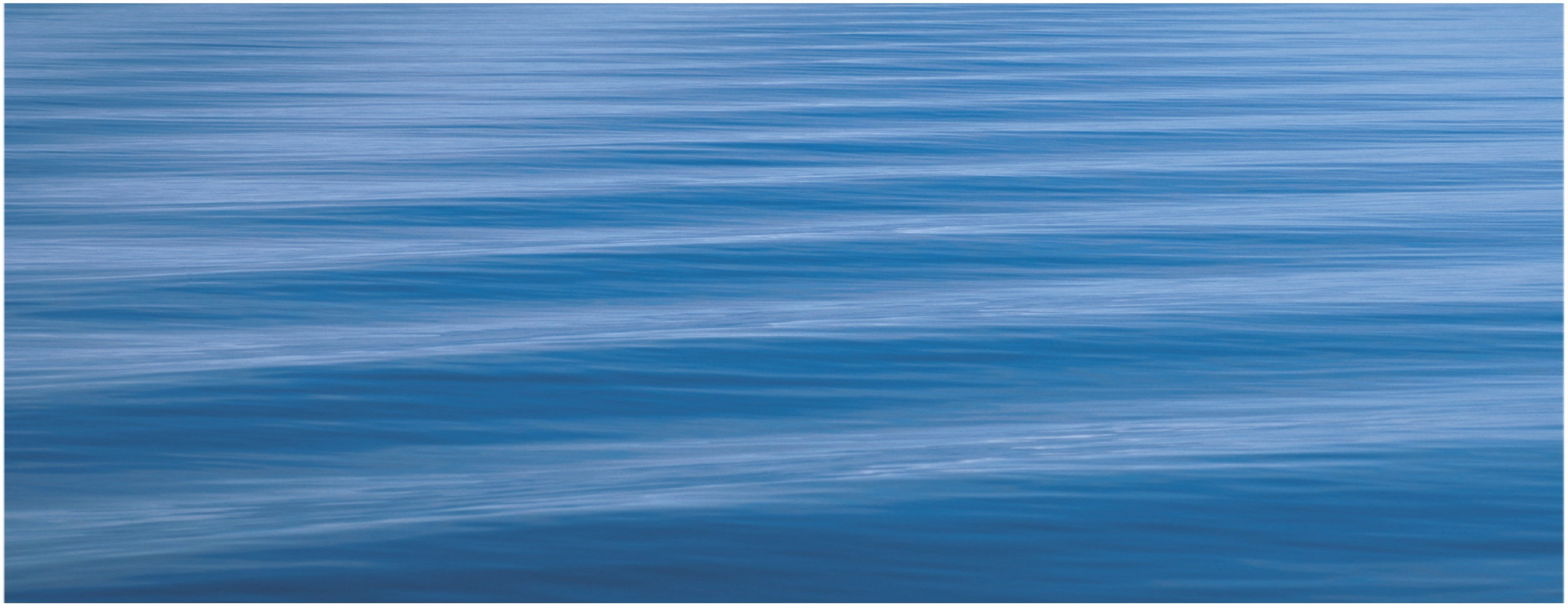
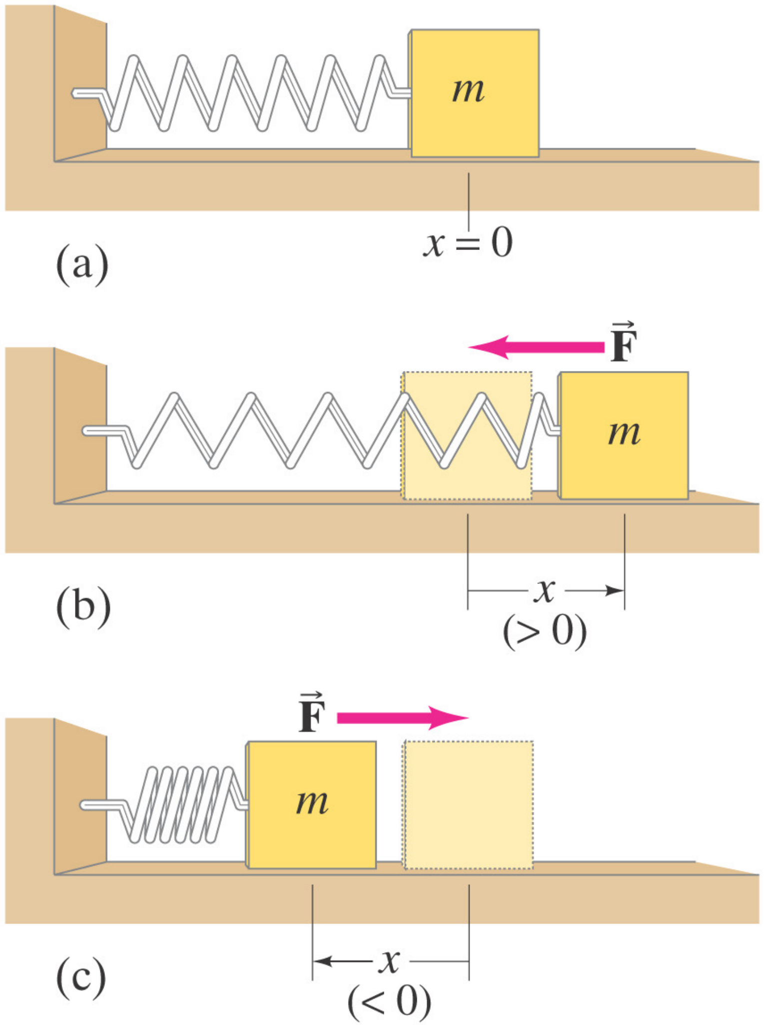
Chapter 15 – Oscillations

Many objects **oscillate/vibrate**
Examples: pendulum, strings of a guitar, atoms in a molecule, etc.

Waves have as a source a vibration.
Examples of waves: ocean waves, waves on a string, sound waves, electromagnetic waves – light.



Simple Harmonic Oscillator - SHO



If an object **vibrates or oscillates** back and forth over the same path, each cycle taking the *same amount of time*, the motion is called ***periodic***.

Assume that the surface is frictionless.

There is a point where the spring is neither stretched nor compressed; this is the ***equilibrium position***. We measure displacement from that point ($x = 0$).

The force exerted by the spring depends on the displacement (Hooke's law):

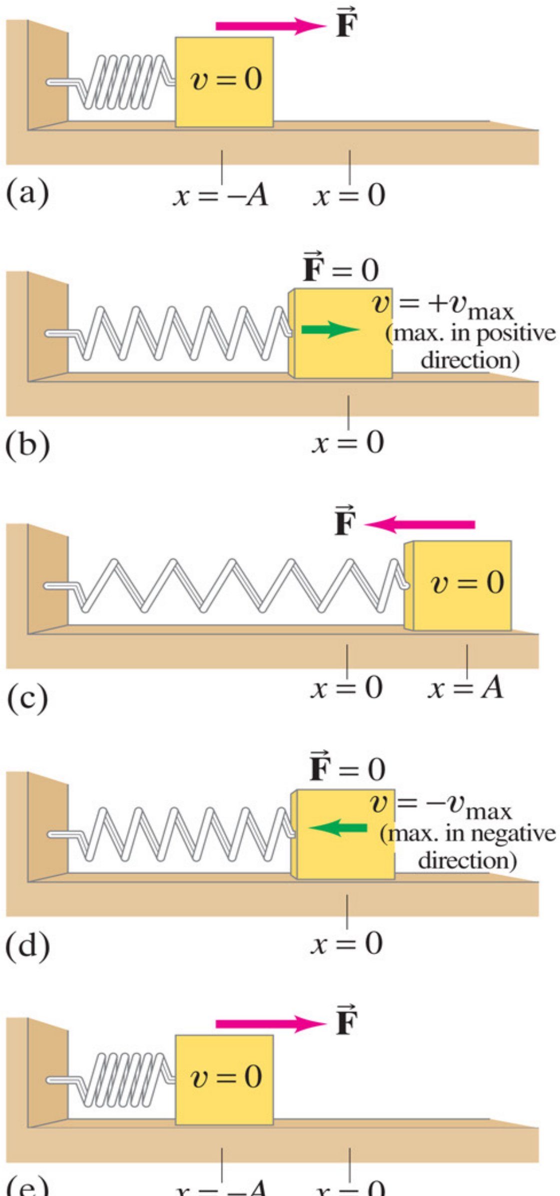
k is the spring constant

$$F = -kx$$

The minus sign on the force indicates that it is a ***restoring force*** – it is directed to restore the mass to its equilibrium position.

The force is not constant, so the **acceleration is not constant** either

Simple Harmonic Motion – SHM



Any vibrating system for which the restoring force is directly proportional to the negative of the displacement is said to exhibit: **simple harmonic motion (SHM).**
Such system called: **simple harmonic oscillator (SHO).**
Many natural **vibrations** are simple harmonic.

