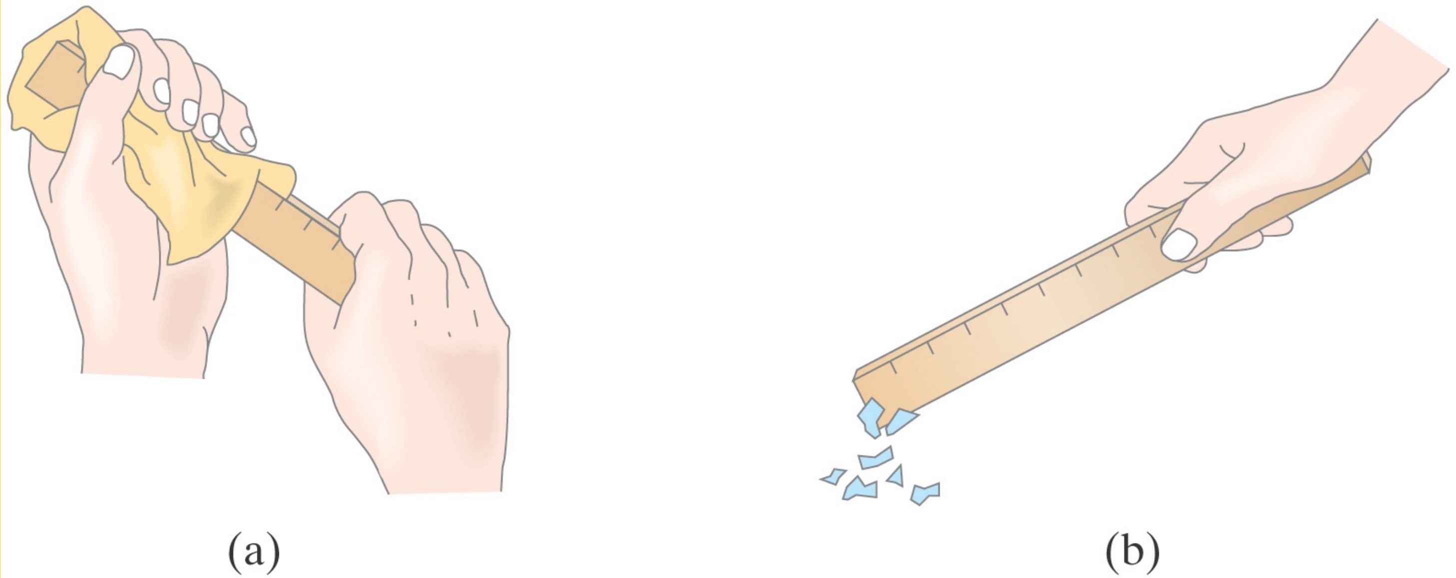
Electric Charge and Force

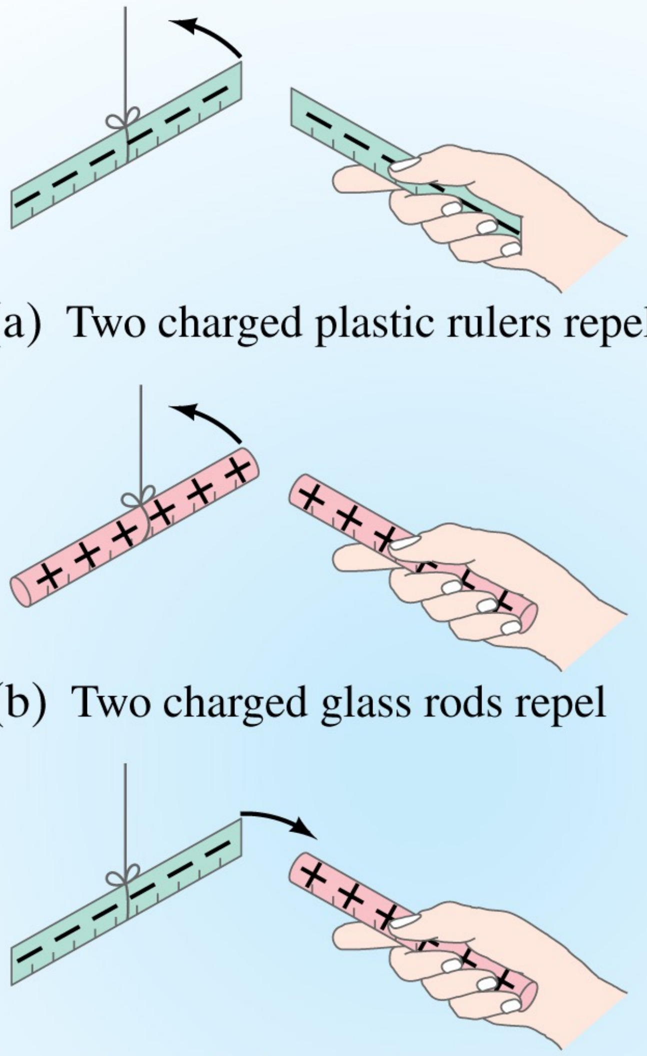
Electric Forces between atoms and molecules hold them together to form liquids and solids

Static Electricity – from Greek: *elektron*=amber (petrified tree resin)
Examples: rubbed plastic ruler, combing hair, synthetic blouse, shock from metal doorknob after sliding over a nylon carpet

Rubbed object possesses a net **electric charge**



Electric Charge



(a) Two charged plastic rulers repel

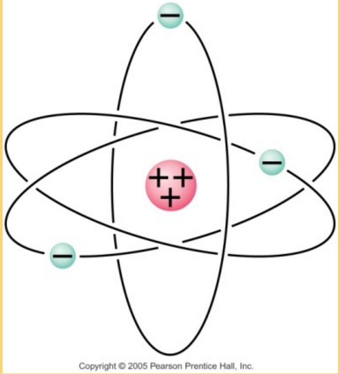
(b) Two charged glass rods repel

(c) Charged glass rod attracts charged plastic ruler

Charge comes in two types, **positive and negative**; *like charges repel and opposite charges attract*

Law of conservation of electric charge
The net amount of electric charge produced in any process is zero

Ex: plastic rulers rubbed with a paper towel acquire negative charge while the towel acquires an equal amount of positive charge.

