Waves

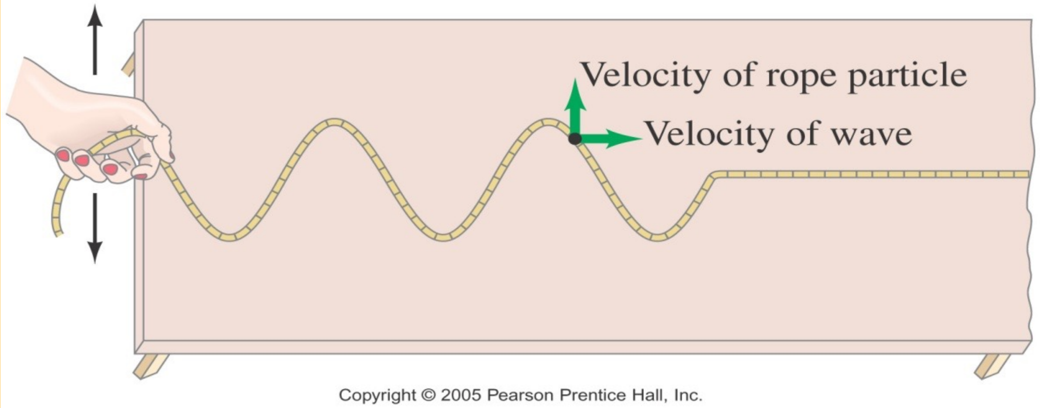
16-2 Types of Waves

Waves are of three main types:

1. ***Mechanical waves.*** These waves are most familiar because we encounter them almost constantly; common examples include water waves, sound waves, and seismic waves. All these waves have two central features: They are governed by Newton's laws, and they can exist only within a material medium, such as water, air, and rock.
2. ***Electromagnetic waves.*** These waves are less familiar, but you use them constantly; common examples include visible and ultraviolet light, radio and television waves, microwaves, x rays, and radar waves. These waves require no material medium to exist. Light waves from stars, for example, travel through the vacuum of space to reach us. All electromagnetic waves travel through a vacuum at the same speed $c = 299\,792\,458$ m/s.
3. ***Matter waves.*** Although these waves are commonly used in modern technology, they are probably very unfamiliar to you. These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. Because we commonly think of these particles as constituting matter, such waves are called matter waves.

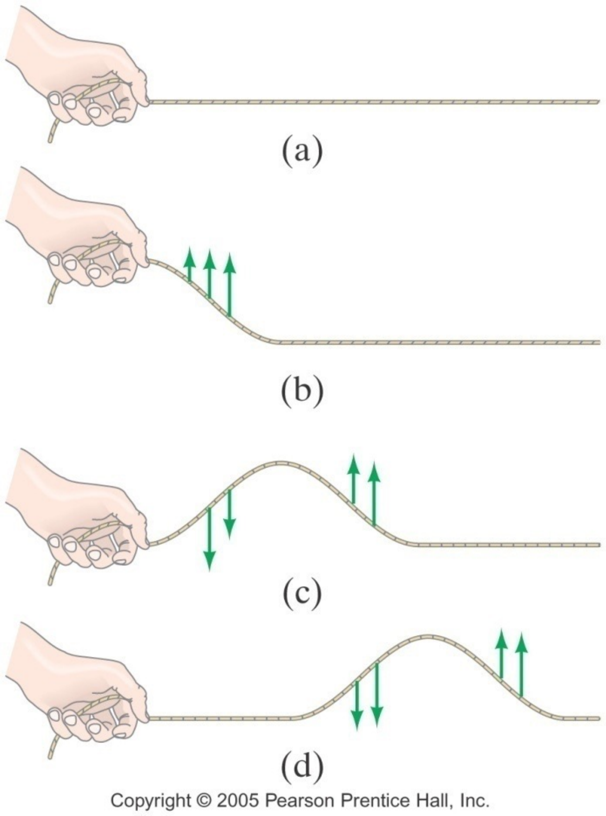
Much of what we discuss in this chapter applies to waves of all kinds. However, for specific examples we shall refer to mechanical waves.

Wave Motion



A wave travels along its medium, but the individual particles just move up and down.

Waves **carry energy** from one place to another.



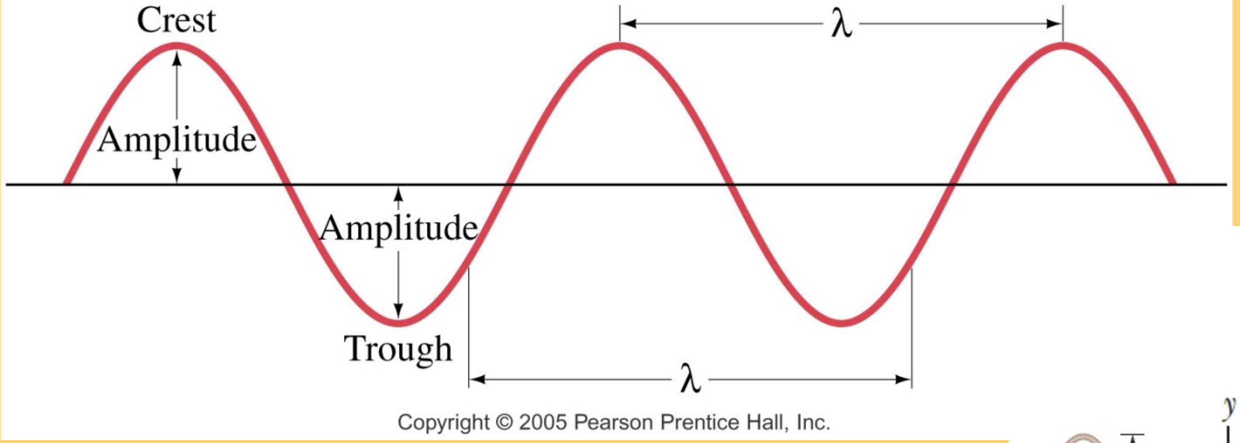
A wave may start with a single **pulse** -- figure. Cohesive forces between adjacent sections of the rope cause the pulse to travel outward. It is similar in other media.

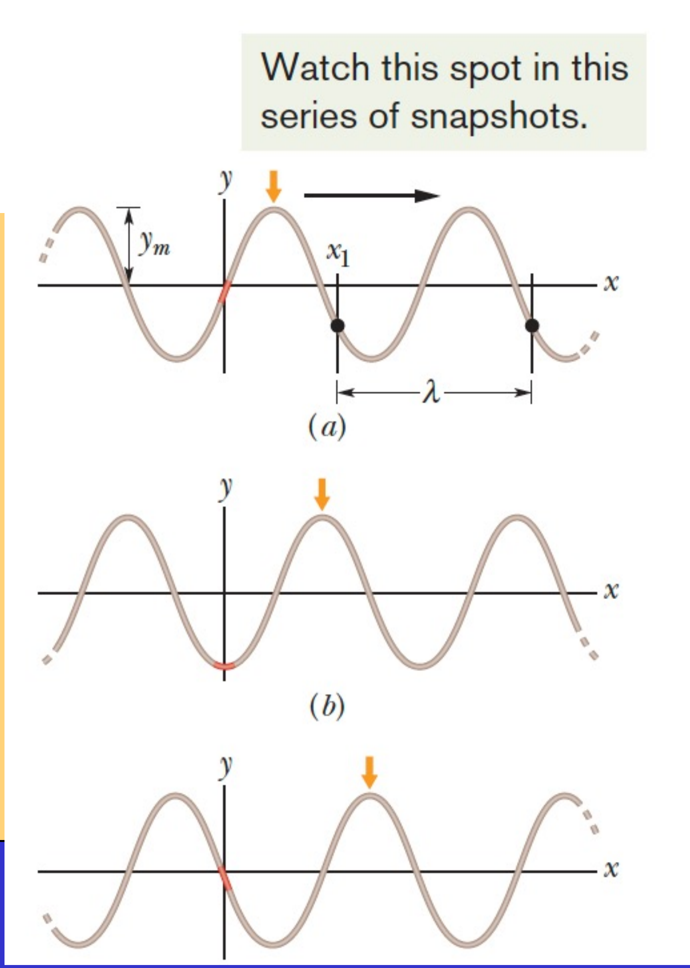
Continuous or periodic waves start with vibrations too, but they are continuous.