To study vibrational motion, we need some definitions:

- **Displacement** is measured from the equilibrium point
- **Amplitude** is the maximum displacement
- A **cycle** is a full to-and-fro motion
- **Period** is the time required to complete one cycle
- **Frequency** is the number of cycles completed per second

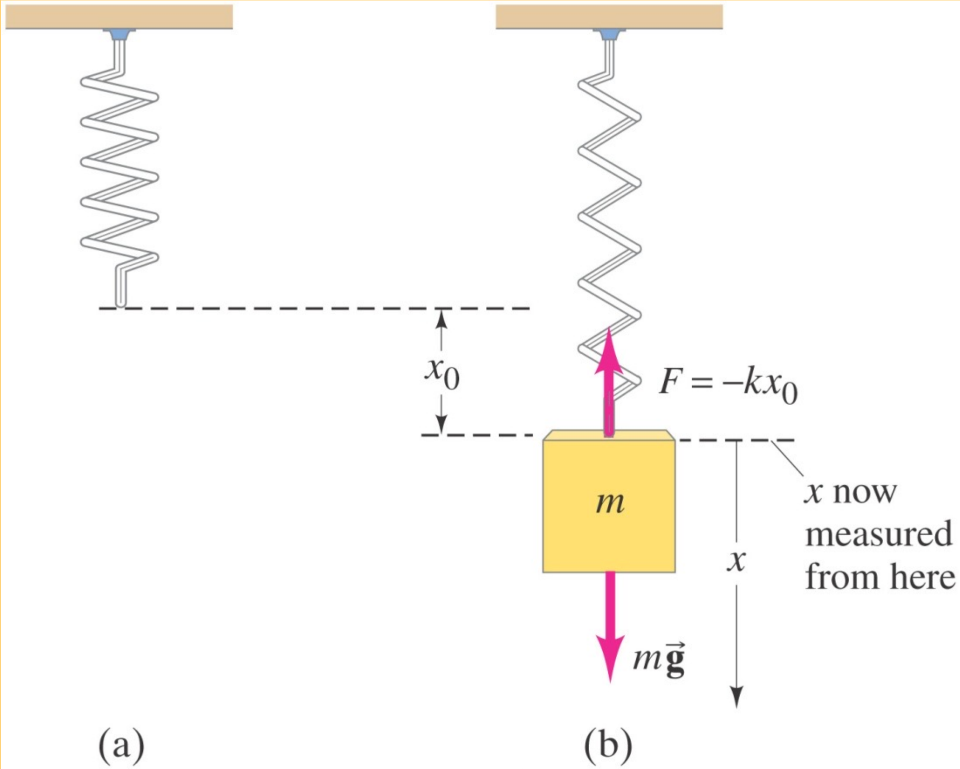
$$T = \frac{1}{f}$$

Simple Harmonic Motion – SHM

If the spring is hung vertically, the only change is in the equilibrium position, which is at the point where the spring force equals the gravitational force.

$$\sum F = 0 = mg - kx_0$$

$$x_0 = mg / k$$



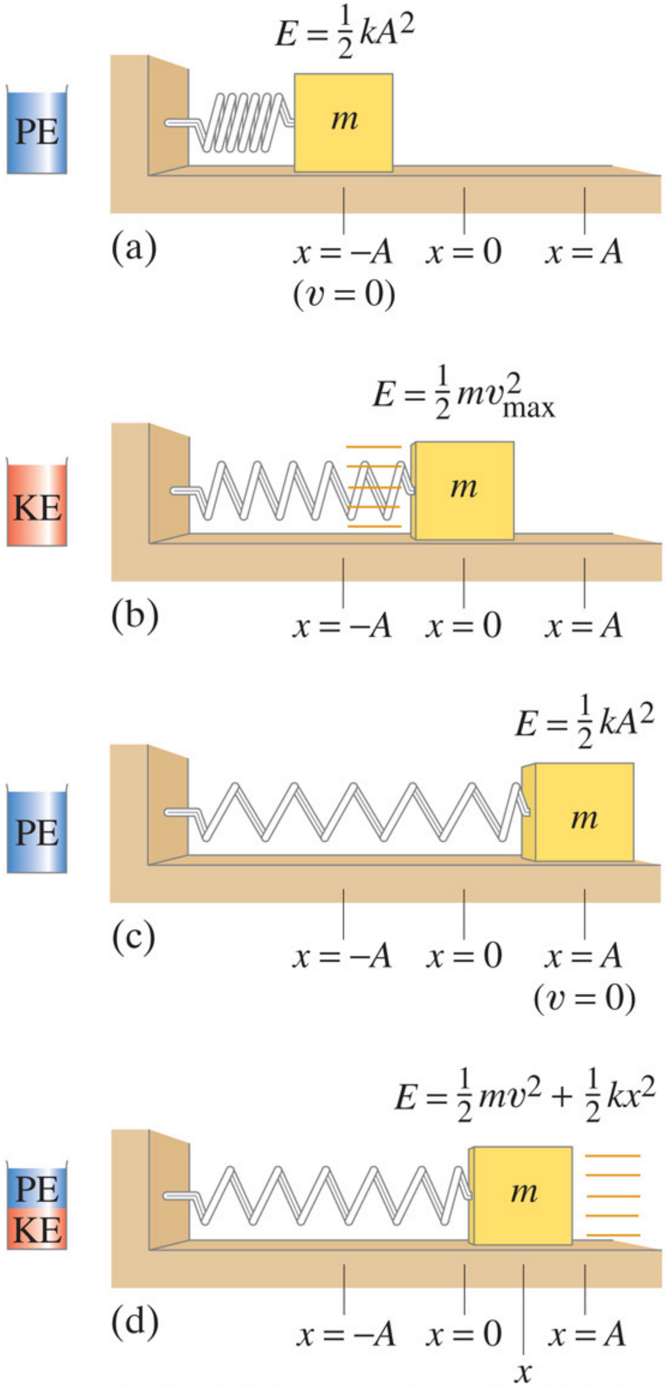
Ex. When a family of four with a total mass of 200 kg step into their 1200-kg car, the car's springs compress 3.0 cm. (a) What is the spring constant of the car's springs, assuming they act as a single spring? (b) How far will the car lower if loaded with 300 kg rather than 200 kg?

$$(a)\ 6.5\times10^4\ N/m \quad (b)\ 4.5\times10^{-2}\ m$$

Ex. Which of the following represent a simple harmonic oscillator:

$(a)\ F = -0.5x^2\ \ (b)\ F = -2.3y\ \ (c)\ F = 8.6c\ \ (d)\ F = -4\theta$?

Only (b) and (d)



Energy in the SHO

The potential energy of a spring: $U = \frac{1}{2}kx^2$

The kinetic energy of the spring: $K = \frac{1}{2}mv^2$

The total mechanical energy:
It is conserved because the system is frictionless

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

If the mass is at the limits of its motion, the energy is all potential.

$$E = \frac{1}{2}kA^2$$

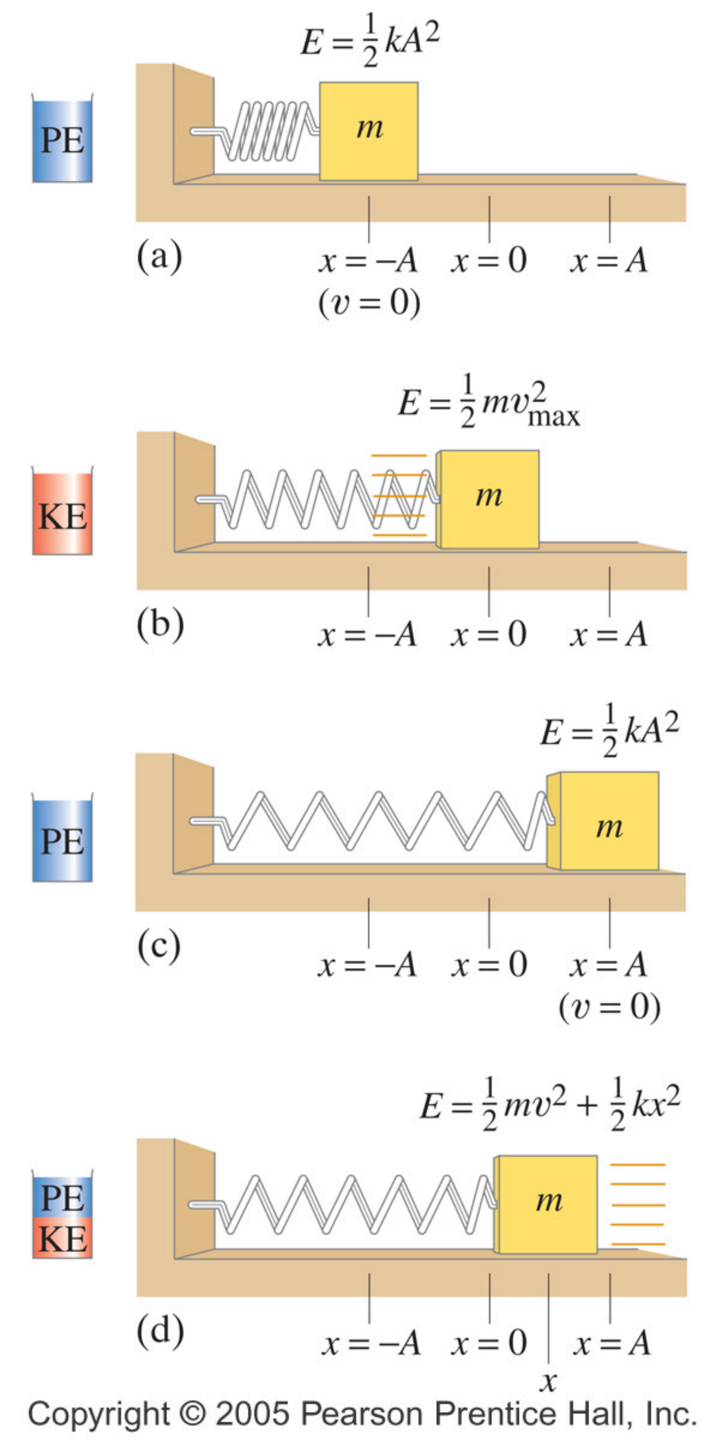
If the mass is at the equilibrium point, the energy is all kinetic.