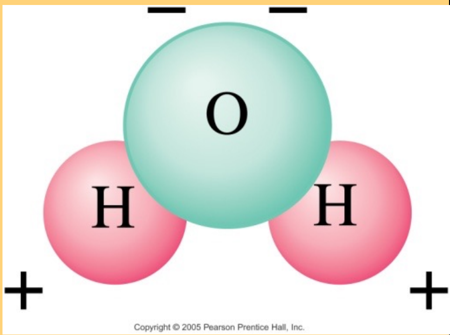


Atom:
Nucleus contains **protons (+)** and neutrons (no net electric charge) and is surrounded by **electrons (-)**
Neutral atom: no net charge
If it loses or gains electrons: **ions**

Insulators and Conductors

Plastic ruler becomes negatively charged by rubbing with a paper towel, that is, electrons are transferred from towel to ruler (towel becomes +)

Objects charged by rubbing return to neutral state, by leaking charge into water molecules in the air. This is more difficult in dry days. Water molecules are **polar**

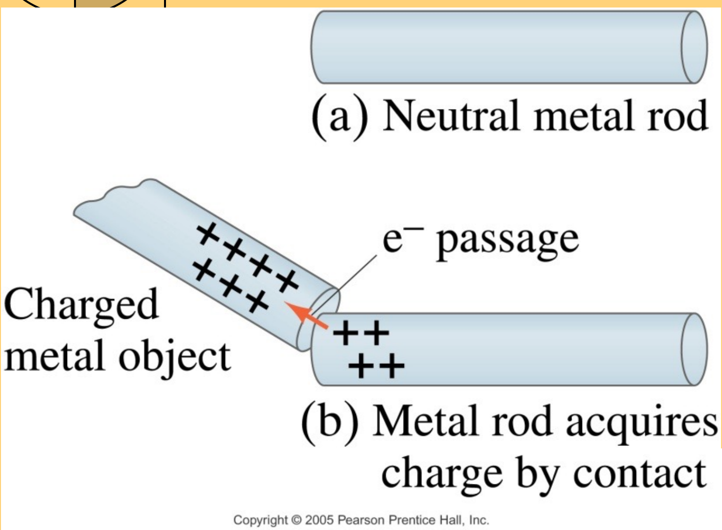


Metal are good **conductors**
Some of the electrons are bound very loosely and can move about freely within the material. *free electrons or conduction electrons*

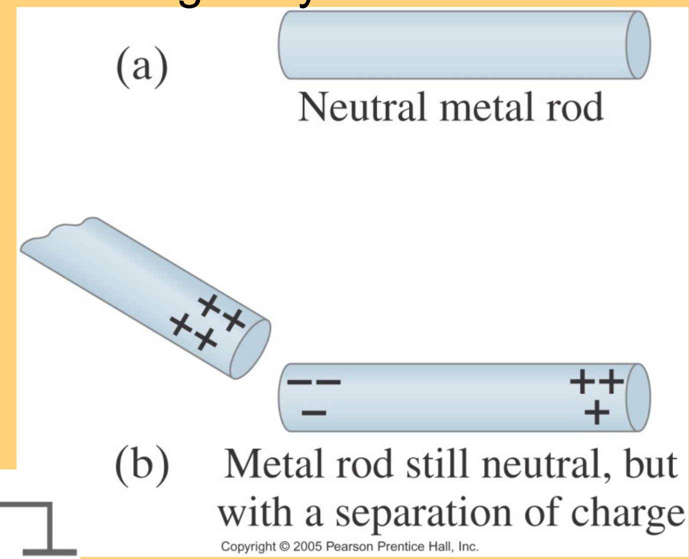
In **insulators** the electrons are bound very tightly to the nuclei.

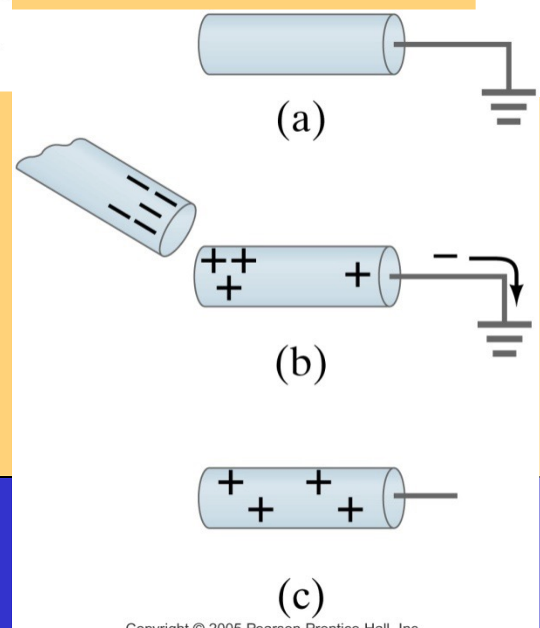
Induced Charge

Metal objects can be charged by **conduction**:



Metal objects can also be charged by **induction**:



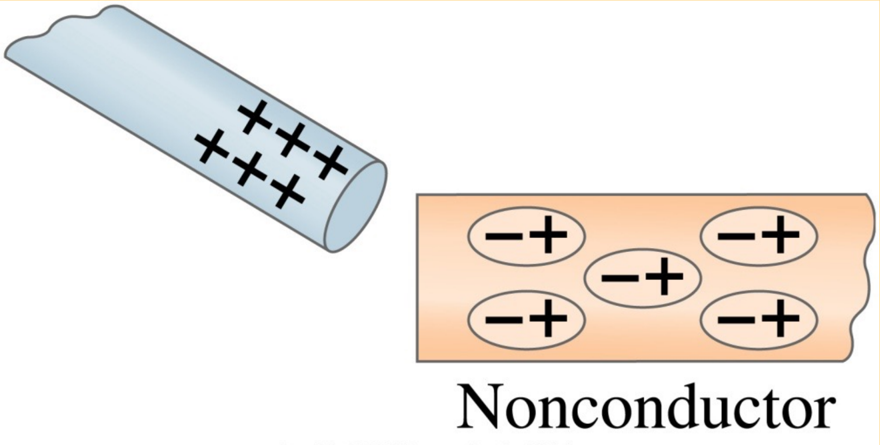


Here the metal is grounded, electrons leave the metal to Earth.