The source of any wave is a vibration.

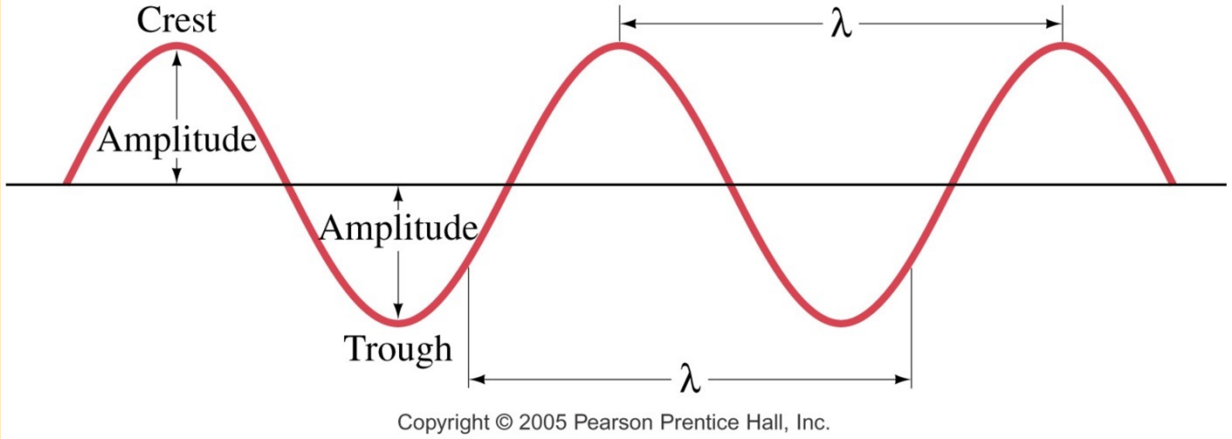
If the vibration is SHM, then the wave will have a sinusoidal shape.

Wave Motion


Crest
λ
Amplitude
Amplitude
Trough
λ
Copyright © 2005 Pearson Prentice Hall, Inc.


Watch this spot in this series of snapshots.
y
y_m
x_1
x
λ
(a)
y
x
(b)
y
x

Wave Motion



Wave characteristics:

•**Amplitude**, *A:* maximum height of a crest or depth of a trough

• **Wavelength**, *λ:* distance between successive crests, or any two successive identical points

• **Frequency** *f* : number of crests (or complete cycles) that pass a given point per unit of time. **Period** *T:* time elapsed between two successive crests

• **Wave velocity**: velocity at which wave crests move. A wave crest travels a distance of one wavelength in a time equal to one period. $\boxed{v = \lambda / T = \lambda f}$

Wave Motion

Wave velocity: velocity at which wave crests move. A wave crest travels a distance of one wavelength in a time equal to one period.

$$\boxed{v = \lambda / T = \lambda f}$$

The speed of a wave on a stretched string or cord depends on the the tension in the cord as

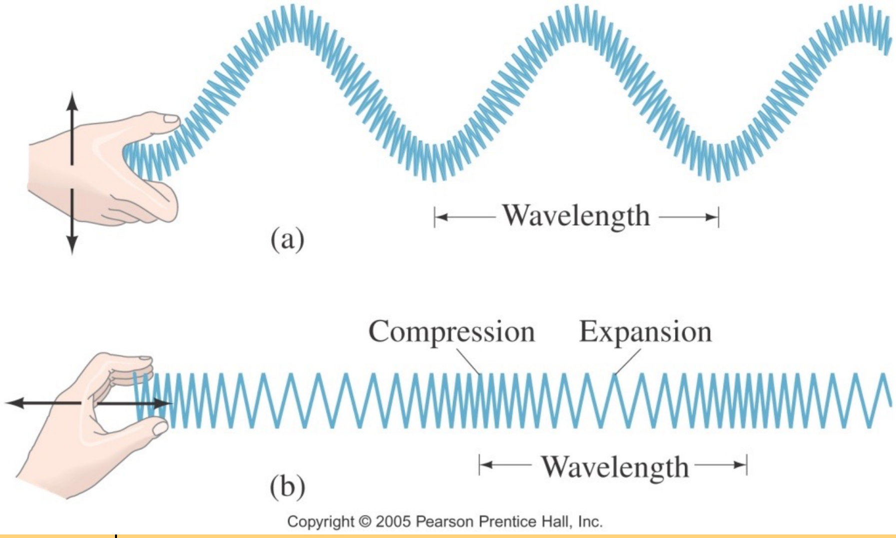
$$\boxed{v = \sqrt{\frac{F_T}{m / L}}}$$

***CAREFUL**: wave velocity is different from the velocity of a particle in the medium!*

Ex. 11-11 A wave whose wavelength is 0.30 m is traveling down a 300-m-long wire whose total mass is 15 kg. If the wire is under tension of 1000 N, what are the speed and frequency of this wave?