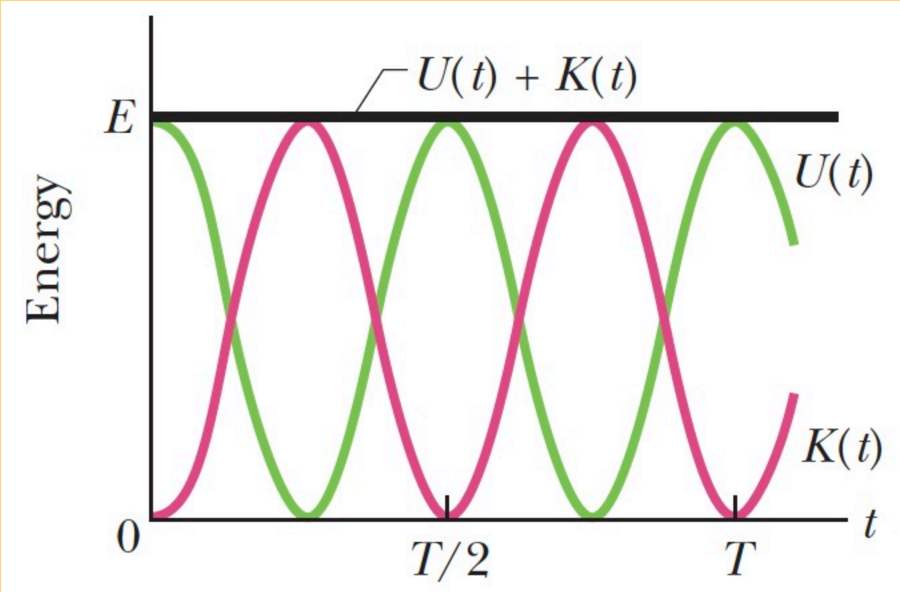
$$E = \frac{1}{2}mv^2_{max}$$

From conservation of energy, at intermediate points:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv^2_{max}$$

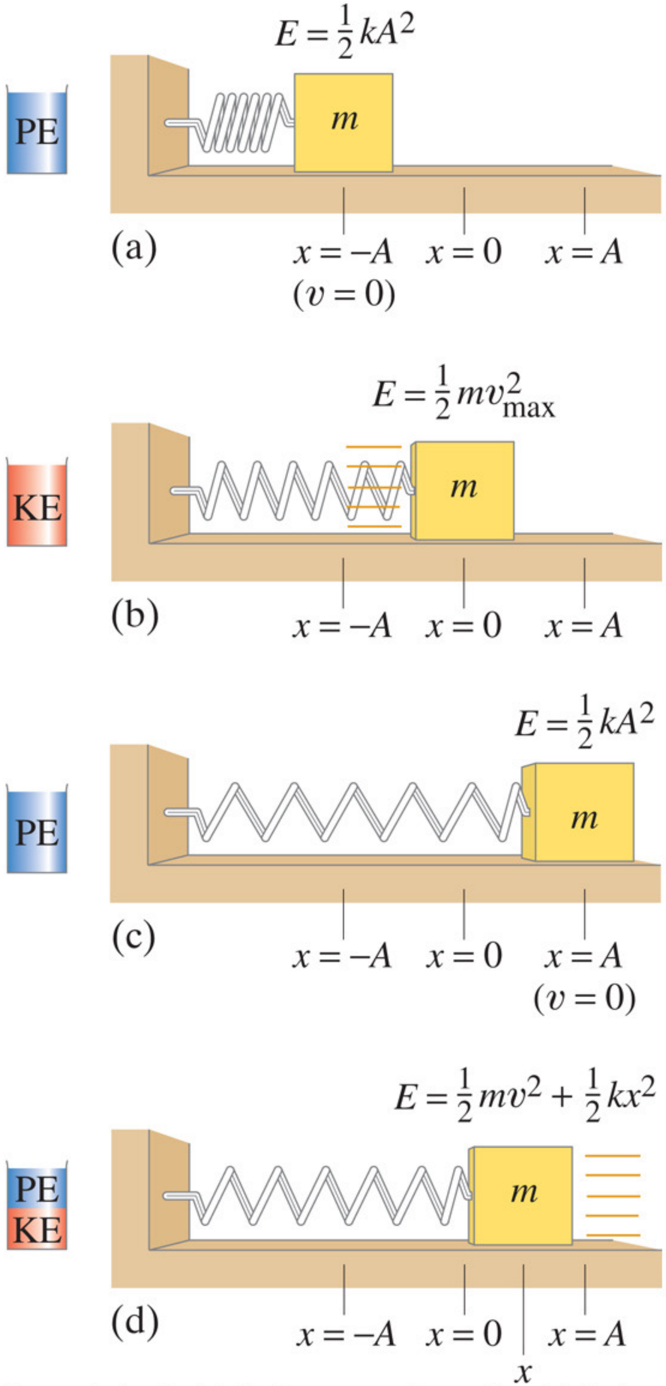
Energy in the SHO

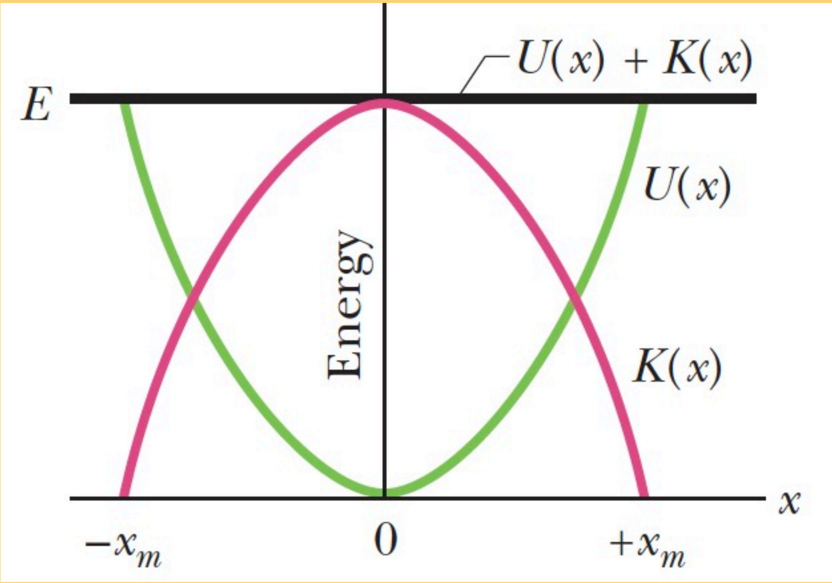




As *time* changes, the energy shifts between the two types, but the total is constant.

Energy in the SHO





NOTE: your book use the notation ***x_m*** for the amplitude, my slides use ***A***

As *position* changes, the energy shifts between the two types, but the total is constant.

Energy in the SHO

From: $$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2 \Rightarrow v_{max}^2 = (k/m)A^2$$

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \Rightarrow v^2 = \frac{k}{m}A^2\left(1 - \frac{x^2}{A^2}\right)$$

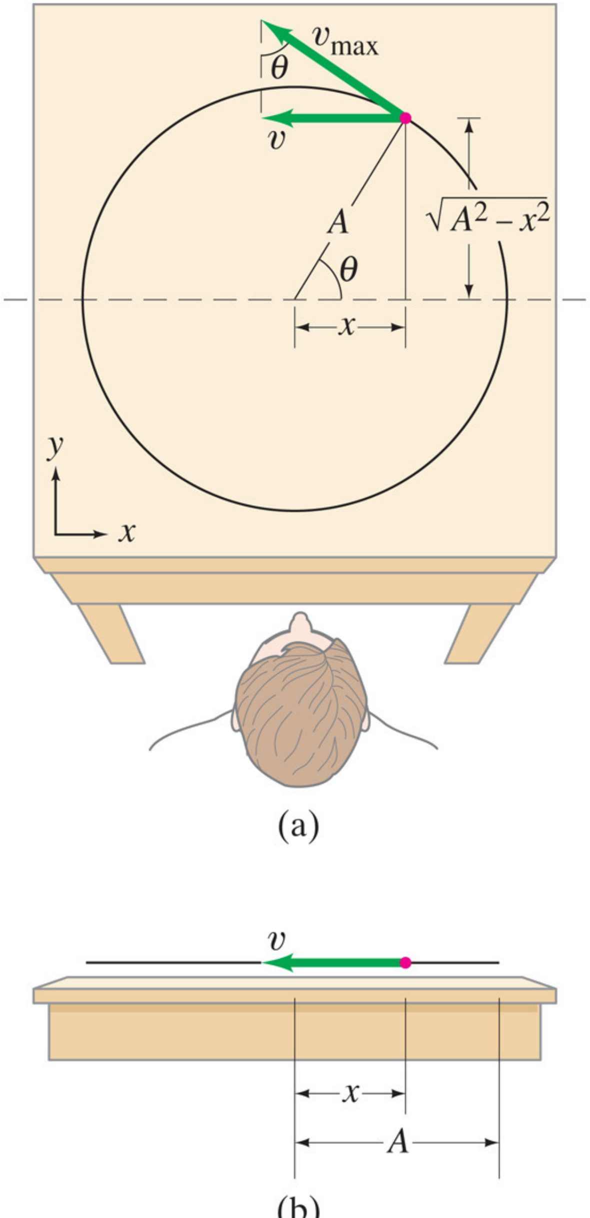
$$v = \pm v_{max}\sqrt{1 - \frac{x^2}{A^2}}$$