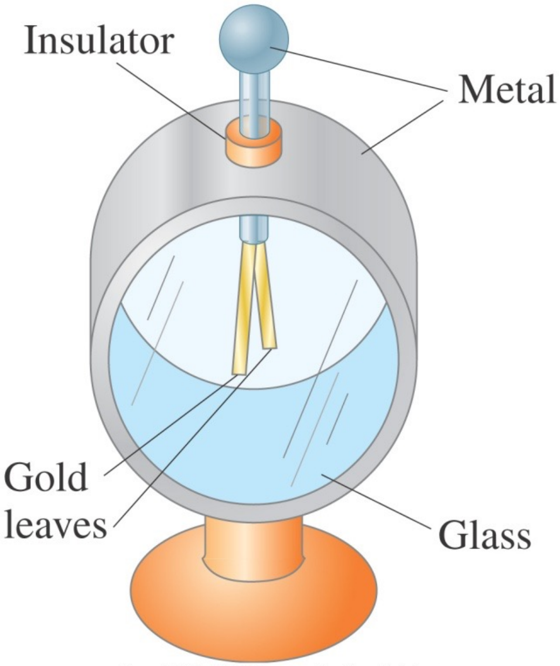
If the wire is cut, metal is positively charged

Charge Separation in nonconductors

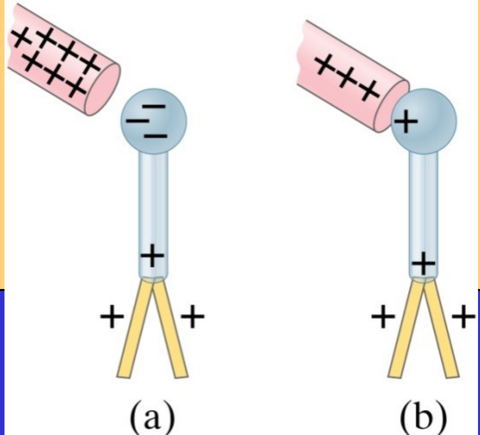
A charged object brought near an insulator causes a charge separation within the insulator's molecules.



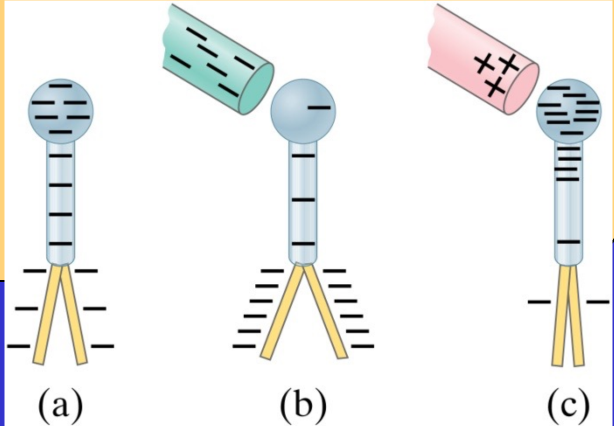
The **electroscope** can be used for detecting charge:



The electroscope can be charged either by conduction or by induction.

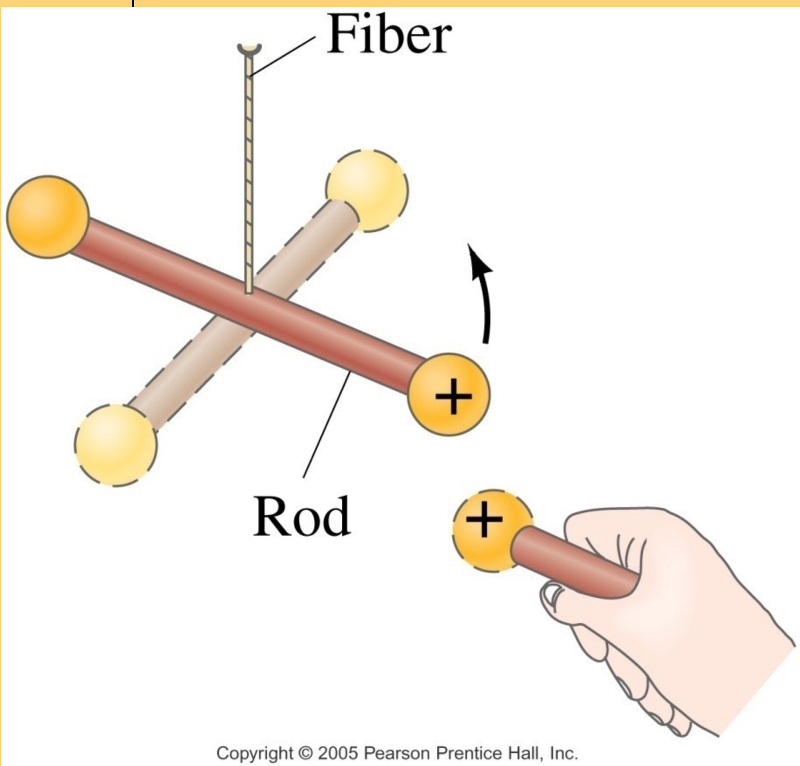


The charged electroscope can then be used to determine the sign of an unknown charge.



Coulomb's Law

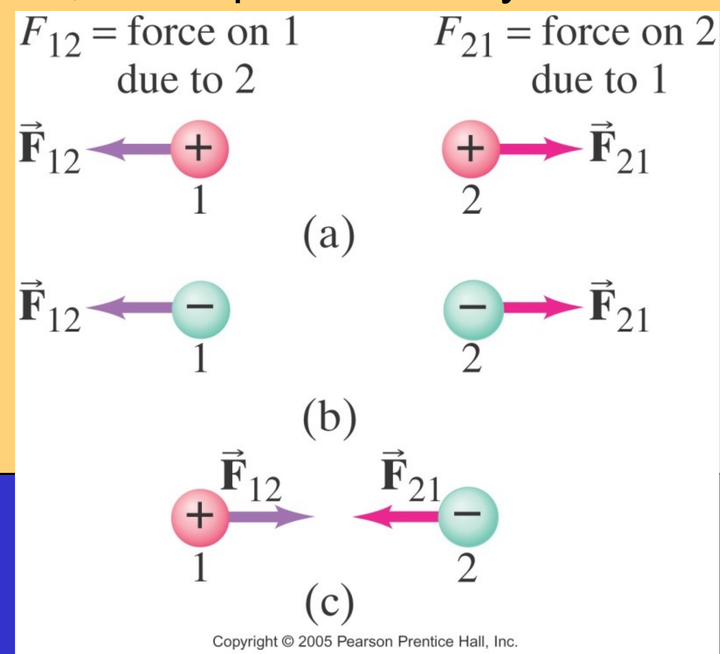
Experiment shows that the electric force between two charges is proportional to the product of the charges and inversely proportional to the distance between them.



$$F = k\frac{Q_1 Q_2}{r^2}$$

This equation gives the magnitude of the force.

