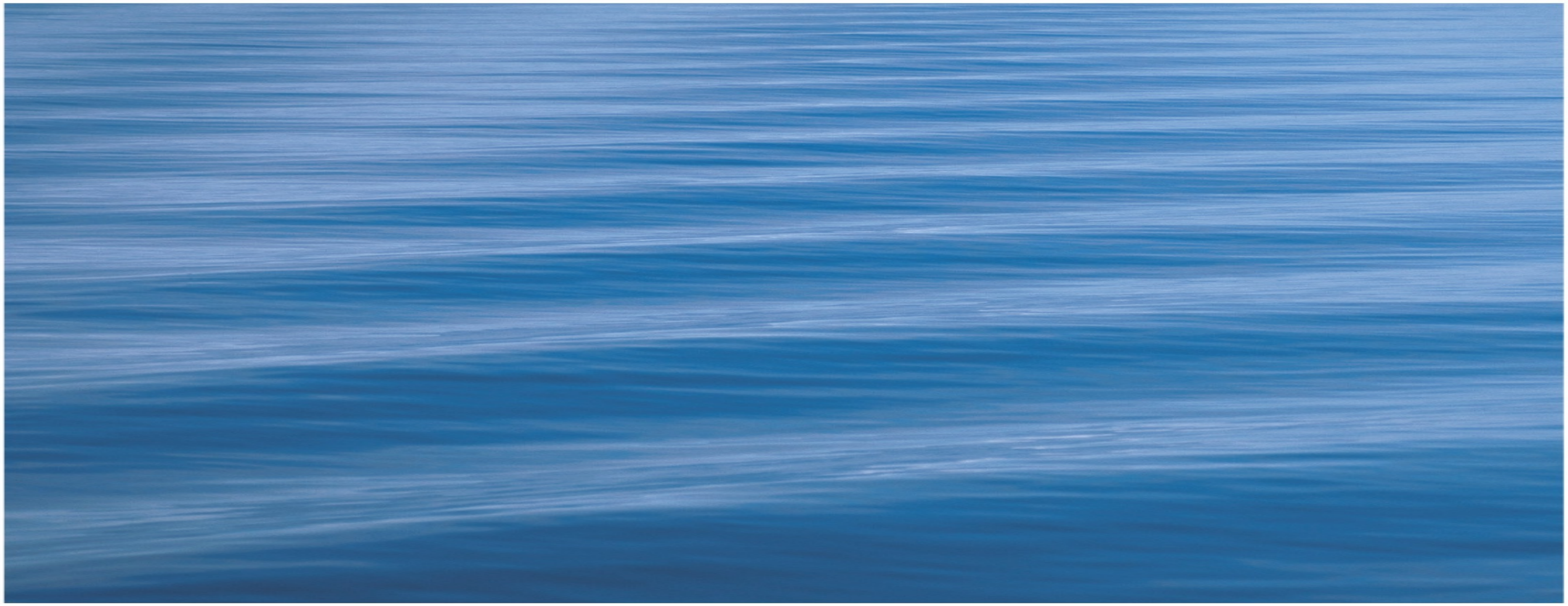
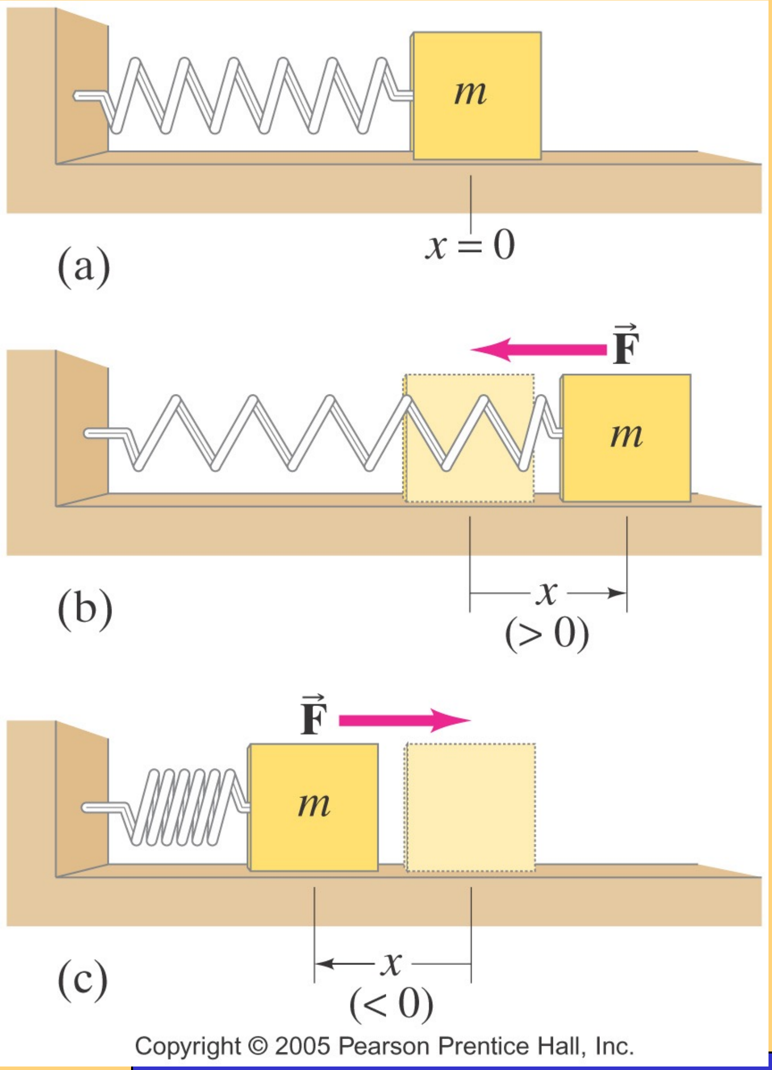
Chapter 11 – Vibrations and Waves