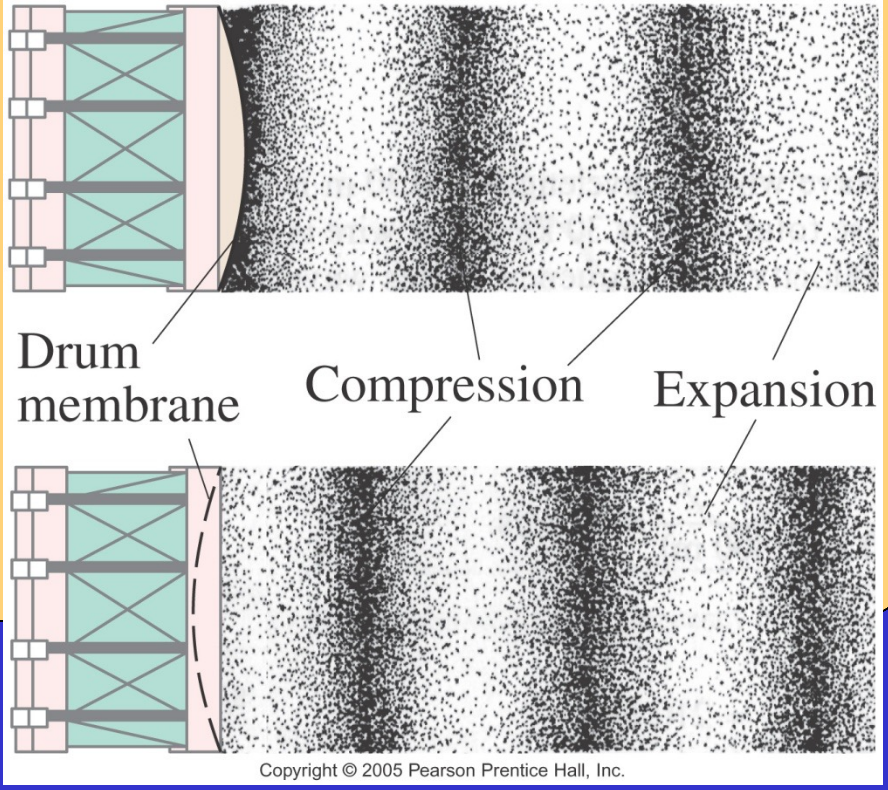
v = 140 m/s and f = 470 Hz

Transverse and Longitudinal Waves

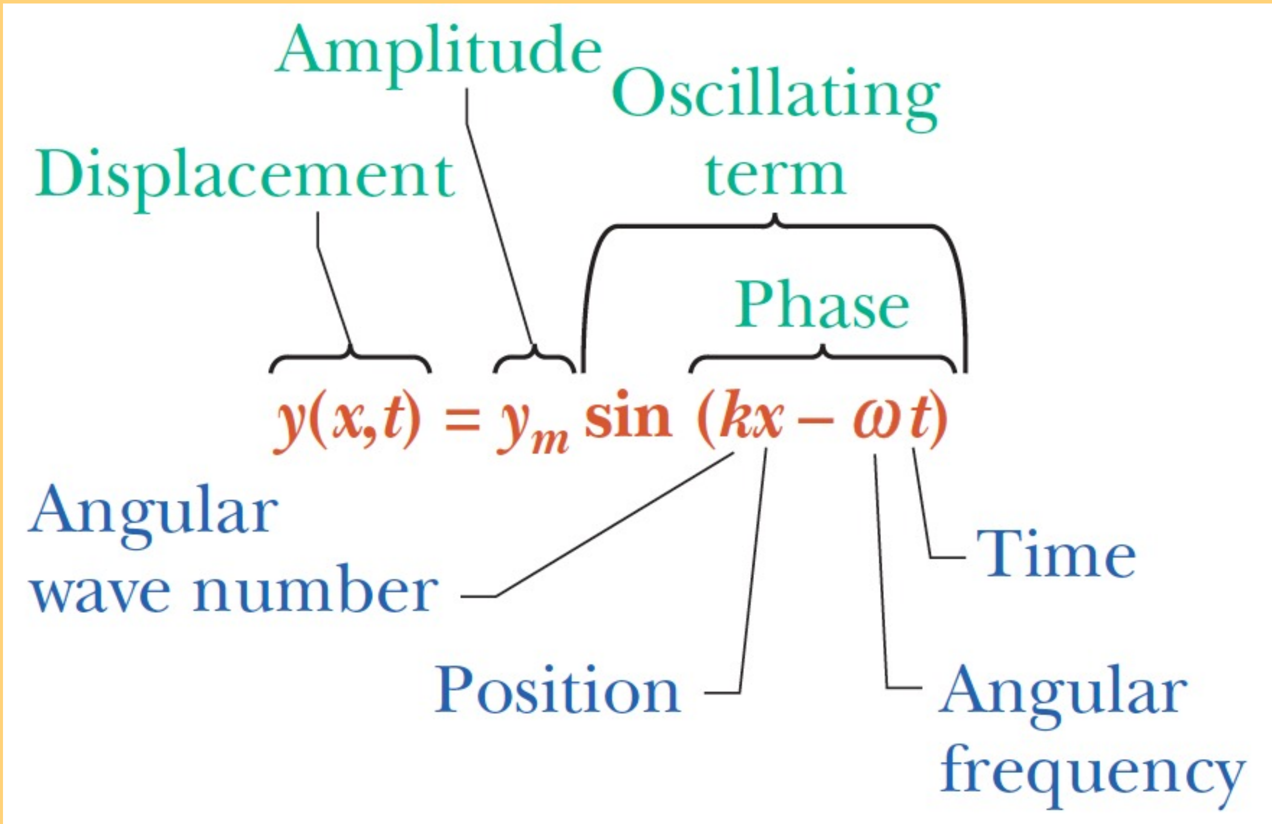


The motion of particles in a wave can either be perpendicular to the wave direction (**transverse**) or parallel to it (**longitudinal**).

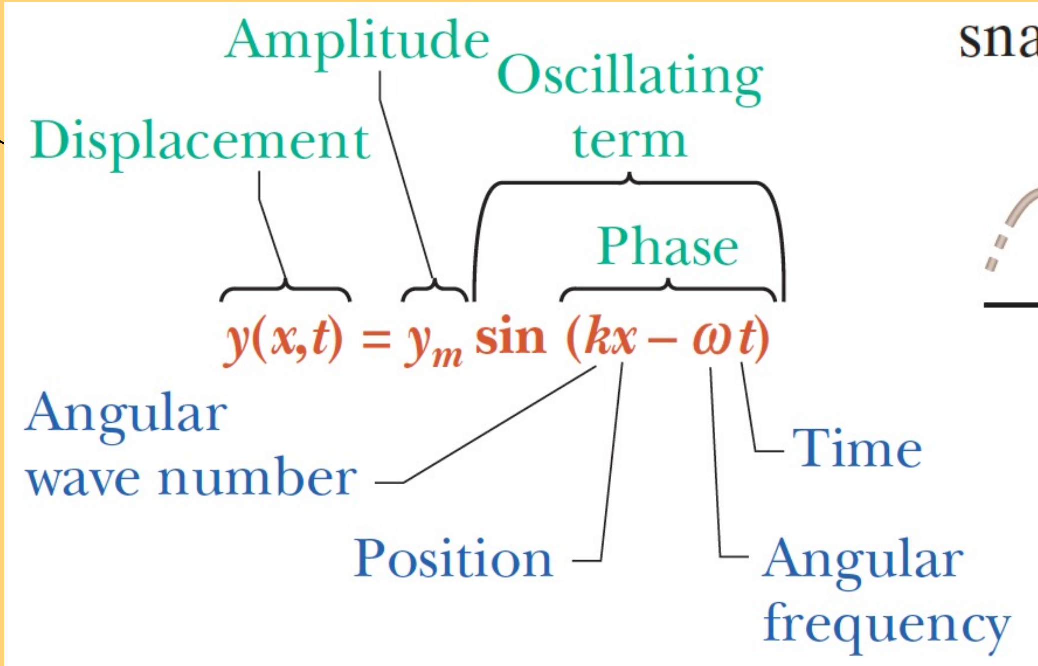
Sound waves are longitudinal waves:



