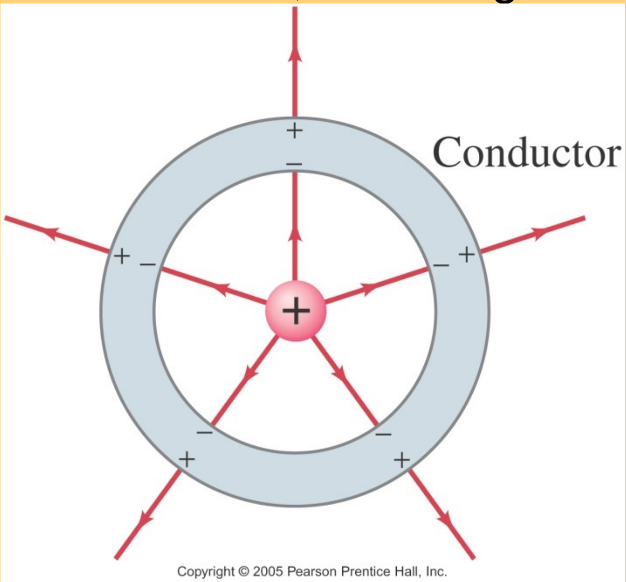
$$E = \frac{\sigma z}{4\varepsilon_0} \left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}} \right]_0^R$$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \qquad (22\text{-}26)$$

Electric Fields and Conductors

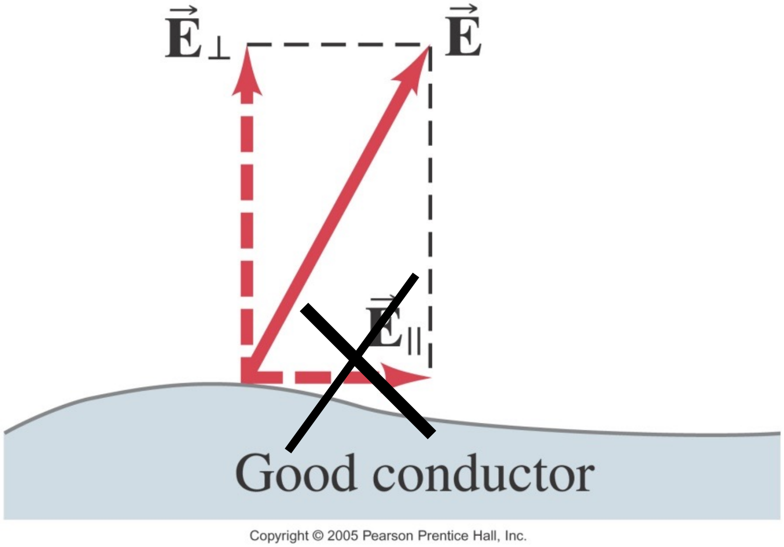
The static electric field inside a conductor is zero – if it were not, the charges would move.

The net charge on a conductor is on its surface.



Suppose a positive charge Q is surrounded by an isolated uncharged metal conductor whose shape is a spherical shell.
There can be no field inside the metal, so –Q is induced on the inner surface and +Q exists on the outer surface.
The shell is neutral

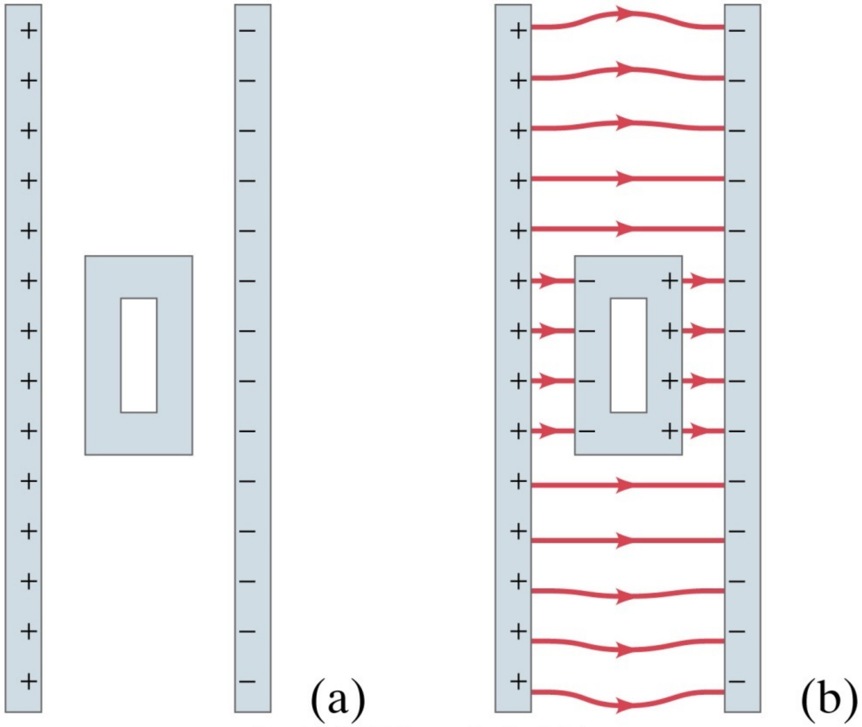
The electric field is perpendicular to the surface of a conductor – if it were not, charges would move.



Faraday Cage

Shielding and safety in a storm

A neutral hollow metal box is placed between two parallel charged plates as in (a). What is the field like inside the box?



The electrons in the metal can move freely to the surface, hence the field inside the hollow metal box is zero. A conducting box used in this way is an effective device for shielding delicate instruments from unwanted external electric fields.

It also explains why it is safe to be inside a car during a lightning storm.

Problems to Solve

Chapter 22:
8, 9, 11, 15, 19, 21, 24, 25, 35, 40, 46