The force is along the line connecting the charges, and is attractive if the charges are opposite, and repulsive if they are the same.



Coulomb's Law

$$F = k\frac{Q_1 Q_2}{r^2}$$

Unit of charge: coulomb, C

The proportionality constant in Coulomb's law is

$$k = 8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

We normally do not find charges as large as a coulomb. Charges produced by rubbing are typically around a microcoulomb: $1\ \mu\mathrm{C} = 10^{-6}\ \mathrm{C}$

Charge on the electron is the **smallest** charge found in nature and is equal to –e, where e is the *elementary charge* given by: $e = 1.602 \times 10^{-19}\ \mathrm{C}$

The charge on a proton is +e

Since an object cannot gain or lose a fraction of an electron, the net charge is an integral multiple of e -- electric charge is **quantized**

Coulomb's law looks a lot like the law of universal gravitation, but gravity is always attractive and the electric force can be either attractive or repulsive.

Coulomb's Law

The proportionality constant k can also be written in terms of ϵ_0, the permittivity of free space:

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}$$

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

Coulomb's law strictly applies to **point charges**.

For finite-sized objects, it is not always clear what value to use for r. If two objects are spheres and the charge is uniformly distributed in each, then r is the distance between their centers.

Coulomb's law describes the force between two charges when they are at rest -- **ELECTROSTATICS**

Additional forces come into play when the charges are in motion: future chapters

Ex. 16-1 Determine the magnitude and direction on the electric force on the electron of a hydrogen atom exerted by the single proton that is the atom's nucleus. Assume the average distance between the revolving electron and the proton is $r = 0.53 \times 10^{-10}\, m,\ Q = 1.6 \times 10^{-19}\, C$

$$F = 8.2 \times 10^{-8}\, N$$

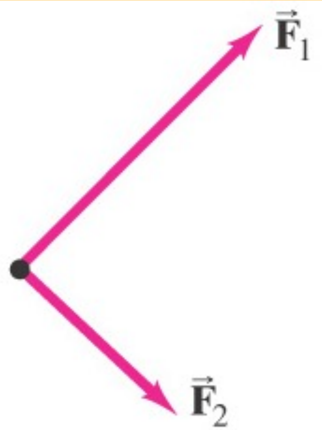
Principle of Superposition: for *multiple point charges*, the forces on each charge is the vector sum of the forces on that charge due to each of the others

Coulomb's Law: vectors

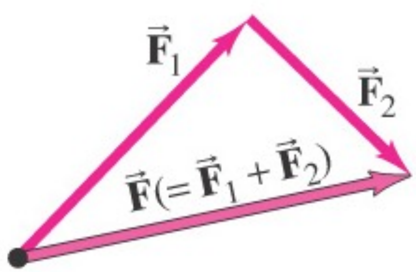
The net force on a charge is the vector sum of all the forces acting on it.

$$\vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots$$

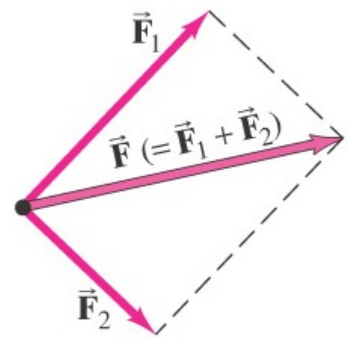
Vector addition review: tail-to-tip and parallelogram methods are good for getting a picture of what is going on.
For calculating the direction and magnitude of the resultant sum is better.



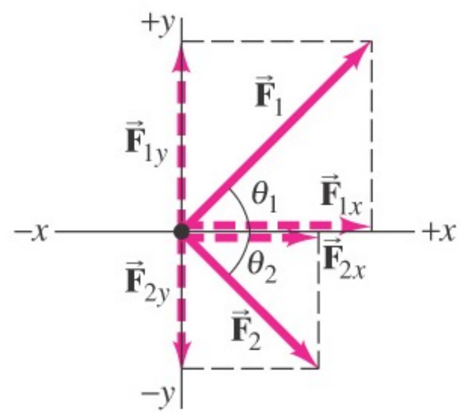
(a) Two forces acting on an object.



(b) The total, or net, force is $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ by the tail-to-tip method of adding vectors.



(c) $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ by the parallelogram method.



(d) $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ resolved into their x and y components.