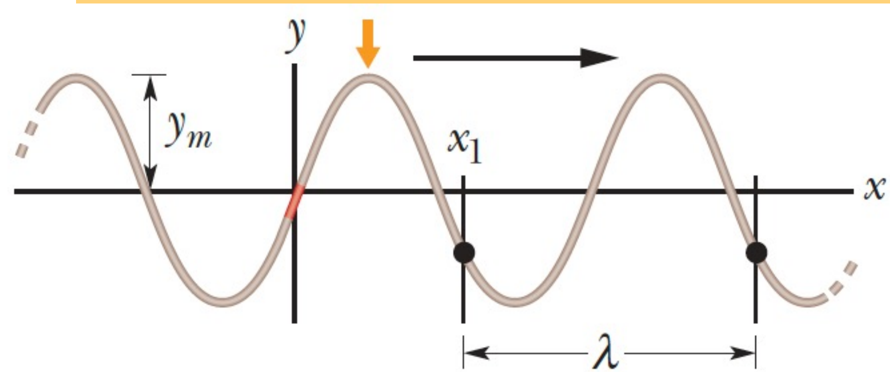
Sinusoidal Wave



Angular Wave Number



snapshot of the wave at time $t = 0$

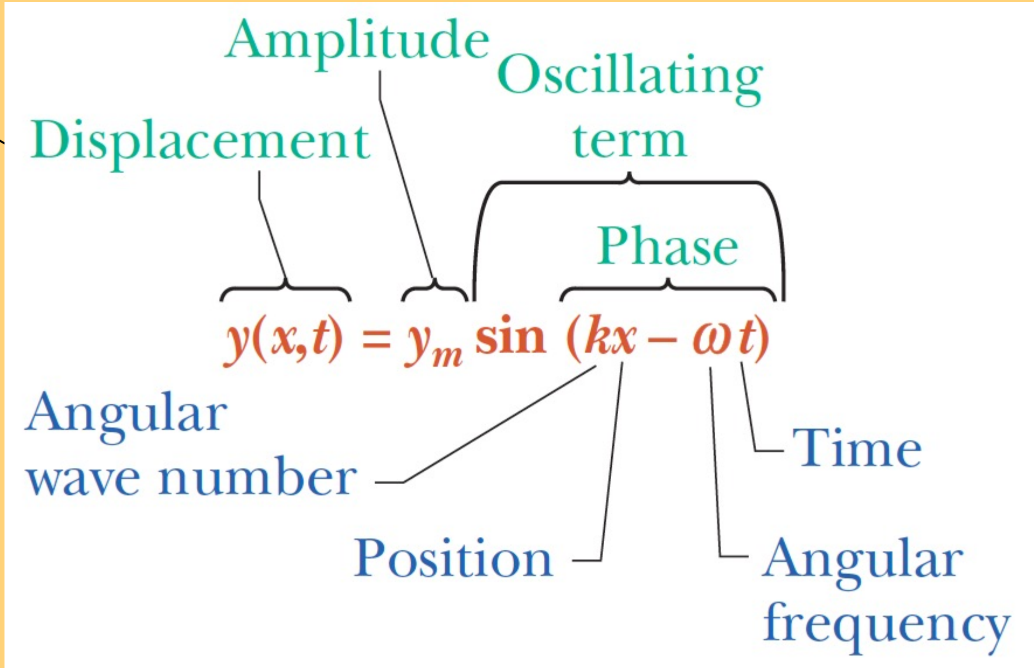


$$
\begin{aligned}
y_m \sin kx_1 &= y_m \sin k(x_1 + \lambda) \\
&= y_m \sin(kx_1 + k\lambda)
\end{aligned}
$$

A sine function begins to repeat itself when its angle (or argument) is increased by 2π rad, so in Eq. 16-4 we must have $k\lambda = 2\pi$, or

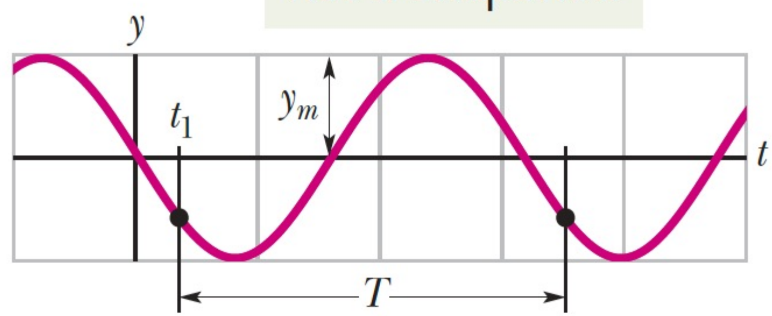
$$k = \frac{2\pi}{\lambda} \qquad \text{(angular wave number)}$$

Angular Frequency



$x = 0$

This is a graph, not a snapshot.



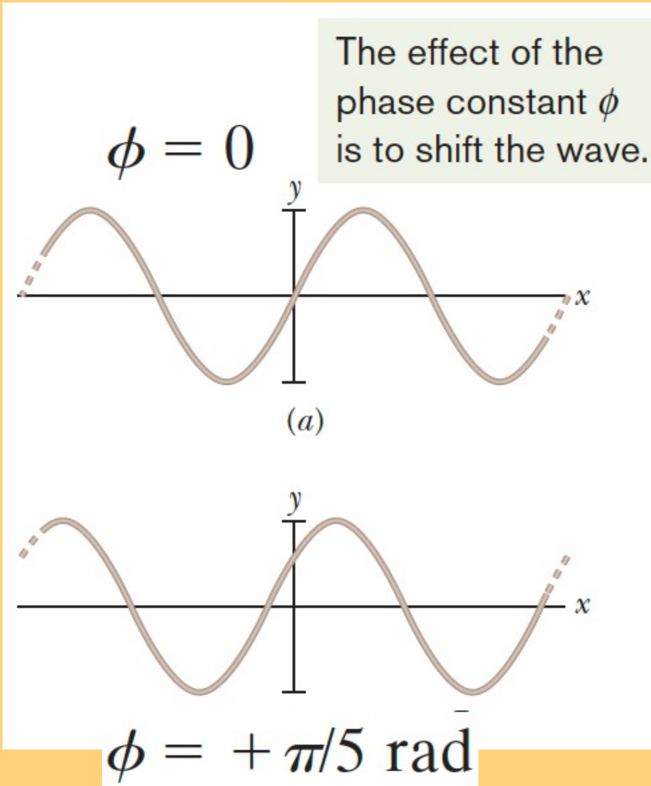
$$y(0,t) = y_m \sin(-\omega t)$$
$$= -y_m \sin \omega t$$

$$-y_m \sin \omega t_1 = -y_m \sin \omega(t_1 + T)$$
$$= -y_m \sin(\omega t_1 + \omega T)$$

This can be true only if $\omega T = 2\pi$,

$$\omega = \frac{2\pi}{T} \qquad \text{(angular frequency)}$$

Phase Constant



$$y = y_m \sin(kx - \omega t + \phi)$$

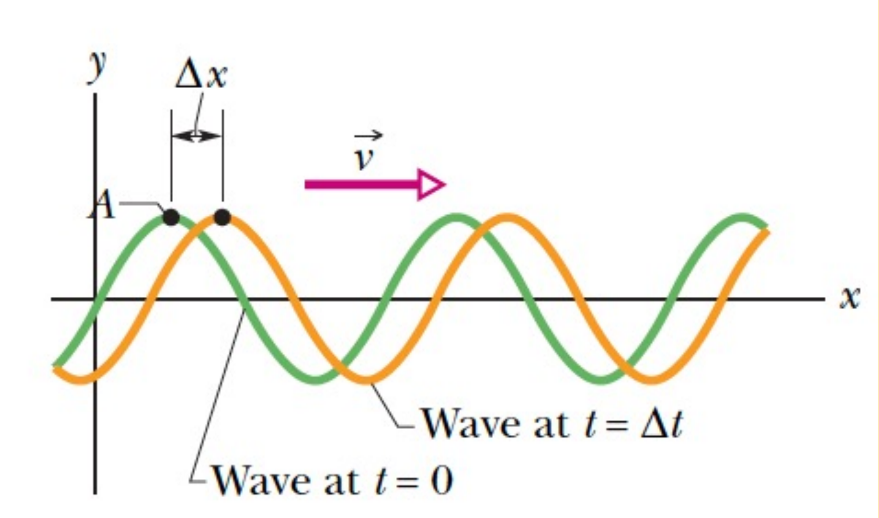
phase constant ϕ

Wave Speed

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \quad \text{(wave speed)}.$$

Wave moves to the right: +v

$$y(x, t) = y_m \sin(kx - \omega t).$$



Wave moves to the left: -v

$$y(x, t) = y_m \sin(kx + \omega t).$$

Transverse wave, amplitude, wavelength, period, velocity

A wave traveling along a string is described by

$$y(x,t) = 0.00327 \sin(72.1x - 2.72t), \qquad (16\text{-}18)$$

in which the numerical constants are in SI units (0.00327 m, 72.1 rad/m, and 2.72 rad/s).

(a) What is the amplitude of this wave?

KEY IDEA

Equation 16-18 is of the same form as Eq. 16-2,

$$y = y_m \sin(kx - \omega t), \qquad (16\text{-}19)$$

so we have a sinusoidal wave. By comparing the two equations, we can find the amplitude.

Calculation: We see that

$$y_m = 0.00327 \text{ m} = 3.27 \text{ mm}. \qquad \text{(Answer)}$$

(b) What are the wavelength, period, and frequency of this wave?