Many objects **vibrate** (or oscillate).
Examples: pendulum, strings of a guitar, atoms in a molecule, etc.

Waves have as a source a vibration.
Examples of waves: ocean waves, waves on a string, sound waves, electromagnetic waves – light.



Simple Harmonic Oscillator - SHO



If an object **vibrates or oscillates** back and forth over the same path, each cycle taking the *same amount of time*, the motion is called ***periodic***.

Assume that the surface is frictionless.

There is a point where the spring is neither stretched nor compressed; this is the ***equilibrium position***. We measure displacement from that point ($x = 0$).

The force exerted by the spring depends on the displacement (Hooke's law):

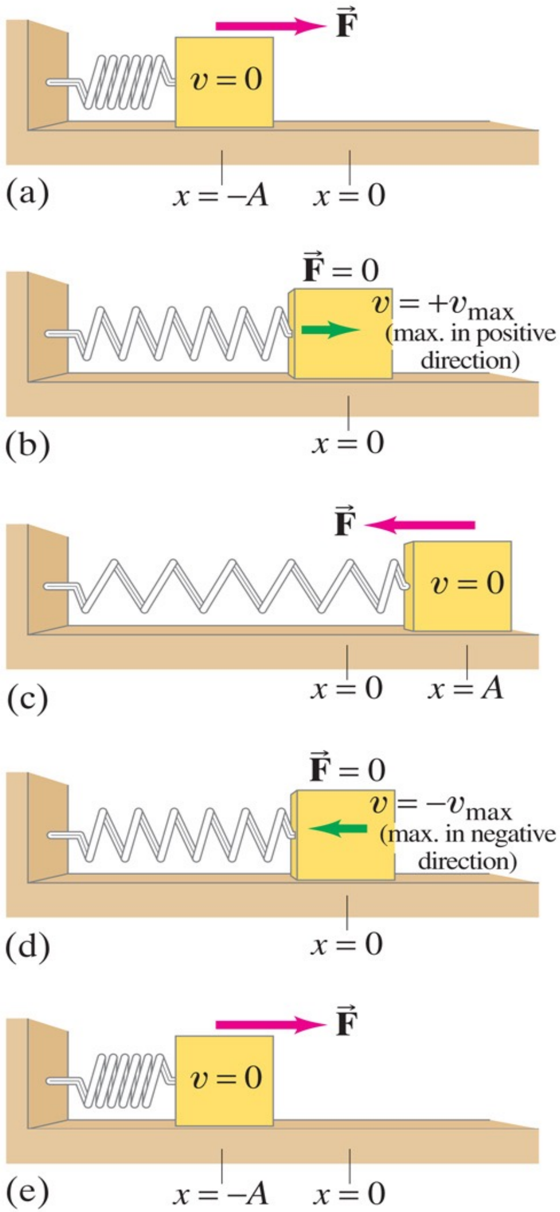
k is the spring constant

$$F = -kx$$

The minus sign on the force indicates that it is a ***restoring force*** – it is directed to restore the mass to its equilibrium position.