$$F = \sqrt{F_x^2 + F_y^2}$$

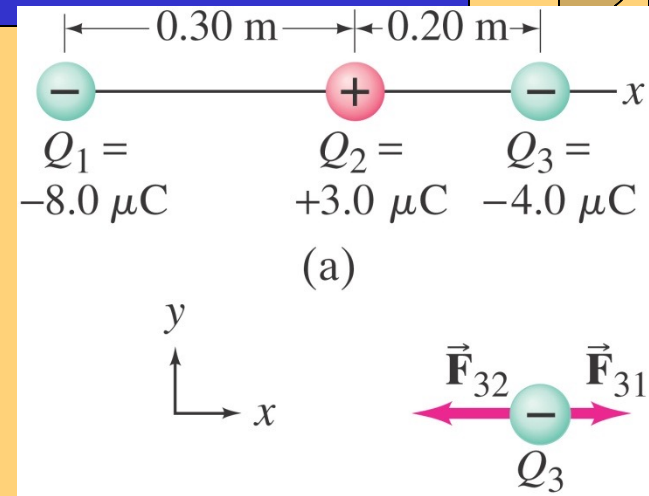
$$\tan\theta = \frac{F_y}{F_x}$$

Exercises

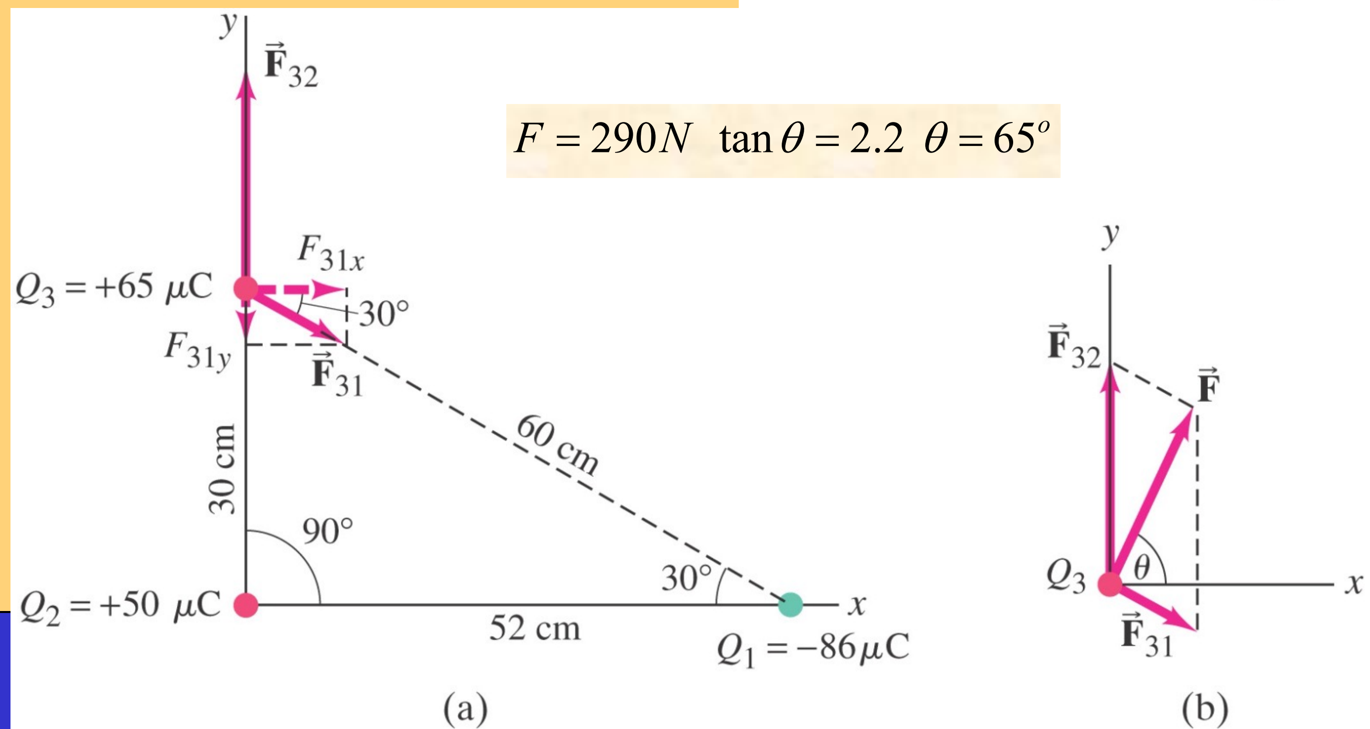
F_{31} = force exerted **ON** particle 3 **BY** particle 1

Ex. Three charged particles are arranged in a line, as in the figure. Calculate the net electrostatic force on particle 3 due to the other two charges.

F_{31}=1.2N F_{32}=2.7 N F=-1.5N



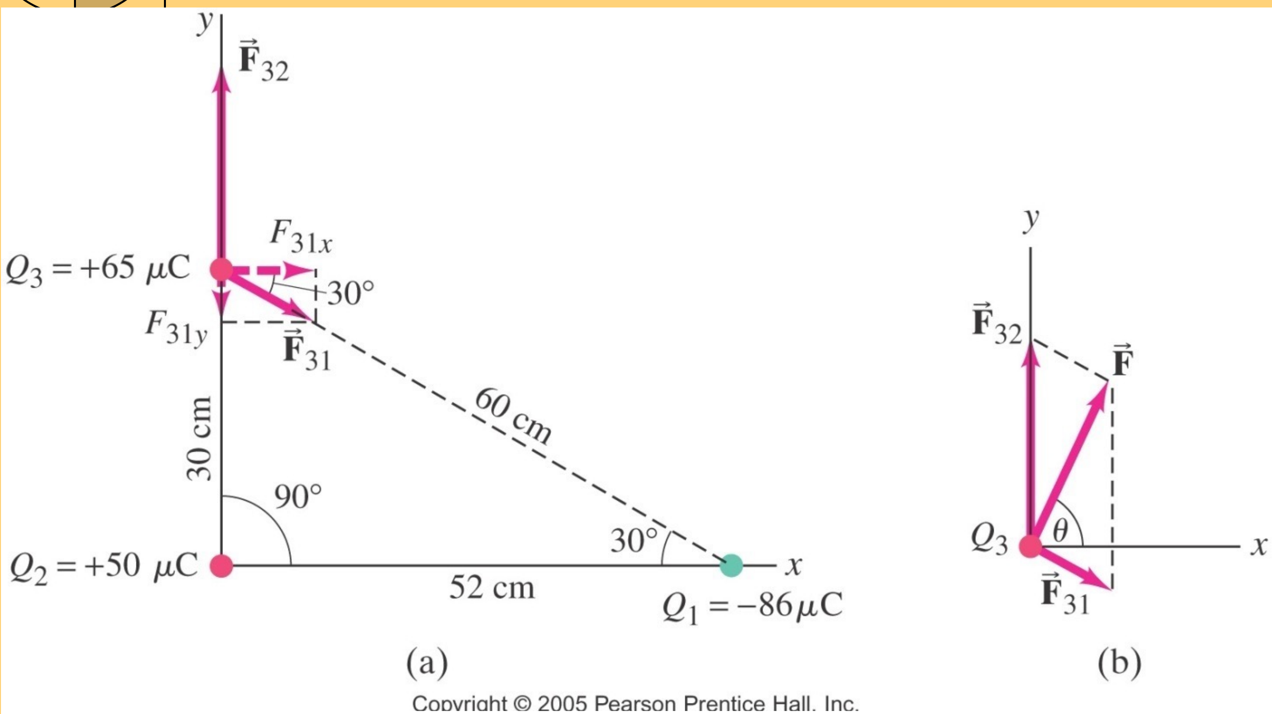
Ex. Calculate the net electrostatic force on charge Q_3 shown in the figure due to the charges Q_1 and Q_2