Calculations: By comparing Eqs. 16-18 and 16-19, we see that the angular wave number and angular frequency are

$$k = 72.1 \text{ rad/m} \quad \text{and} \quad \omega = 2.72 \text{ rad/s}.$$

We then relate wavelength λ to k via Eq. 16-5:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi \text{ rad}}{72.1 \text{ rad/m}}$$
$$= 0.0871 \text{ m} = 8.71 \text{ cm}. \qquad \text{(Answer)}$$

Next, we relate T to ω with Eq. 16-8:

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{2.72 \text{ rad/s}} = 2.31 \text{ s}, \qquad \text{(Answer)}$$

and from Eq. 16-9 we have

$$f = \frac{1}{T} = \frac{1}{2.31 \text{ s}} = 0.433 \text{ Hz}. \qquad \text{(Answer)}$$

(c) What is the velocity of this wave?

Calculation: The speed of the wave is given by Eq. 16-13:

$$v = \frac{\omega}{k} = \frac{2.72 \text{ rad/s}}{72.1 \text{ rad/m}} = 0.0377 \text{ m/s}$$
$$= 3.77 \text{ cm/s}. \qquad \text{(Answer)}$$

Because the phase in Eq. 16-18 contains the position variable x, the wave is moving along the x axis. Also, because the wave equation is written in the form of Eq. 16-2, the *minus* sign in front of the ωt term indicates that the wave is moving in the *positive* direction of the x axis. (Note that the quantities calculated in (b) and (c) are independent of the amplitude of the wave.)

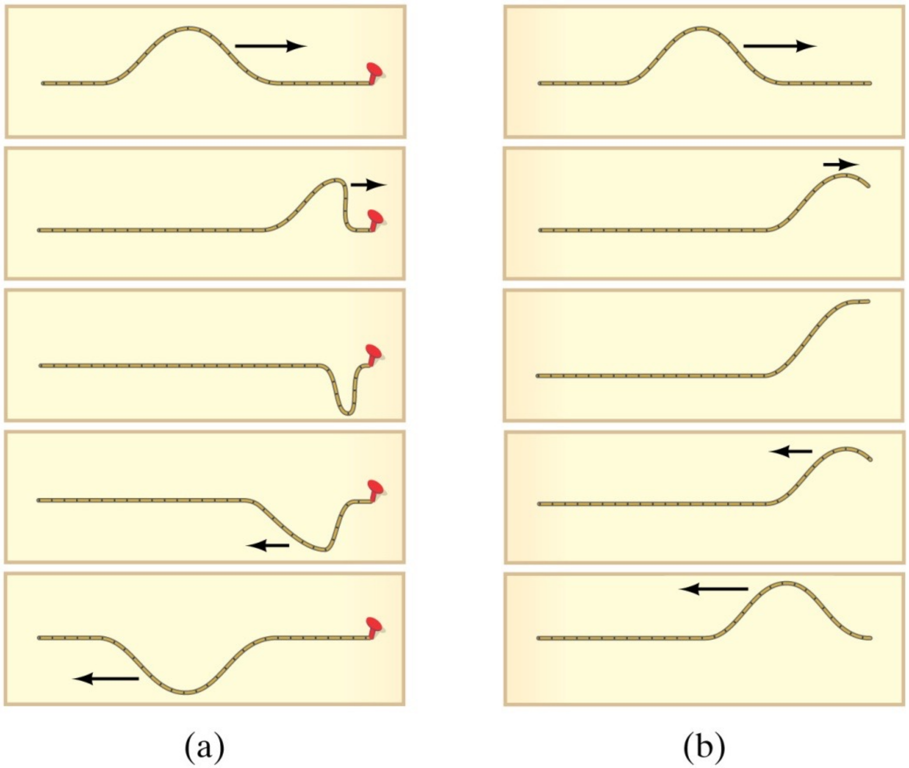
(d) What is the displacement y of the string at $x = 22.5$ cm and $t = 18.9$ s?

Calculation: Equation 16-18 gives the displacement as a function of position x and time t. Substituting the given values into the equation yields

$$y = 0.00327 \sin(72.1 \times 0.225 - 2.72 \times 18.9)$$
$$= (0.00327 \text{ m}) \sin(-35.1855 \text{ rad})$$
$$= (0.00327 \text{ m})(0.588)$$
$$= 0.00192 \text{ m} = 1.92 \text{ mm}. \qquad \text{(Answer)}$$

Thus, the displacement is positive. (Be sure to change your calculator mode to radians before evaluating the sine. Also, note that we do *not* round off the sine's argument before evaluating the sine. Also note that both terms in the argument are properly in radians, a dimensionless quantity.)

Reflection and Transmission of Waves

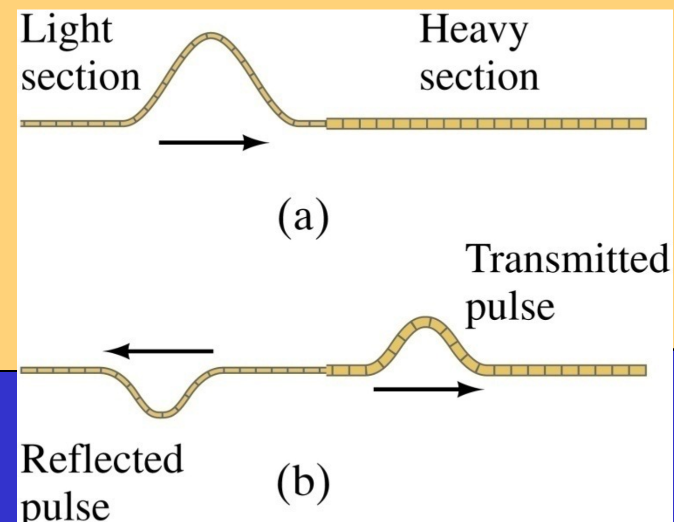


When a wave strikes an obstacle or come to the end of the medium it is traveling in, at least part of it is reflected. Echo is an example of reflection

http://www.kettering.edu/~drussell/Demos/reflect/reflect.html

A wave hitting an **obstacle** will be reflected (a), and its reflection will be **inverted**. (action and reaction)

A wave reaching the end of its medium, but where the medium is still **free** to move, will be reflected (b), and its reflection will be **upright.**