The force is not constant, so the **acceleration is not constant** either

Simple Harmonic Motion – SHM



Any vibrating system for which the restoring force is directly proportional to the negative of the displacement is said to exhibit: **simple harmonic motion (SHM).**
Such system called: **simple harmonic oscillator (SHO).**
Many natural **vibrations** are simple harmonic.

To study vibrational motion, we need some definitions:

- **Displacement** is measured from the equilibrium point
- **Amplitude** is the maximum displacement
- A **cycle** is a full to-and-fro motion
- **Period** is the time required to complete one cycle
- **Frequency** is the number of cycles completed per second

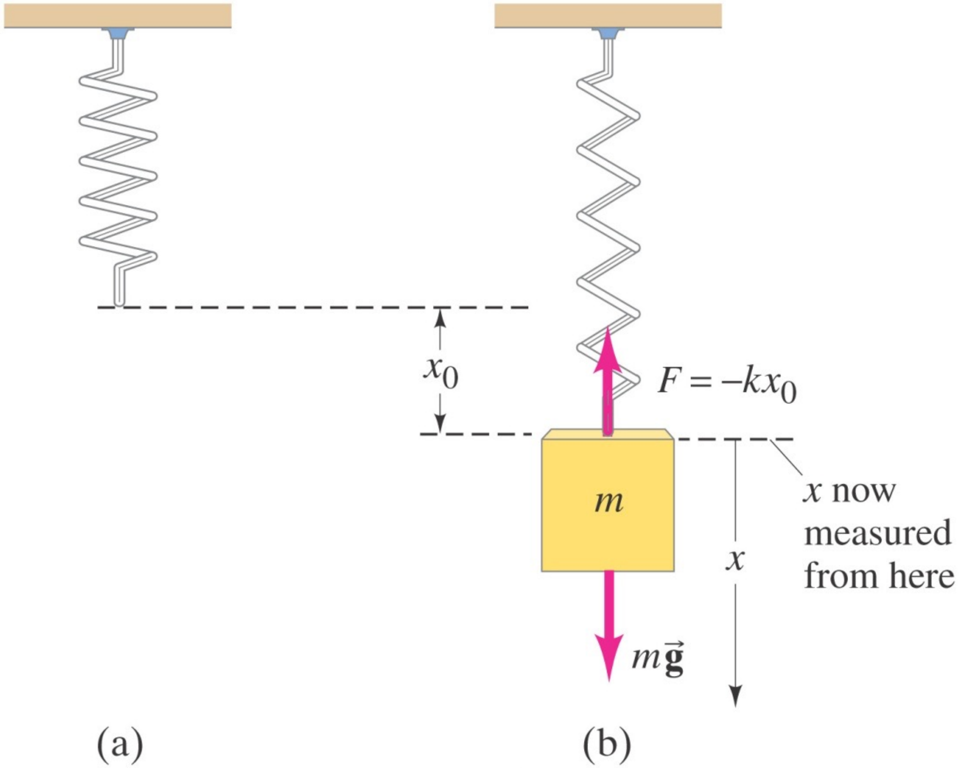
$$T = \frac{1}{f}$$

Simple Harmonic Motion – SHM

If the spring is hung vertically, the only change is in the equilibrium position, which is at the point where the spring force equals the gravitational force.