Ex. A spring of stiffness constant 19.6N/m has a 0.300-kg mass attached to it. It is on a frictionless table. The mass is pulled so that the spring is stretched 0.100 m from the equilibrium point, and released from rest. Determine: (a) the amplitude of the horizontal oscillation, (b) the magnitude of the maximum velocity, (c) the magnitude of the velocity when the displacement is 0.050 m from equilibrium, (d) the magnitude of the maximum acceleration of the mass, (e) the total energy, and (f) the kinetic and potential energies at half amplitude.

$$(a)0.100m \quad (b)0.808m/s \quad (c)0.70m/s \quad (d)a_{max} = F_{max}/m = kA/m = 6.53m/s^2$$

$$(e)E = 9.80\times10^{-2}J \quad PE = 2.5\times10^{-2}J \quad K = 7.3\times10^{-2}J$$

Period and Sinusoidal Nature of SHM



https://www.youtube.com/watch?v=9r0HexjGRE4

The projection onto the *x* axis of an object moving in a circle of radius *A* is identical to the SHM.

Therefore, we can use the period and frequency of a particle moving in a circle to find the period and frequency of a SHO:

$$v_{max} = \frac{2\pi A}{T} \Rightarrow T = \frac{2\pi A}{v_{max}}$$

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2 \Rightarrow A / v_{max} = \sqrt{m / k}$$

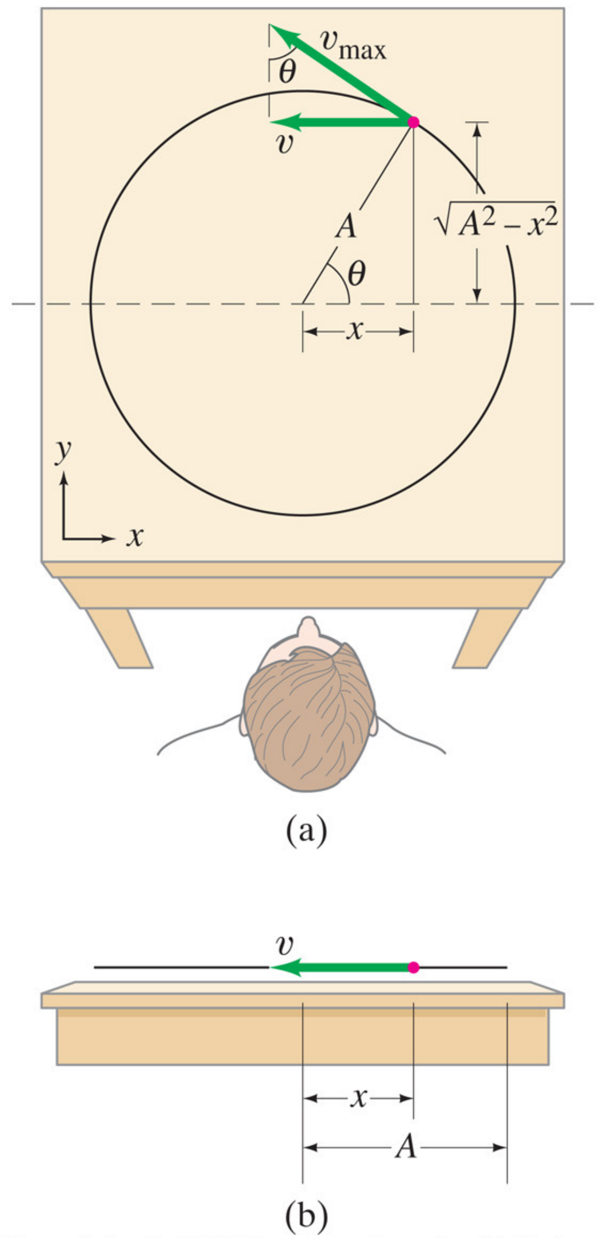
$$T = 2\pi\sqrt{\frac{m}{k}}$$

Notice that the period does not depend on A

Ex. A spider of mass 0.30 g waits in its web of negligible mass. A slight movement causes the web to vibrate with a frequency of about 15 Hz. Estimate the value of the spring stiffness constant for the web.

$$k = 2.7 N / m$$

Period and Sinusoidal Nature of SHM



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$$T = 2\pi\sqrt{\frac{m}{k}}$$

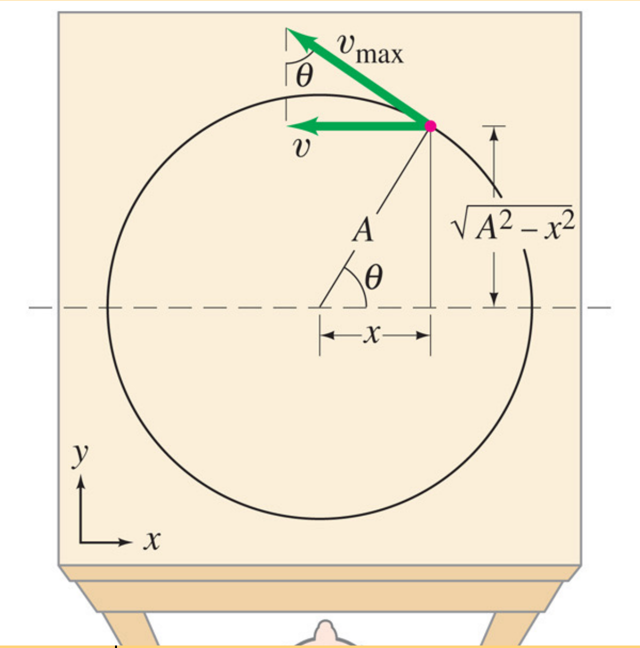
Notice that the period does not depend on A

$$\omega = 2\pi f$$

$$\omega = \frac{2\pi}{T}$$

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{(angular frequency)}$$

Position as a Function of Time - SHM



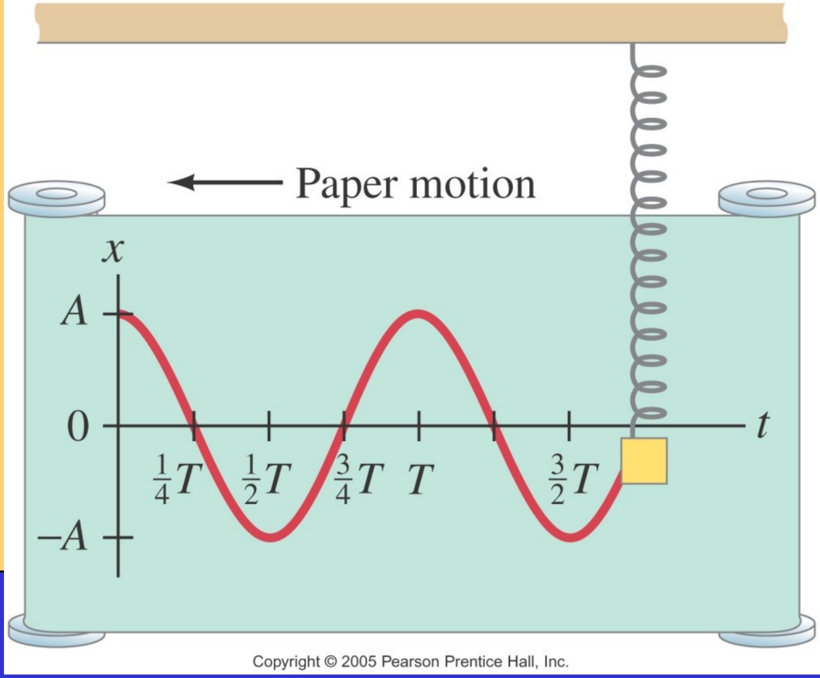
From the figure:

$$\cos\theta = x / A \Rightarrow x = A\cos\theta$$

$$\theta = \omega t \Rightarrow x = A\cos(\omega t)$$

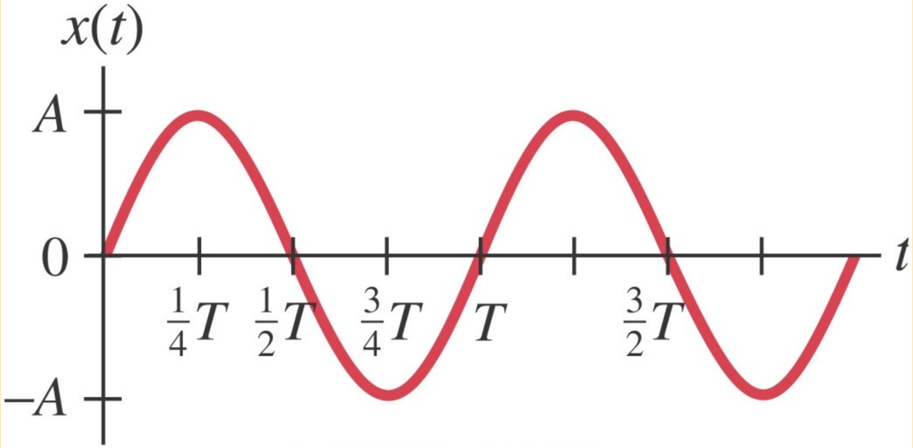
$$\omega = 2\pi f \Rightarrow x = A\cos(2\pi f t) = A\cos(2\pi t / T)$$

Notice that at t=0 we have x=A, and also at t=T. The oscillating object starts from rest (v=0) at its maximum displacement (x=A) at t=0.
The cosine function varies between 1 and –1, so x varies between A and –A.