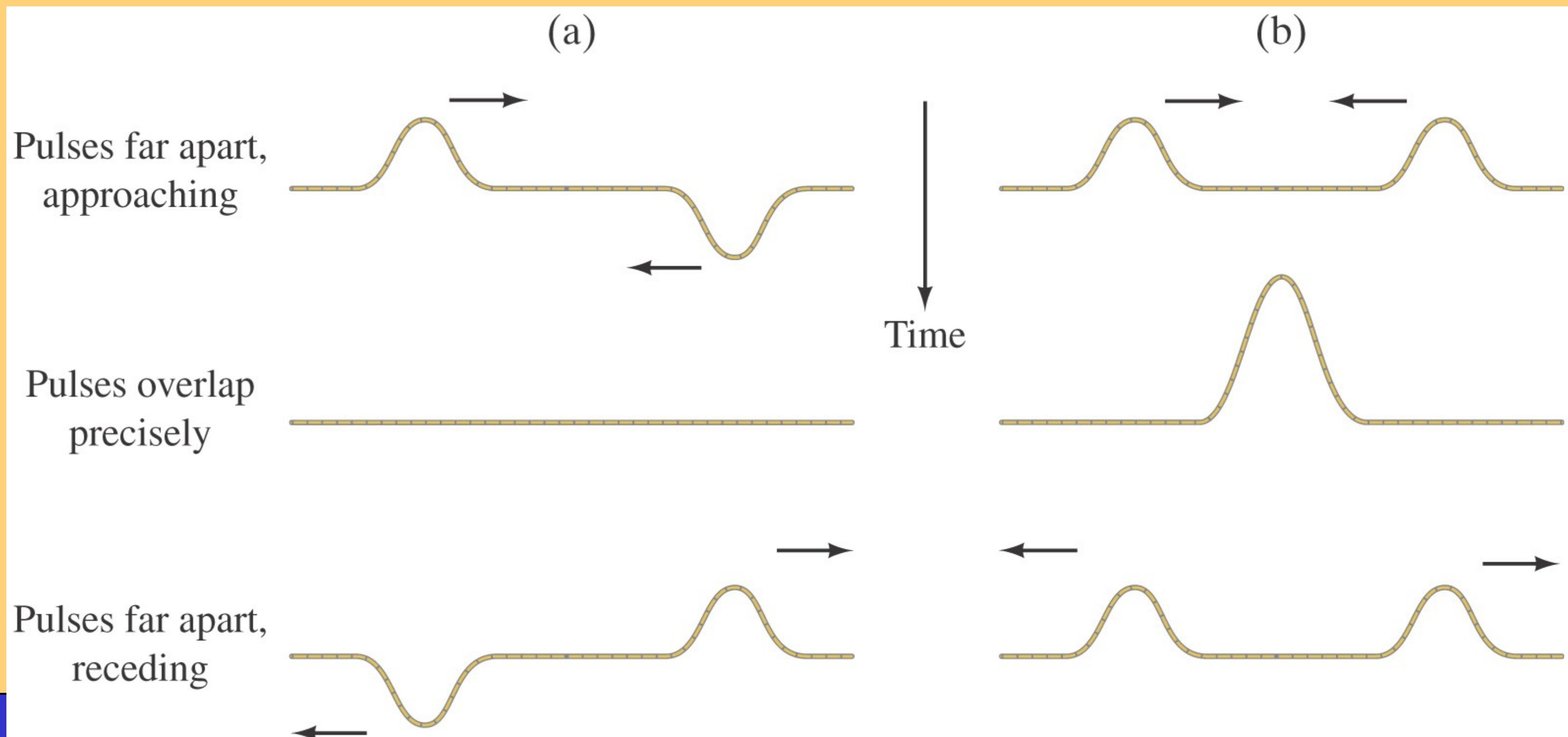
A wave encountering a denser medium will be partly reflected and partly transmitted; if the wave speed is less in the denser medium, the wavelength will be shorter, because f does not change

$$\lambda = v / f$$

Interference;Principle of Superposition

The **superposition principle** says that when two waves pass through the same point, the displacement is the arithmetic sum of the individual displacements.

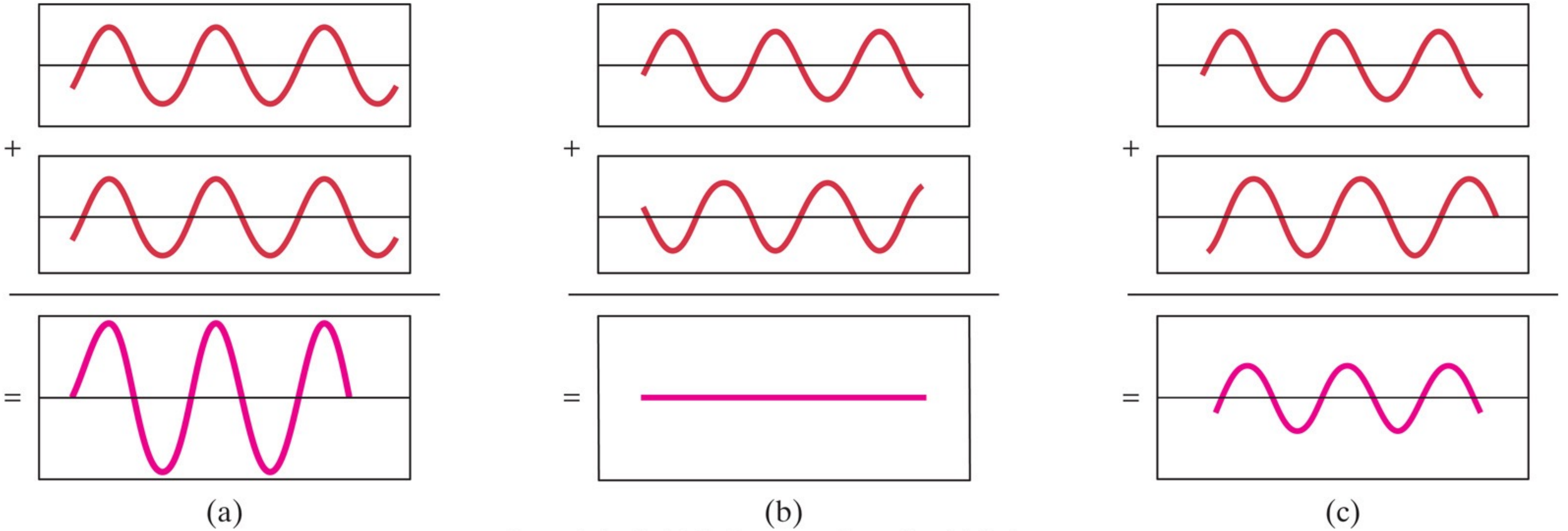
In the figure below, (a) exhibits **destructive** interference and (b) exhibits **constructive** interference.



Interference

These figures show the sum of two waves.
(a) they add **constructively**, the two waves are said to be **in phase**;
(b) they add **destructively,** the two waves are said to be **out phase**;
(c) they add partially destructively.

If the amplitudes of two interfering waves are not equal, fully destructive interference does not occur.



Interference

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

$$y_2(x, t) = y_m \sin(kx - \omega t + \phi)$$

$$\begin{aligned} y'(x, t) &= y_1(x, t) + y_2(x, t) \\ &= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) \end{aligned}$$

$$\sin \alpha + \sin \beta = 2 \sin \tfrac{1}{2}(\alpha + \beta) \cos \tfrac{1}{2}(\alpha - \beta)$$

$$y'(x, t) = [2y_m \cos \tfrac{1}{2}\phi] \sin(kx - \omega t + \tfrac{1}{2}\phi)$$

$|2y_m \cos \tfrac{1}{2}\phi|$ (amplitude)

phase constant is $\tfrac{1}{2}\phi$

$$y'(x,t) = [2y_m \cos \tfrac{1}{2}\phi] \sin(kx - \omega t + \tfrac{1}{2}\phi)$$