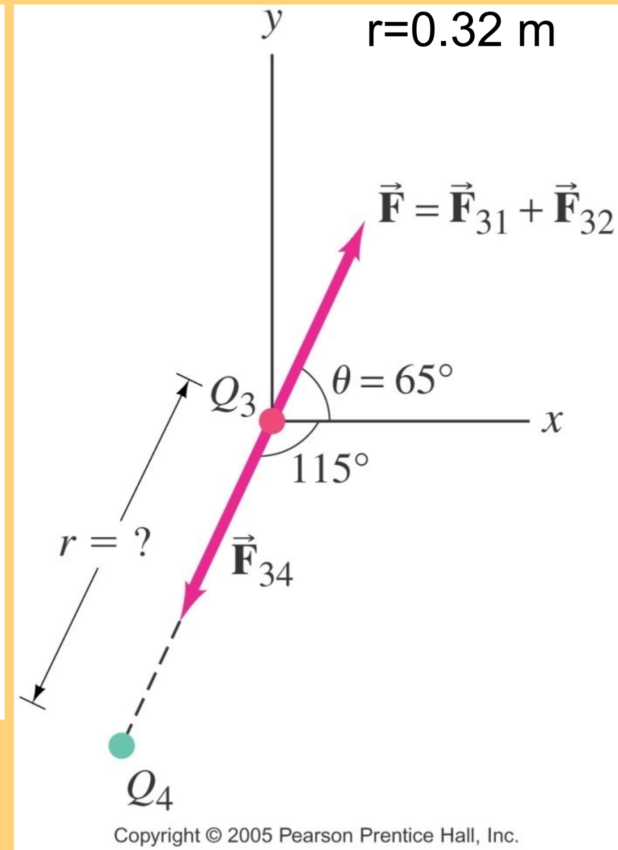
$F = 290N \quad \tan\theta = 2.2 \quad \theta = 65^o$

Exercises

Ex. Where could you place a fourth charge Q4 so that the net force on Q3 would be zero? What distance r must Q4 be from Q3? $Q_4 = -50\,\mu C$



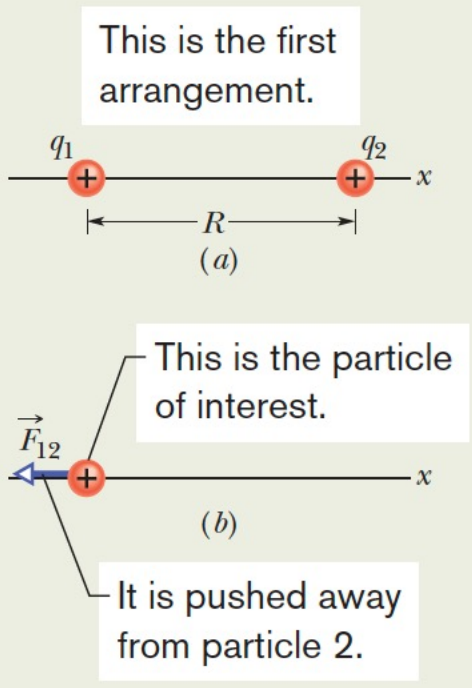
r=0.32 m



EX. 16-5 Consider two point charges of the same magnitude but opposite sign (+Q and –Q) fixed at a distance d apart. (a) Can you find a location where a 3rd positive charge could be placed so that the net electric force on this 3rd charge is zero? (b) What if the two charges were both +Q?

(a) NO (b) in the middle

(a) Figure 21-8*a* shows two positively charged particles fixed in place on an x axis. The charges are $q_1 = 1.60 \times 10^{-19}$ C and $q_2 = 3.20 \times 10^{-19}$ C, and the particle separation is $R = 0.0200$ m. What are the magnitude and direction of the electrostatic force $\vec{F}_{12}$ on particle 1 from particle 2?



Because both particles are positively charged, particle 1 is repelled by particle 2, with a force magnitude given by Eq. 21-4. Thus, the direction of force $\vec{F}_{12}$ on particle 1 is *away from* particle 2, in the negative direction of the x axis, as indicated in the free-body diagram of Fig. 21-8*b*.

Two particles: Using Eq. 21-4 with separation R substituted for r, we can write the magnitude F_{12} of this force as

$$
\begin{aligned}
F_{12} &= \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{R^2} \\
&= (8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2}) \\
&\quad \times \frac{(1.60 \times 10^{-19}\ \mathrm{C})(3.20 \times 10^{-19}\ \mathrm{C})}{(0.0200\ \mathrm{m})^2} \\
&= 1.15 \times 10^{-24}\ \mathrm{N}.
\end{aligned}
$$

Thus, force $\vec{F}_{12}$ has the following magnitude and direction (relative to the positive direction of the x axis):

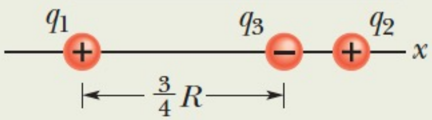
$$1.15 \times 10^{-24}\ \mathrm{N} \quad \text{and} \quad 180°. \qquad \text{(Answer)}$$

We can also write $\vec{F}_{12}$ in unit-vector notation as

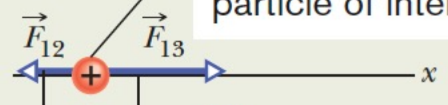
$$\vec{F}_{12} = -(1.15 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}}. \qquad \text{(Answer)}$$

(b) Figure 21-8*c* is identical to Fig. 21-8*a* except that particle 3 now lies on the x axis between particles 1 and 2. Particle 3 has charge $q_3 = -3.20 \times 10^{-19}$ C and is at a distance $\frac{3}{4}R$ from particle 1. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 3?

This is the second arrangement.



(*c*)



(*d*)

This is still the particle of interest.

It is pulled toward particle 3.

It is pushed away from particle 2.

The presence of particle 3 does not alter the electrostatic force on particle 1 from particle 2. Thus, force $\vec{F}_{12}$ still acts on particle 1. Similarly, the force $\vec{F}_{13}$ that acts on particle 1 due to particle 3 is not affected by the presence of particle 2. Because particles 1 and 3 have charge of opposite signs, particle 1 is attracted to particle 3. Thus, force $\vec{F}_{13}$ is directed *toward* particle 3, as indicated in the free-body diagram of Fig. 21-8*d*.