Sinusoidal Motion - SHM

Other equations for SHM are also possible, depending on the initial conditions. For example, if at t=0 the object is at the equilibrium position and the oscillations are begun by giving the object a push to the right (+x), then



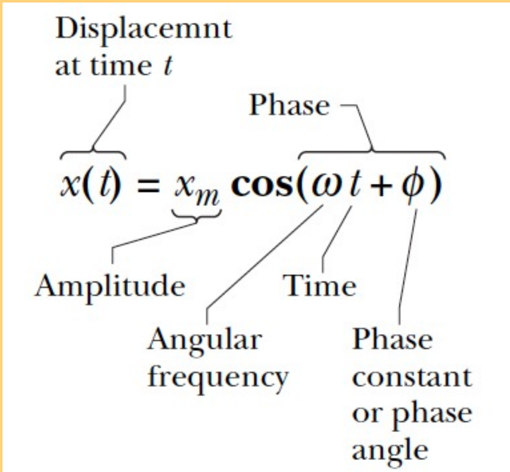
$$x = A\sin(\omega t) = A\sin(2\pi t / T)$$

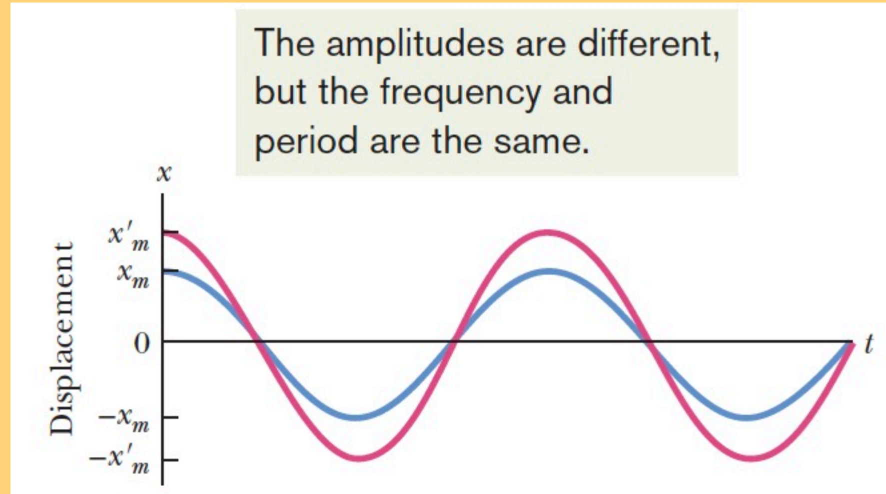
Both sine and cosine functions are referred to as being **SINUSOIDAL.**

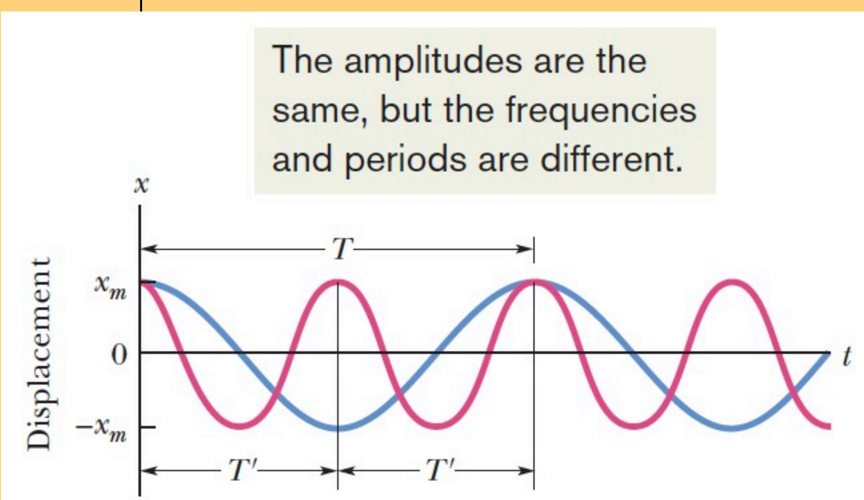
Ex. The displacement of an object is described by the following equation, where x is in meters and t in seconds: $x = (0.30m)\cos(8.0t)$
Determine (a) the amplitude, (b) frequency, (c) period, (d) maximum speed, and (e) maximum acceleration.

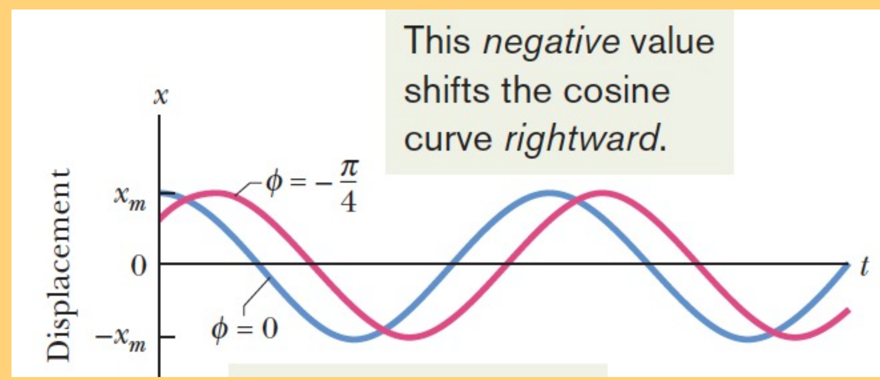
$(a) A = 0.30m \quad (b) f = 1.27Hz \quad (c) T = 0.79s \quad v_{max} = 2.4m/s \quad a_{max} = 19m/s^2$

Sinusoidal Motion - SHM









Velocity and acceleration

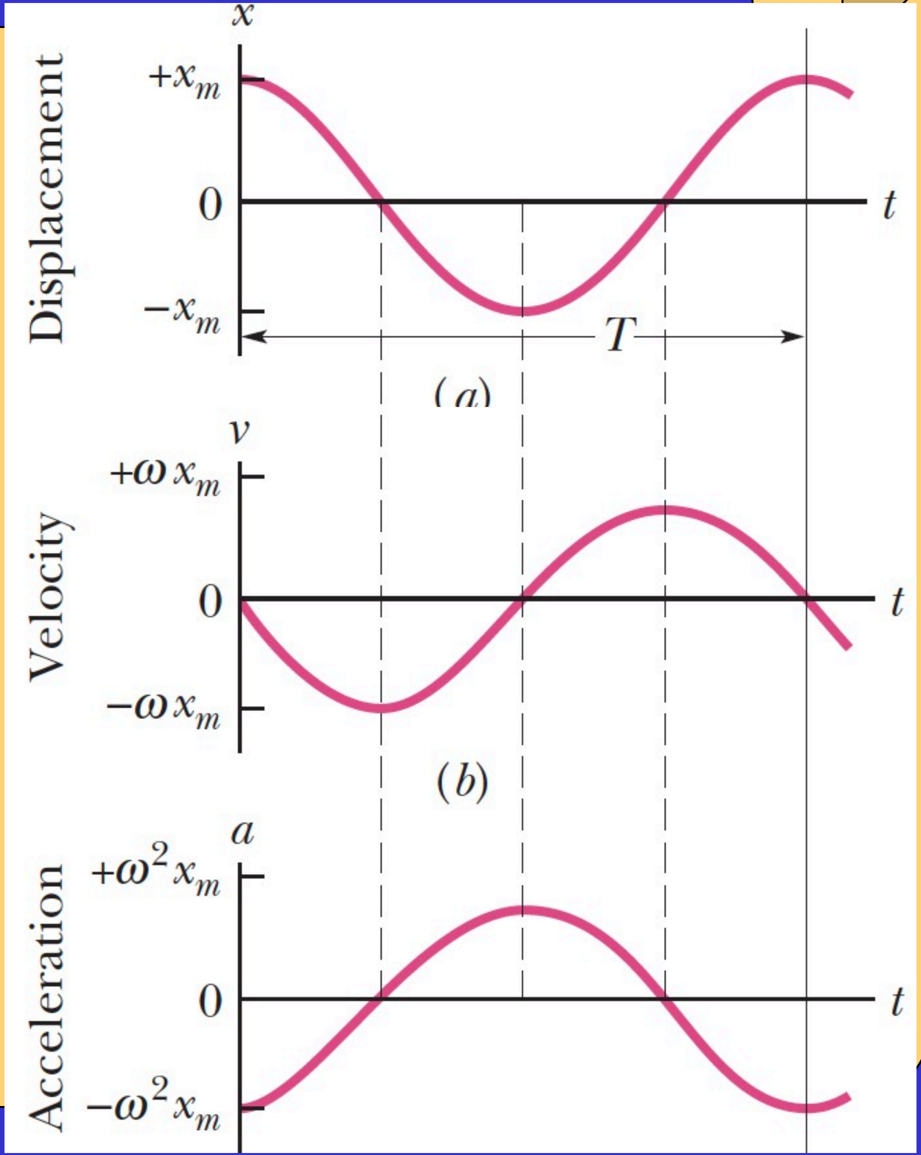
$$x(t) = x_m \cos(\omega t + \phi) \qquad \text{(displacement)}$$

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}[x_m \cos(\omega t + \phi)]$$

$$v(t) = -\omega x_m \sin(\omega t + \phi) \qquad \text{(velocity).}$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}[-\omega x_m \sin(\omega t + \phi)]$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi) \qquad \text{(acceleration)}$$



In SHM, the acceleration is proportional to the displacement but opposite in sign, and the two quantities are related by the square of the angular frequency.

$$a(t) = -\omega^2 x(t)$$

Block–spring SHM, amplitude, acceleration, phase constant

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(a) What are the angular frequency, the frequency, and the period of the resulting motion?