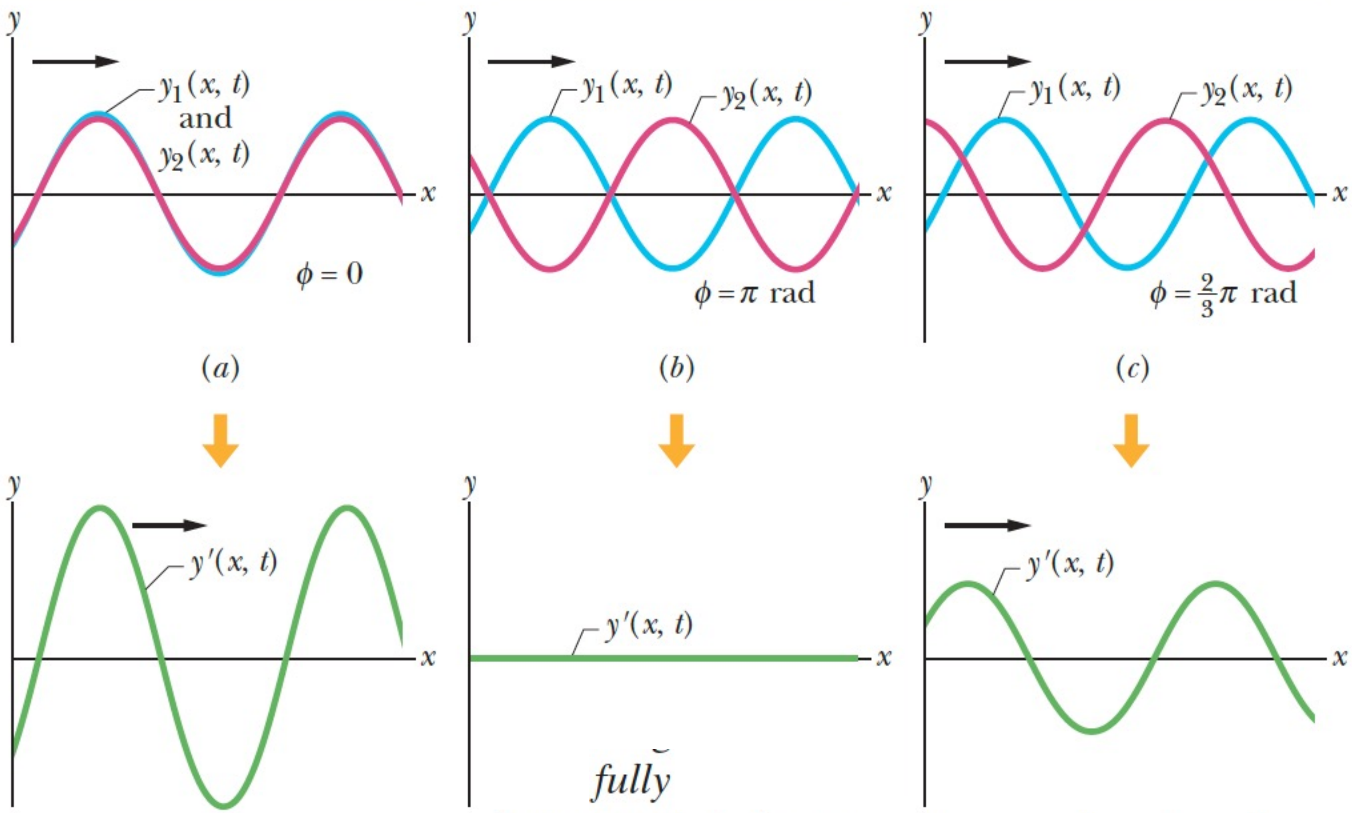
$|2y_m \cos \tfrac{1}{2}\phi|$ (amplitude)

phase constant is $\tfrac{1}{2}\phi$

Being exactly in phase, the waves produce a large resultant wave.

Being exactly out of phase, they produce a flat string.

This is an intermediate situation, with an intermediate result.



fully constructive interference. *fully destructive interference.* *intermediate interference.*

Interference

Interference of two waves, same direction, same amplitude

Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude y_m of each wave is 9.8 mm, and the phase difference ϕ between them is 100°.

(a) What is the amplitude y'_m of the resultant wave due to the interference, and what is the type of this interference?

KEY IDEA

These are identical sinusoidal waves traveling in the *same direction* along a string, so they interfere to produce a sinusoidal traveling wave.

Calculations: Because they are identical, the waves have the *same amplitude*. Thus, the amplitude y'_m of the resultant wave is given by Eq. 16-52:

$$y'_m = |2y_m \cos \tfrac{1}{2}\phi| = |(2)(9.8 \text{ mm}) \cos(100°/2)|$$
$$= 13 \text{ mm.} \qquad \text{(Answer)}$$

We can tell that the interference is *intermediate* in two ways. The phase difference is between 0 and 180°, and, correspondingly, the amplitude y'_m is between 0 and $2y_m$ (= 19.6 mm).

(b) What phase difference, in radians and wavelengths, will give the resultant wave an amplitude of 4.9 mm?

Calculations: Now we are given y'_m and seek ϕ. From Eq. 16-52,

$$y'_m = |2y_m \cos \tfrac{1}{2}\phi|,$$

we now have

$$4.9 \text{ mm} = (2)(9.8 \text{ mm}) \cos \tfrac{1}{2}\phi,$$

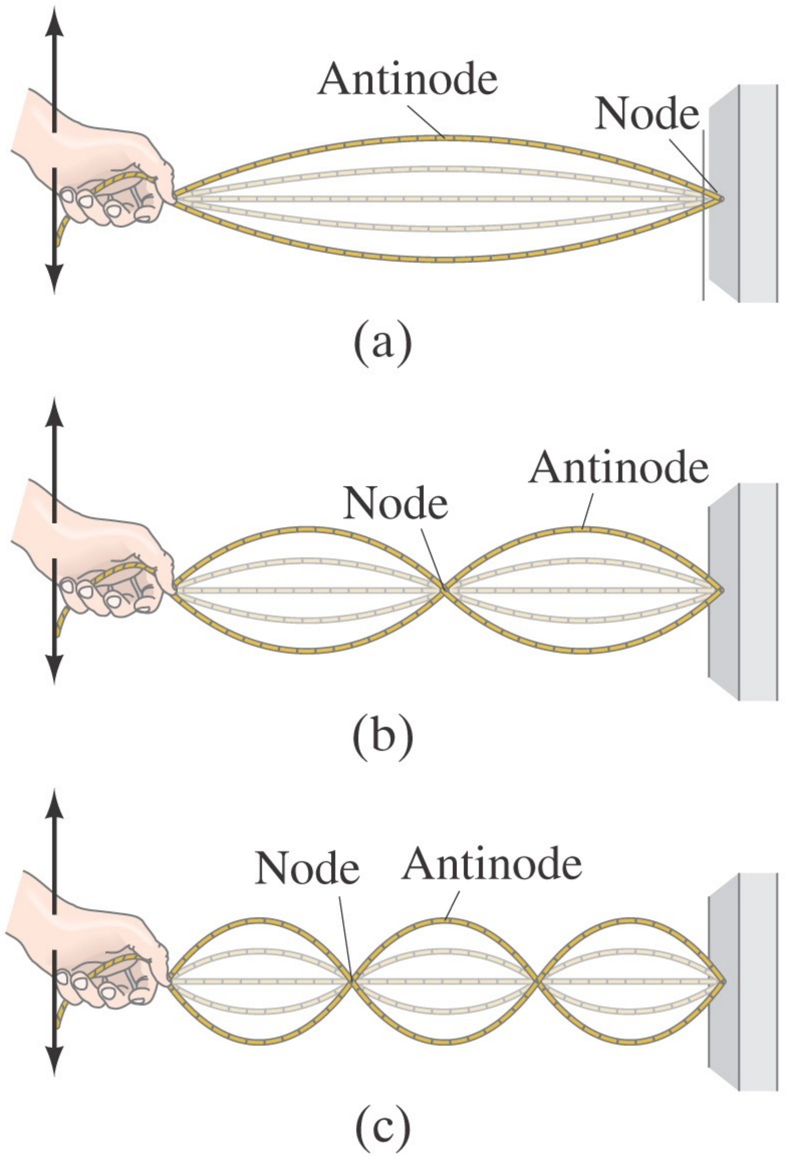
which gives us (with a calculator in the radian mode)

$$\phi = 2 \cos^{-1} \frac{4.9 \text{ mm}}{(2)(9.8 \text{ mm})}$$
$$= \pm 2.636 \text{ rad} \approx \pm 2.6 \text{ rad.} \qquad \text{(Answer)}$$

There are two solutions because we can obtain the same resultant wave by letting the first wave *lead* (travel ahead of) or *lag* (travel behind) the second wave by 2.6 rad. In wavelengths, the phase difference is

$$\frac{\phi}{2\pi \text{ rad/wavelength}} = \frac{\pm 2.636 \text{ rad}}{2\pi \text{ rad/wavelength}}$$
$$= \pm 0.42 \text{ wavelength.} \qquad \text{(Answer)}$$

Standing Waves, Resonance



If you shake one end of a cord and the other is kept fixed, waves will travel in both directions.

If you vibrate at the right frequency, the two traveling waves will interfere in such a way that a large-amplitude **standing wave** is produced.