Three particles: To find the magnitude of $\vec{F}_{13}$, we can rewrite Eq. 21-4 as

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{(\frac{3}{4}R)^2}$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)$$

$$\times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2}$$

$$= 2.05 \times 10^{-24}\ \text{N}.$$

We can also write $\vec{F}_{13}$ in unit-vector notation:

$$\vec{F}_{13} = (2.05 \times 10^{-24}\ \text{N})\hat{\text{i}}.$$

The net force $\vec{F}_{1,\text{net}}$ on particle 1 is the vector sum of $\vec{F}_{12}$ and $\vec{F}_{13}$; that is, from Eq. 21-7, we can write the net force $\vec{F}_{1,\text{net}}$ on particle 1 in unit-vector notation as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13}$$

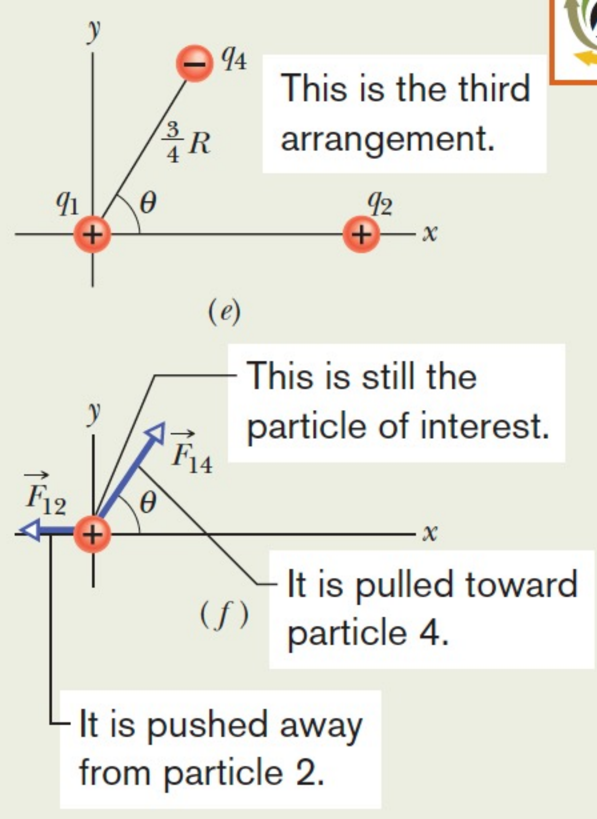
$$= -(1.15 \times 10^{-24}\ \text{N})\hat{\text{i}} + (2.05 \times 10^{-24}\ \text{N})\hat{\text{i}}$$

$$= (9.00 \times 10^{-25}\ \text{N})\hat{\text{i}}. \qquad \text{(Answer)}$$

Thus, $\vec{F}_{1,\text{net}}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$9.00 \times 10^{-25}\ \text{N} \quad \text{and} \quad 0°. \qquad \text{(Answer)}$$

(c) Figure 21-8e is identical to Fig. 21-8a except that particle 4 is now included. It has charge $q_4 = -3.20 \times 10^{-19}$ C, is at a distance $\frac{3}{4}R$ from particle 1, and lies on a line that makes an angle $\theta = 60°$ with the x axis. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 4?



The net force $\vec{F}_{1,\text{net}}$ is the vector sum of $\vec{F}_{12}$ and a new force $\vec{F}_{14}$ acting on particle 1 due to particle 4. Because particles 1 and 4 have charge of opposite signs, particle 1 is attracted to particle 4. Thus, force $\vec{F}_{14}$ on particle 1 is directed *toward* particle 4, at angle $\theta = 60°$, as indicated in the free-body diagram of Fig. 21-8f.

Four particles: We can rewrite Eq. 21-4 as

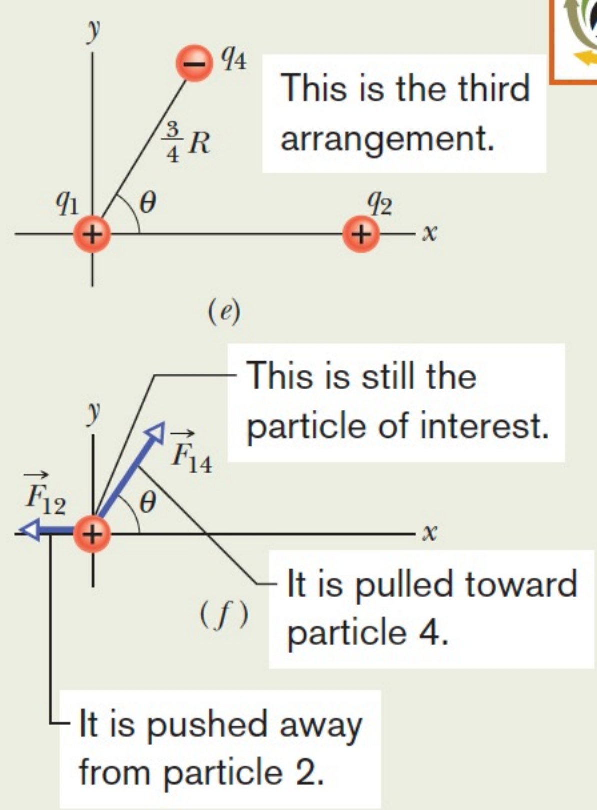
$$
\begin{aligned}
F_{14} &= \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_4|}{(\frac{3}{4}R)^2} \\
&= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \\
&\quad \times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2} \\
&= 2.05 \times 10^{-24}\ \text{N}.
\end{aligned}
$$

Then from Eq. 21-7, we can write the net force $\vec{F}_{1,\text{net}}$ on particle 1 as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{14}.$$

Because the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are not directed along the same axis, we *cannot* sum simply by combining their magnitudes. Instead, we must add them as vectors, using one of the following methods.

(c) Figure 21-8*e* is identical to Fig. 21-8*a* except that particle 4 is now included. It has charge $q_4 = -3.20 \times 10^{-19}$ C, is at a distance $\frac{3}{4}R$ from particle 1, and lies on a line that makes an angle $\theta = 60°$ with the x axis. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 4?