$$\sum F = 0 = mg - kx_0$$

$$x_0 = mg / k$$



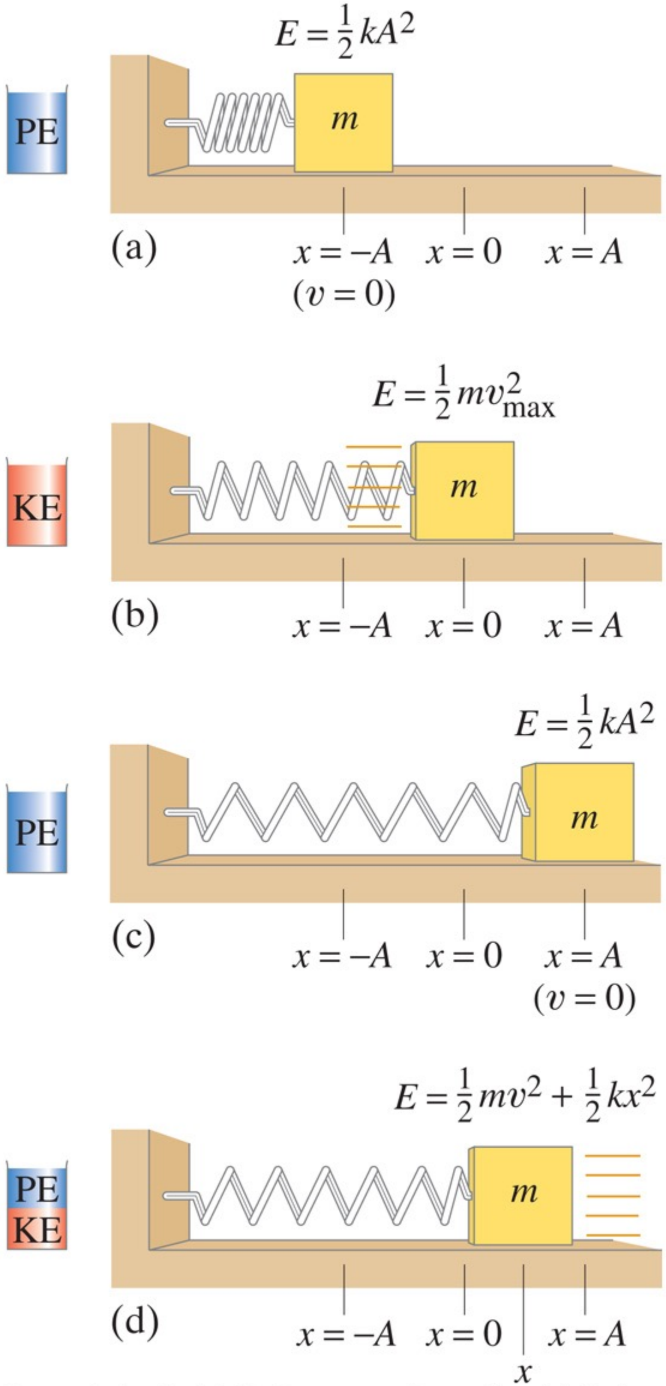
Ex. 11-1 When a family of four with a total mass of 200 kg step into their 1200-kg car, the car's springs compress 3.0 cm. (a) What is the spring constant of the car's springs, assuming they act as a single spring? (b) How far will the car lower if loaded with 300 kg rather than 200 kg?

$$(a)\ 6.5\times10^4\ N/m \quad (b)\ 4.5\times10^{-2}\ m$$

Ex. 11-2 Which of the following represent a simple harmonic oscillator:

$$(a)\ F = -0.5x^2\ (b)\ F = -2.3y\ (c)\ F = 8.6c\ (d)\ F = -4\theta ?$$

Only (b) and (d)



Energy in the SHO

The potential energy of a spring: $PE = \frac{1}{2}kx^2$

The total mechanical energy: It is conserved because the system is frictionless

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

If the mass is at the limits of its motion, the energy is all potential.

$$E = \frac{1}{2}kA^2$$

If the mass is at the equilibrium point, the energy is all kinetic.

$$E = \frac{1}{2}mv_{max}^2$$

From conservation of energy, at intermediate points:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$

Energy in the SHO

From: $$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2$$

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2 \Rightarrow v_{\max}^2 = (k/m)A^2$$