(b) What is the amplitude of the oscillation?

(c) What is the maximum speed v_m of the oscillating block and where is the block when it has this speed?

(d) What is the magnitude a_m of the maximum acceleration of the block?

(e) What is the phase constant ϕ for the motion?

(f) What is the displacement function $x(t)$ for the spring–block system?

Block–spring SHM, amplitude, acceleration, phase constant

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(a) What are the angular frequency, the frequency, and the period of the resulting motion?

KEY IDEA

The block–spring system forms a linear simple harmonic oscillator, with the block undergoing SHM.

Calculations: The angular frequency is given by Eq. 15-12:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65\ \text{N/m}}{0.68\ \text{kg}}} = 9.78\ \text{rad/s}$$

$$\approx 9.8\ \text{rad/s.} \qquad \text{(Answer)}$$

The frequency follows from Eq. 15-5, which yields

$$f = \frac{\omega}{2\pi} = \frac{9.78\ \text{rad/s}}{2\pi\ \text{rad}} = 1.56\ \text{Hz} \approx 1.6\ \text{Hz.} \qquad \text{(Answer)}$$

The period follows from Eq. 15-2, which yields

$$T = \frac{1}{f} = \frac{1}{1.56\ \text{Hz}} = 0.64\ \text{s} = 640\ \text{ms.} \qquad \text{(Answer)}$$

Block–spring SHM, amplitude, acceleration, phase constant

(b) What is the amplitude of the oscillation?

KEY IDEA

With no friction involved, the mechanical energy of the spring–block system is conserved.

Reasoning: The block is released from rest 11 cm from its equilibrium position, with zero kinetic energy and the elastic potential energy of the system at a maximum. Thus, the block will have zero kinetic energy whenever it is again 11 cm from its equilibrium position, which means it will never be farther than 11 cm from that position. Its maximum displacement is 11 cm:

$$x_m = 11 \text{ cm}. \qquad \text{(Answer)}$$

(c) What is the maximum speed v_m of the oscillating block, and where is the block when it has this speed?

KEY IDEA

The maximum speed v_m is the velocity amplitude ωx_m in Eq. 15-6.

Calculation: Thus, we have

$$v_m = \omega x_m = (9.78 \text{ rad/s})(0.11 \text{ m})$$
$$= 1.1 \text{ m/s.} \qquad \text{(Answer)}$$

This maximum speed occurs when the oscillating block is rushing through the origin; compare Figs. 15-4*a* and 15-4*b*, where you can see that the speed is a maximum whenever $x = 0$.

Block–spring SHM, amplitude, acceleration, phase constant

(d) What is the magnitude a_m of the maximum acceleration of the block?

KEY IDEA

The magnitude a_m of the maximum acceleration is the acceleration amplitude $\omega^2 x_m$ in Eq. 15-7.

Calculation: So, we have

$$a_m = \omega^2 x_m = (9.78 \text{ rad/s})^2(0.11 \text{ m})$$
$$= 11 \text{ m/s}^2. \quad \text{(Answer)}$$

This maximum acceleration occurs when the block is at the ends of its path. At those points, the force acting on the block has its maximum magnitude; compare Figs. 15-4*a* and 15-4*c*, where you can see that the magnitudes of the displacement and acceleration are maximum at the same times.

Block–spring SHM, amplitude, acceleration, phase constant

(e) What is the phase constant ϕ for the motion?

Calculations: Equation 15-3 gives the displacement of the block as a function of time. We know that at time $t = 0$, the block is located at $x = x_m$. Substituting these *initial conditions,* as they are called, into Eq. 15-3 and canceling x_m give us

$$1 = \cos\phi. \qquad (15\text{-}14)$$

Taking the inverse cosine then yields

$$\phi = 0 \text{ rad}. \qquad \text{(Answer)}$$

(Any angle that is an integer multiple of 2π rad also satisfies Eq. 15-14; we chose the smallest angle.)

(f) What is the displacement function $x(t)$ for the spring–block system?

Calculation: The function $x(t)$ is given in general form by Eq. 15-3. Substituting known quantities into that equation gives us

$$\begin{aligned} x(t) &= x_m \cos(\omega t + \phi) \\ &= (0.11 \text{ m}) \cos[(9.8 \text{ rad/s})t + 0] \\ &= 0.11 \cos(9.8t), \qquad \text{(Answer)} \end{aligned}$$

where x is in meters and t is in seconds.

Finding SHM phase constant from displacement and velocity

At $t = 0$, the displacement $x(0)$ of the block in a linear oscillator like that of Fig. 15-5 is -8.50 cm. (Read $x(0)$ as "x at time zero.") The block's velocity $v(0)$ then is -0.920 m/s, and its acceleration $a(0)$ is $+47.0\ \text{m/s}^2$.

(a) What is the angular frequency ω of this system?

KEY IDEA

With the block in SHM, Eqs. 15-3, 15-6, and 15-7 give its displacement, velocity, and acceleration, respectively, and each contains ω.

Calculations: Let's substitute $t = 0$ into each to see whether we can solve any one of them for ω. We find

$$x(0) = x_m \cos \phi, \qquad (15\text{-}15)$$

$$v(0) = -\omega x_m \sin \phi, \qquad (15\text{-}16)$$

and

$$a(0) = -\omega^2 x_m \cos \phi. \qquad (15\text{-}17)$$

In Eq. 15-15, ω has disappeared. In Eqs. 15-16 and 15-17, we know values for the left sides, but we do not know x_m and ϕ. However, if we divide Eq. 15-17 by Eq. 15-15, we neatly eliminate both x_m and ϕ and can then solve for ω as

$$\omega = \sqrt{-\frac{a(0)}{x(0)}} = \sqrt{-\frac{47.0\ \text{m/s}^2}{-0.0850\ \text{m}}}$$

$$= 23.5\ \text{rad/s}. \qquad \text{(Answer)}$$

(b) What are the phase constant ϕ and amplitude x_m?

Calculations: We know ω and want ϕ and x_m. If we divide Eq. 15-16 by Eq. 15-15, we eliminate one of those unknowns and reduce the other to a single trig function:

$$\frac{v(0)}{x(0)} = \frac{-\omega x_m \sin \phi}{x_m \cos \phi} = -\omega \tan \phi.$$

Solving for $\tan \phi$, we find

$$\tan \phi = -\frac{v(0)}{\omega x(0)} = -\frac{-0.920\ \text{m/s}}{(23.5\ \text{rad/s})(-0.0850\ \text{m})}$$

$$= -0.461.$$

This equation has two solutions:

$$\phi = -25^\circ \quad \text{and} \quad \phi = 180^\circ + (-25^\circ) = 155^\circ.$$

Normally only the first solution here is displayed by a calculator, but it may not be the physically possible solution. To choose the proper solution, we test them both by using them to compute values for the amplitude x_m. From Eq. 15-15, we find that if $\phi = -25^\circ$, then

$$x_m = \frac{x(0)}{\cos \phi} = \frac{-0.0850\ \text{m}}{\cos(-25^\circ)} = -0.094\ \text{m}.$$

We find similarly that if $\phi = 155^\circ$, then $x_m = 0.094$ m. Because the amplitude of SHM must be a positive constant, the correct phase constant and amplitude here are

$$\phi = 155^\circ \quad \text{and} \quad x_m = 0.094\ \text{m} = 9.4\ \text{cm}. \qquad \text{(Answer)}$$

SHM potential energy, kinetic energy, mass dampers

Many tall buildings have *mass dampers*, which are anti-sway devices to prevent them from oscillating in a wind. The device might be a block oscillating at the end of a spring and on a lubricated track. If the building sways, say, eastward, the block also moves eastward but delayed enough so that when it finally moves, the building is then moving back westward. Thus, the motion of the oscillator is out of step with the motion of the building.

Suppose the block has mass $m = 2.72 \times 10^5$ kg and is designed to oscillate at frequency $f = 10.0$ Hz and with amplitude $x_m = 20.0$ cm.

(a) What is the total mechanical energy E of the spring–block system?

KEY IDEA

The mechanical energy E (the sum of the kinetic energy $K = \frac{1}{2}mv^2$ of the block and the potential energy $U = \frac{1}{2}kx^2$ of the spring) is constant throughout the motion of the oscillator. Thus, we can evaluate E at any point during the motion.