Method 3. *Summing components axis by axis.* The sum of the x components gives us

$$\begin{aligned} F_{1,\text{net},x} &= F_{12,x} + F_{14,x} = F_{12} + F_{14}\cos 60° \\ &= -1.15 \times 10^{-24}\text{ N} + (2.05 \times 10^{-24}\text{ N})(\cos 60°) \\ &= -1.25 \times 10^{-25}\text{ N}. \end{aligned}$$

The sum of the y components gives us

$$\begin{aligned} F_{1,\text{net},y} &= F_{12,y} + F_{14,y} = 0 + F_{14}\sin 60° \\ &= (2.05 \times 10^{-24}\text{ N})(\sin 60°) \\ &= 1.78 \times 10^{-24}\text{ N}. \end{aligned}$$

The net force $\vec{F}_{1,\text{net}}$ has the magnitude

$$F_{1,\text{net}} = \sqrt{F^2_{1,\text{net},x} + F^2_{1,\text{net},y}} = 1.78 \times 10^{-24}\text{ N}. \quad \text{(Answer)}$$

To find the direction of $\vec{F}_{1,\text{net}}$, we take

$$\theta = \tan^{-1}\frac{F_{1,\text{net},y}}{F_{1,\text{net},x}} = -86.0°.$$

However, this is an unreasonable result because $\vec{F}_{1,\text{net}}$ must have a direction between the directions of $\vec{F}_{12}$ and $\vec{F}_{14}$. To correct θ, we add 180°, obtaining

$$-86.0° + 180° = 94.0°. \quad \text{(Answer)}$$

Equilibrium of two forces on a particle

Figure 21-9*a* shows two particles fixed in place: a particle of charge $q_1 = +8q$ at the origin and a particle of charge $q_2 = -2q$ at $x = L$. At what point (other than infinitely far away) can a proton be placed so that it is in *equilibrium* (the net force on it is zero)? Is that equilibrium *stable* or *unstable*? (That is, if the proton is displaced, do the forces drive it back to the point of equilibrium or drive it farther away?)

KEY IDEA

If $\vec{F}_1$ is the force on the proton due to charge q_1 and $\vec{F}_2$ is the force on the proton due to charge q_2, then the point we seek is where $\vec{F}_1 + \vec{F}_2 = 0$. Thus,

$$\vec{F}_1 = -\vec{F}_2. \tag{21-8}$$

This tells us that at the point we seek, the forces acting on the proton due to the other two particles must be of equal magnitudes,

$$F_1 = F_2, \tag{21-9}$$

and that the forces must have opposite directions.

Reasoning: Because a proton has a positive charge, the proton and the particle of charge q_1 are of the same sign, and force $\vec{F}_1$ on the proton must point away from q_1. Also, the proton and the particle of charge q_2 are of opposite signs, so force $\vec{F}_2$ on the proton must point toward q_2. "Away from q_1" and "toward q_2" can be in opposite directions only if the proton is located on the x axis.

If the proton is on the x axis at any point between q_1 and q_2, such as point P in Fig. 21-9*b*, then $\vec{F}_1$ and $\vec{F}_2$ are in the same direction and not in opposite directions as required. If the proton is at any point on the x axis to the left of q_1, such as point S in Fig. 21-9*c*, then $\vec{F}_1$ and $\vec{F}_2$ are in opposite directions. However, Eq. 21-4 tells us that $\vec{F}_1$ and $\vec{F}_2$ cannot have equal magnitudes there: F_1 must be greater than F_2, because F_1 is produced by a closer charge (with lesser r) of greater magnitude ($8q$ versus $2q$).

Finally, if the proton is at any point on the x axis to the right of q_2, such as point R in Fig. 21-9*d*, then $\vec{F}_1$ and $\vec{F}_2$ are again in opposite directions. However, because now the charge of greater magnitude (q_1) is *farther* away from the proton than the charge of lesser magnitude, there is a point at which F_1 is equal to F_2. Let x be the coordinate of this point, and let q_p be the charge of the proton.

Calculations: With the aid of Eq. 21-4, we can now rewrite Eq. 21-9 (which says that the forces have equal magnitudes):

$$\frac{1}{4\pi\varepsilon_0}\frac{8qq_p}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{(x-L)^2}. \tag{21-10}$$

(Note that only the charge magnitudes appear in Eq. 21-10. We already decided about the directions of the forces in drawing Fig. 21-9*d* and do not want to include any positive or negative signs here.) Rearranging Eq. 21-10 gives us

$$\left(\frac{x-L}{x}\right)^2 = \frac{1}{4}.$$

After taking the square roots of both sides, we have

$$\frac{x-L}{x} = \frac{1}{2},$$

which gives us

$$x = 2L. \qquad \text{(Answer)}$$

The equilibrium at $x = 2L$ is unstable; that is, if the proton is displaced leftward from point R, then F_1 and F_2 both increase but F_2 increases more (because q_2 is closer than q_1), and a net force will drive the proton farther leftward. If the proton is displaced rightward, both F_1 and F_2 decrease but F_2 decreases more, and a net force will then drive the proton farther rightward. In a stable equilibrium, if the proton is displaced slightly, it returns to the equilibrium position.

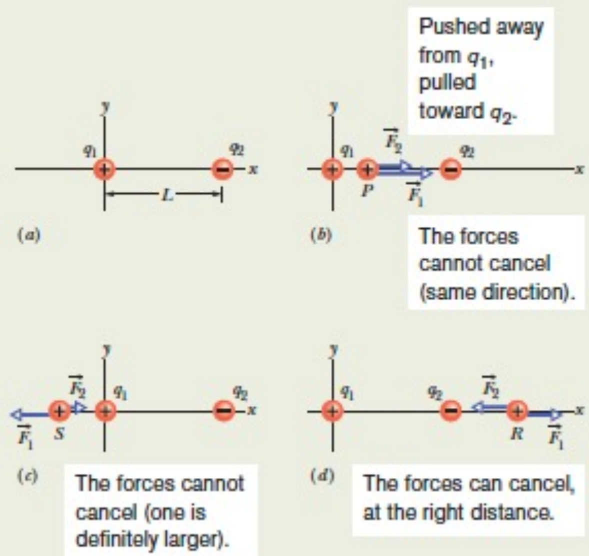


Fig. 21-9 (*a*) Two particles of charges q_1 and q_2 are fixed in place on an x axis, with separation L. (*b*)–(*d*) Three possible locations P, S, and R for a proton. At each location, $\vec{F}_1$ is the force on the proton from particle 1 and $\vec{F}_2$ is the force on the proton from particle 2.