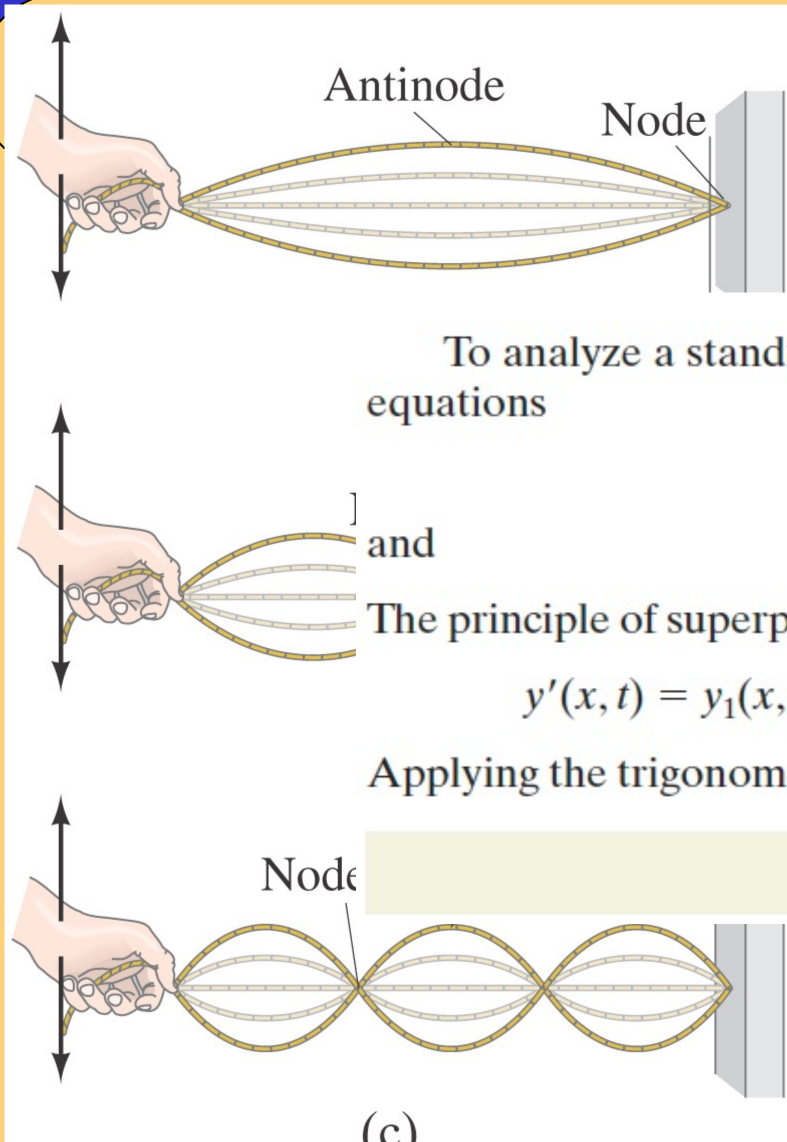
Standing waves do not appear to travel.

Nodes are points of destructive interference, where the cord remains still all the time

Antinodes are points of constructive interference, where the cord oscillates with maximum amplitude.

(a) Lowest frequency
(b) Twice the lowest frequency
(c) Three times the lowest frequency

Standing Waves, Resonance



To analyze a standing wave, we represent the two combining waves with the equations

$$y_1(x,t) = y_m \sin(kx - \omega t) \tag{16-58}$$

and

$$y_2(x,t) = y_m \sin(kx + \omega t). \tag{16-59}$$

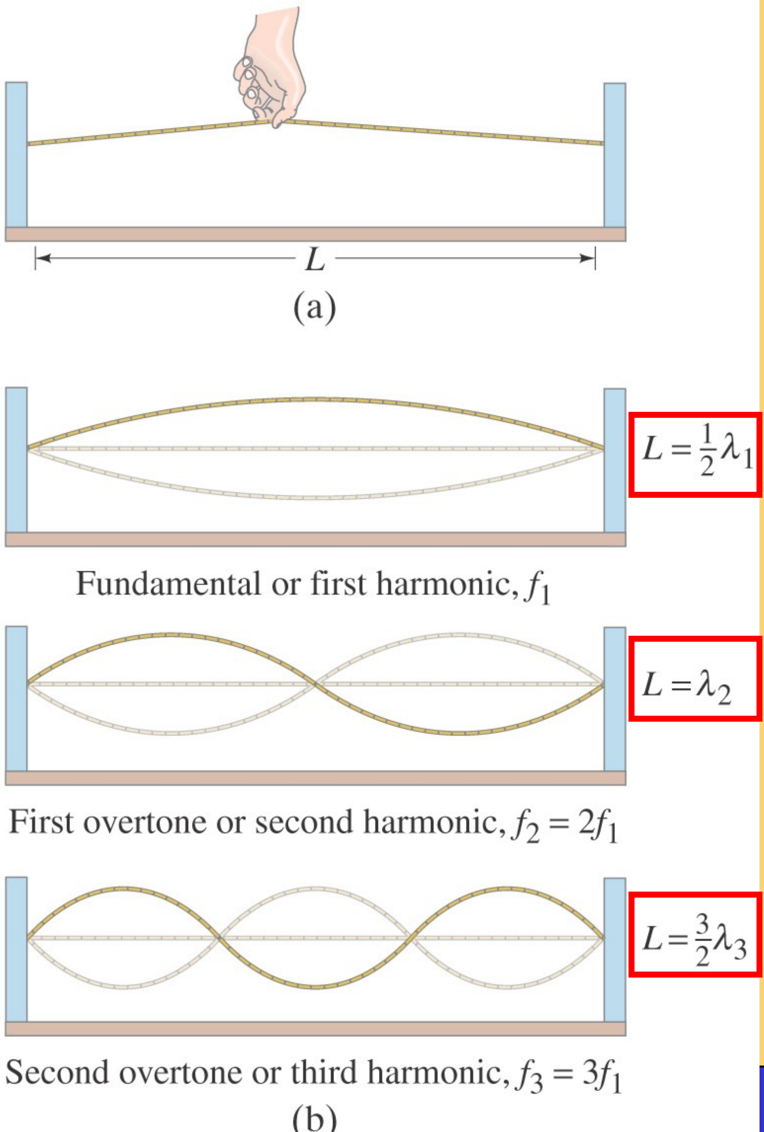
The principle of superposition gives, for the combined wave,

$$y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t).$$

Applying the trigonometric relation of Eq. 16-50 leads to Fig. 16-17 and

$$y'(x,t) = [2y_m \sin kx] \cos \omega t. \tag{16-60}$$

Standing Waves, Resonance



The frequencies at which standing waves are produced are called **natural frequencies** or **resonant frequencies**.

The lowest frequency is called fundamental frequency. It corresponds to one antinode (or loop). The next mode of vibration has two loops. See wavelength in figure.

Natural frequencies are also called **harmonics**.
First harmonic = **fundamental**
Second harmonic or first **overtone** = twice the fundamental; etc

For a vibrating string **overtones** are whole-number (integral) multiples of the fundamental.

Standing Waves, Resonance

In general we can write: $L = \dfrac{n\lambda_n}{2} \quad n = 1,2,3,\ldots$

Therefore, the wavelengths and frequencies of standing waves are:

$$\lambda_n = \frac{2L}{n}, \qquad n = 1, 2, 3, \cdots$$

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = nf_1, \qquad n = 1, 2, 3, \cdots$$

Because a standing wave is equivalent to two traveling waves moving in opposite directions, the concept of wave velocity still makes sense

$$\boxed{v = \sqrt{\frac{F_T}{m/L}}}$$

Ex 11-14 A piano string is 1.10 m long and has a mass of 9.00 g. (a) How much tension must the string be under if it is to vibrate at a fundamental frequency of 131 Hz? (b) What are the frequencies of the first four harmonics?

(a) 679 N (b) 131, 262, 393, and 524 Hz