Calculations: Because we are given amplitude x_m of the oscillations, let's evaluate E when the block is at position $x = x_m$, where it has velocity $v = 0$. However, to evaluate U at that point, we first need to find the spring constant k. From Eq. 15-12 ($\omega = \sqrt{k/m}$) and Eq. 15-5 ($\omega = 2\pi f$), we find

$$\begin{aligned} k &= m\omega^2 = m(2\pi f)^2 \\ &= (2.72 \times 10^5 \text{ kg})(2\pi)^2(10.0 \text{ Hz})^2 \\ &= 1.073 \times 10^9 \text{ N/m}. \end{aligned}$$

We can now evaluate E as

$$\begin{aligned} E &= K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \\ &= 0 + \tfrac{1}{2}(1.073 \times 10^9 \text{ N/m})(0.20 \text{ m})^2 \\ &= 2.147 \times 10^7 \text{ J} \approx 2.1 \times 10^7 \text{ J}. \end{aligned} \quad \text{(Answer)}$$

(b) What is the block's speed as it passes through the equilibrium point?

Calculations: We want the speed at $x = 0$, where the potential energy is $U = \frac{1}{2}kx^2 = 0$ and the mechanical energy is entirely kinetic energy. So, we can write

$$E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

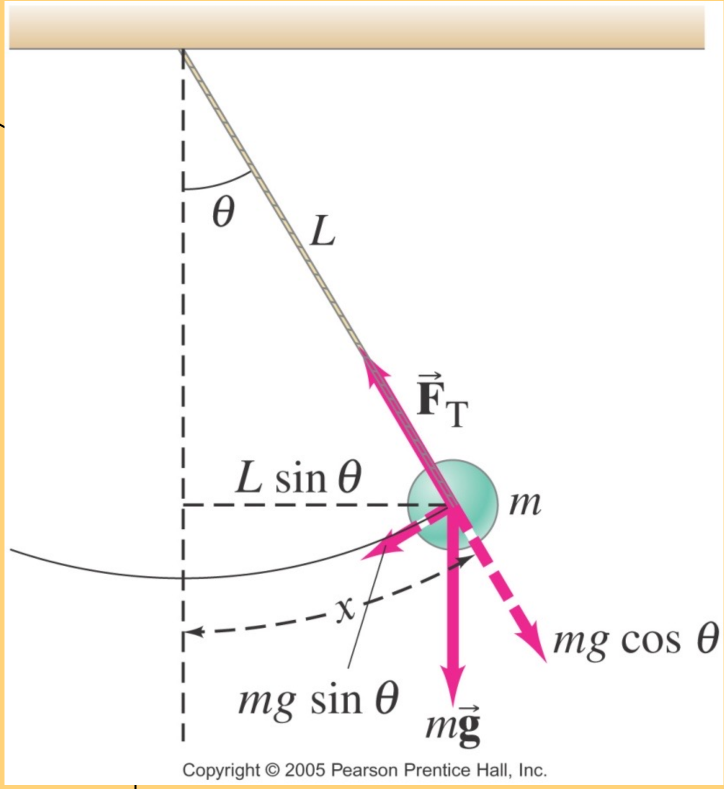
$$2.147 \times 10^7 \text{ J} = \tfrac{1}{2}(2.72 \times 10^5 \text{ kg})v^2 + 0,$$

or

$$v = 12.6 \text{ m/s}. \quad \text{(Answer)}$$

Because E is entirely kinetic energy, this is the maximum speed v_m.

The Simple Pendulum



A simple pendulum consists of a mass at the end of a lightweight cord. We assume that the cord does not stretch, and that its mass is negligible.

The restoring force is $F = -mg\sin\theta$

However, if the angle is small, $\sin\theta \approx \theta$, and the restoring force becomes proportional to the displacement. We then have **SHM.**

$$F = -mg\sin\theta \approx -mg\theta$$

$$x = L\theta \Rightarrow F \approx -\frac{mg}{L}x$$

which fits Hooke's law, the effective force constant being k=mg/L

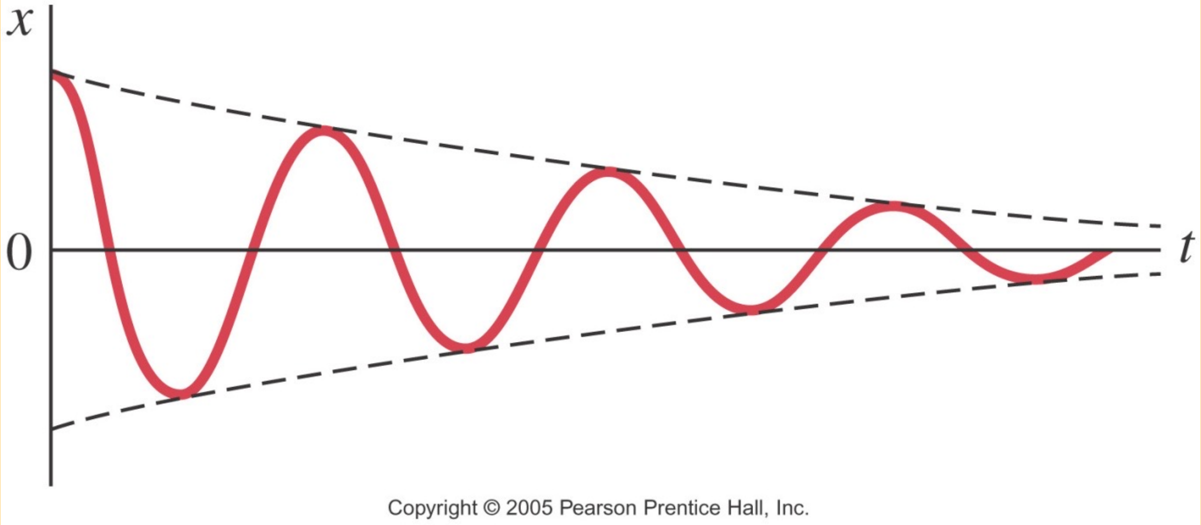
$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}} \Rightarrow T = 2\pi\sqrt{\frac{L}{g}} = \frac{1}{f}$$

The period and frequency do not depend on the mass

Ex.11-9 A geologist uses a simple pendulum that has length of 37.10 cm and a frequency of 0.8190 Hz at a particular location on Earth. What is the acceleration of gravity at that place? $g = 9.824 m/s^2$

Damped Harmonic Motion

Damped harmonic motion is harmonic motion with a frictional or drag force. If the damping is small, we can treat it as an “envelope” that modifies the undamped oscillation.



There are systems where damping is unwanted, such as clocks and watches. Then there are systems in which it is wanted, such as automobile shock absorbers and earthquake protection for buildings

Damped Harmonic Motion

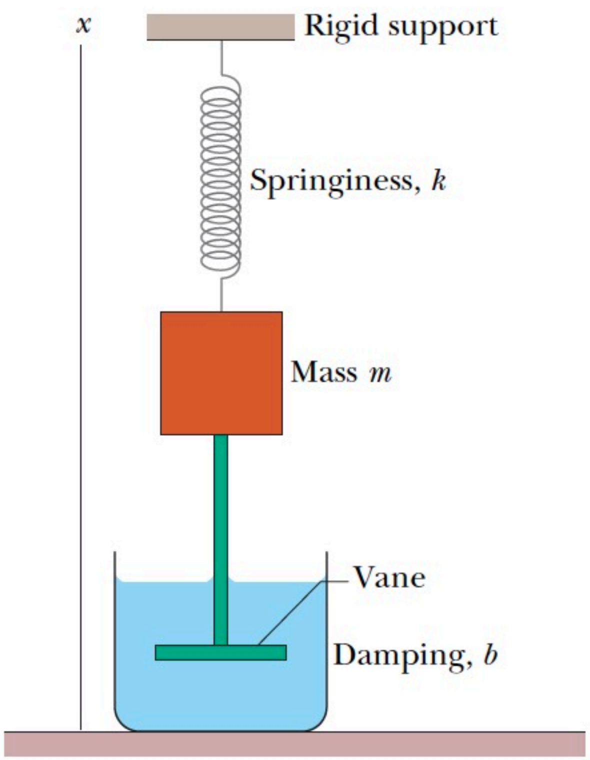


Fig. 15-14 An idealized damped simple harmonic oscillator. A vane immersed in a liquid exerts a damping force on the block as the block oscillates parallel to the x axis.

The force on the block from the spring

$$F_s = -kx$$

damping force $F_d = -bv$,

where b is a **damping constant**

gravitational force on the block is negligible relative to F_d and F_s

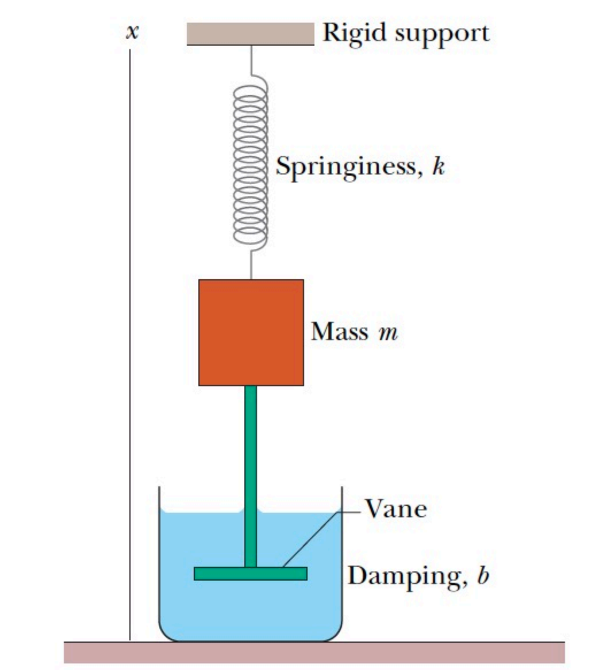
Newton's second law $-bv - kx = ma$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0.$$

$$x(t) = x_m e^{-bt/2m}\cos(\omega' t + \phi) \qquad (15\text{-}42)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Damped Harmonic Motion



$$x(t) = x_m e^{-bt/2m} \cos(\omega' t + \phi) \quad (15\text{-}42)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

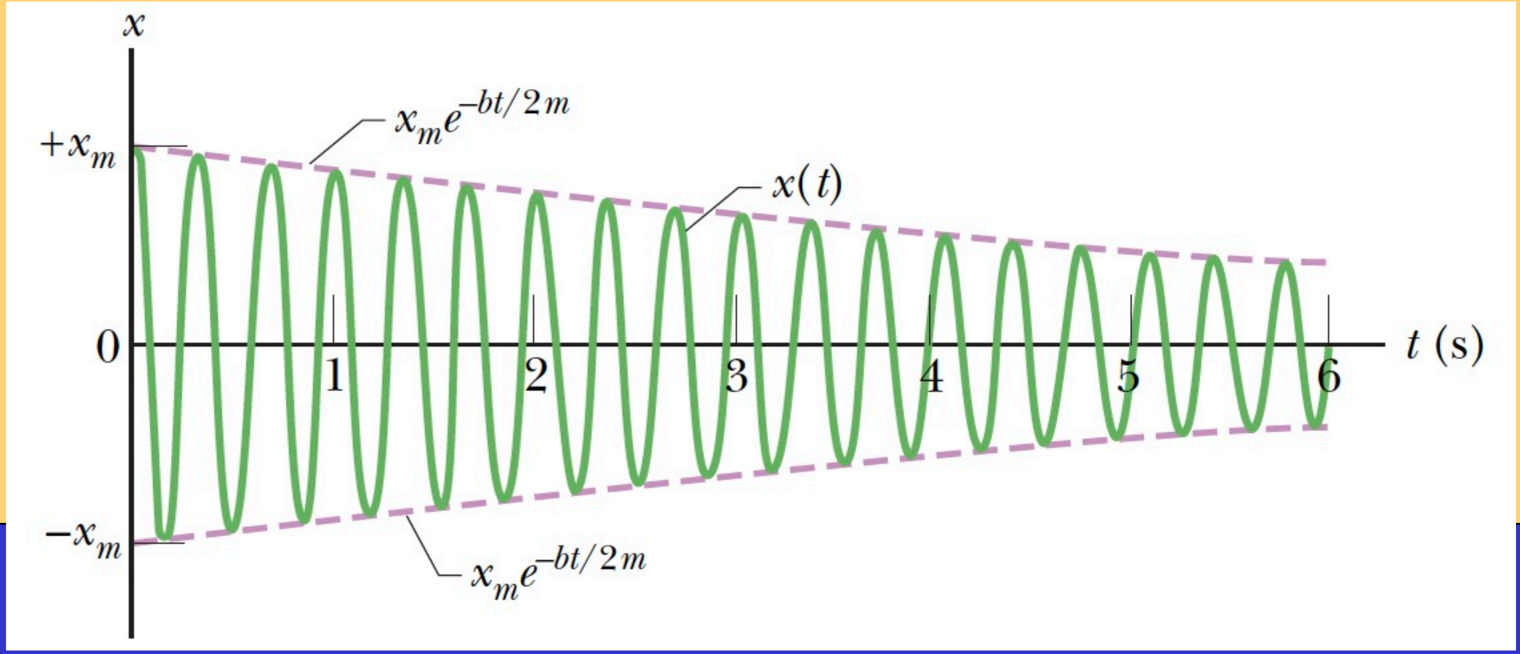
We can regard Eq. 15-42 as a cosine function whose amplitude, which is $x_m e^{-bt/2m}$, gradually decreases with time, as Fig. 15-15 suggests. For an undamped oscillator, the mechanical energy is constant and is given by Eq. 15-21 ($E = \frac{1}{2}kx_m^2$). If the oscillator is damped, the mechanical energy is not constant but decreases with time. If the damping is small, we can find $E(t)$ by replacing x_m in Eq. 15-21 with $x_m e^{-bt/2m}$, the amplitude of the damped oscillations. By doing so, we find that

$$E(t) \approx \tfrac{1}{2}kx_m^2 e^{-bt/m}, \quad (15\text{-}44)$$

Damped Harmonic Motion

$$x(t) = x_m e^{-bt/2m} \cos(\omega' t + \phi) \qquad (15\text{-}42)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$



Damped harmonic oscillator, time to decay, energy

For the damped oscillator of Fig. 15-14, $m = 250$ g, $k = 85$ N/m, and $b = 70$ g/s.

(a) What is the period of the motion?

KEY IDEA

Because $b \ll \sqrt{km} = 4.6$ kg/s, the period is approximately that of the undamped oscillator.

Calculation: From Eq. 15-13, we then have

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.25\text{ kg}}{85\text{ N/m}}} = 0.34\text{ s.} \qquad \text{(Answer)}$$

(b) How long does it take for the amplitude of the damped oscillations to drop to half its initial value?

KEY IDEA

The amplitude at time t is displayed in Eq. 15-42 as $x_m e^{-bt/2m}$.

Calculations: The amplitude has the value x_m at $t = 0$. Thus, we must find the value of t for which

$$x_m e^{-bt/2m} = \tfrac{1}{2}x_m.$$

Canceling x_m and taking the natural logarithm of the equation that remains, we have $\ln\frac{1}{2}$ on the right side and

$$\ln(e^{-bt/2m}) = -bt/2m$$

on the left side. Thus,

$$t = \frac{-2m\ln\frac{1}{2}}{b} = \frac{-(2)(0.25\text{ kg})(\ln\frac{1}{2})}{0.070\text{ kg/s}}$$

$$= 5.0\text{ s.} \qquad \text{(Answer)}$$

Because $T = 0.34$ s, this is about 15 periods of oscillation.

(c) How long does it take for the mechanical energy to drop to one-half its initial value?

KEY IDEA

From Eq. 15-44, the mechanical energy at time t is $\frac{1}{2}kx_m^2 e^{-bt/m}$.

Calculations: The mechanical energy has the value $\frac{1}{2}kx_m^2$ at $t = 0$. Thus, we must find the value of t for which

$$\tfrac{1}{2}kx_m^2 e^{-bt/m} = \tfrac{1}{2}(\tfrac{1}{2}kx_m^2).$$

If we divide both sides of this equation by $\frac{1}{2}kx_m^2$ and solve for t as we did above, we find

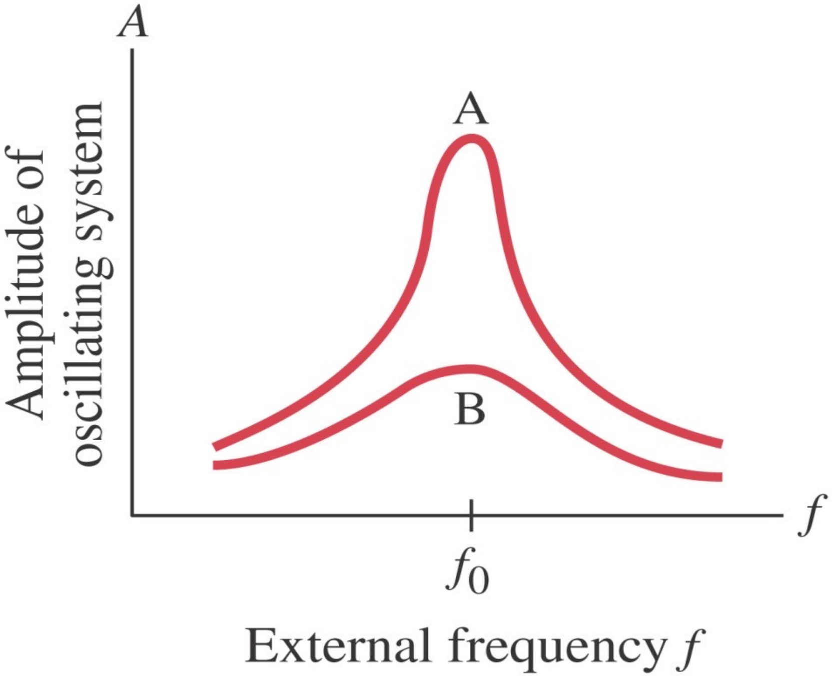
$$t = \frac{-m\ln\frac{1}{2}}{b} = \frac{-(0.25\text{ kg})(\ln\frac{1}{2})}{0.070\text{ kg/s}} = 2.5\text{ s.} \qquad \text{(Answer)}$$

This is exactly half the time we calculated in (b), or about 7.5 periods of oscillation. Figure 15-15 was drawn to illustrate this sample problem.

Forced Vibrations, Resonance

Forced vibrations occur when there is a **periodic driving force**. This force may or may not have the same period as the natural frequency f_0 of the system.

If the frequency is the same as the natural frequency, the amplitude becomes quite large. This is called **resonance.**



The sharpness of the resonant peak depends on the damping. If the damping is small (A), it can be quite sharp; if the damping is larger (B), it is less sharp.

Examples of resonance: child on a swing, singer shattering a crystal, Tacoma Narrows Bridge.

Tacoma Bridge: http://br.youtube.com/watch?v=3mclp9QmCGs

PROBLEMS TO SOLVE

14, 15, 17, 18, 20

28-33, 35

58-59