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \Rightarrow v^2 = \frac{k}{m}A^2\left(1 - \frac{x^2}{A^2}\right)$$

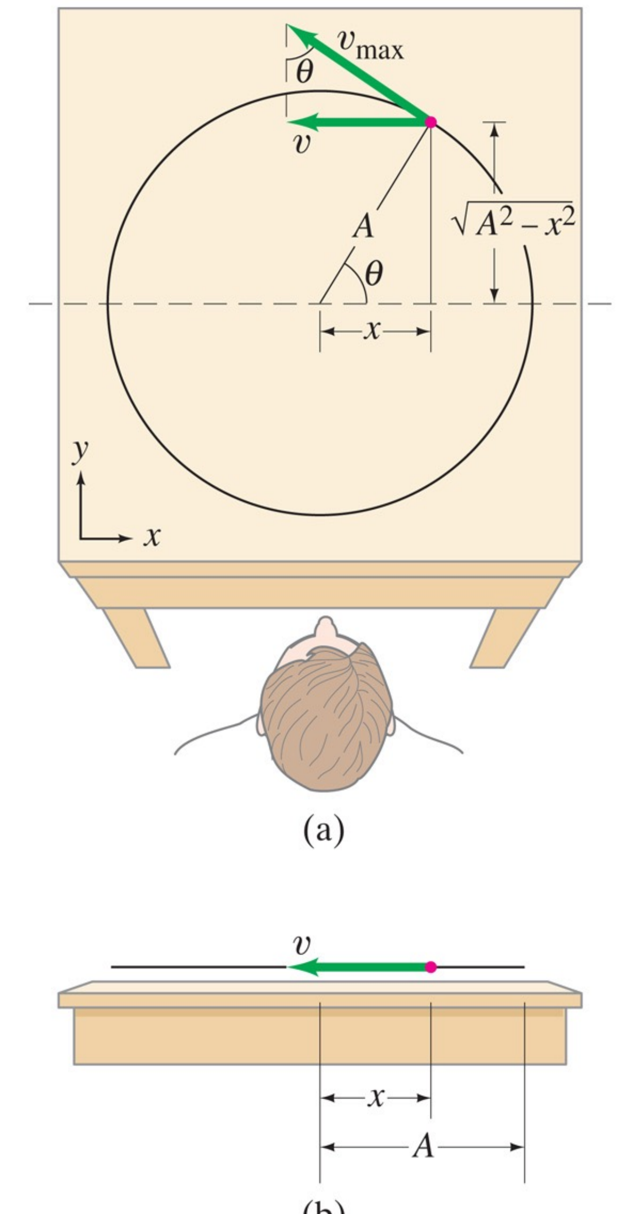
$$v = \pm v_{\max}\sqrt{1 - \frac{x^2}{A^2}}$$

Ex.11-4 A spring of stiffness constant 19.6N/m has a 0.300-kg mass attached to it. It is on a frictionless table. The mass is pulled so that the spring is stretched 0.100 m from the equilibrium point, and released from rest. Determine:
(a) the amplitude of the horizontal oscillation, (b) the magnitude of the maximum velocity, (c) the magnitude of the velocity when the displacement is 0.050 m from equilibrium, (d) the magnitude of the maximum acceleration of the mass, (e) the total energy, and (f) the kinetic and potential energies at half amplitude.

$$(a) 0.100m \quad (b) 0.808m/s \quad (c) 0.70m/s \quad (d) a_{\max} = F_{\max}/m = kA/m = 6.53m/s^2$$

$$(e) E = 9.80\times10^{-2}J \quad PE = 2.5\times10^{-2}J \quad K = 7.3\times10^{-2}J$$

Period and Sinusoidal Nature of SHM



https://www.youtube.com/watch?v=9r0HexjGRE4

The projection onto the *x* axis of an object moving in a circle of radius *A* is identical to the SHM.

Therefore, we can use the period and frequency of a particle moving in a circle to find the period and frequency of a SHO:

$$v_{max} = \frac{2\pi A}{T} \Rightarrow T = \frac{2\pi A}{v_{max}}$$

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2 \Rightarrow A / v_{max} = \sqrt{m/k}$$

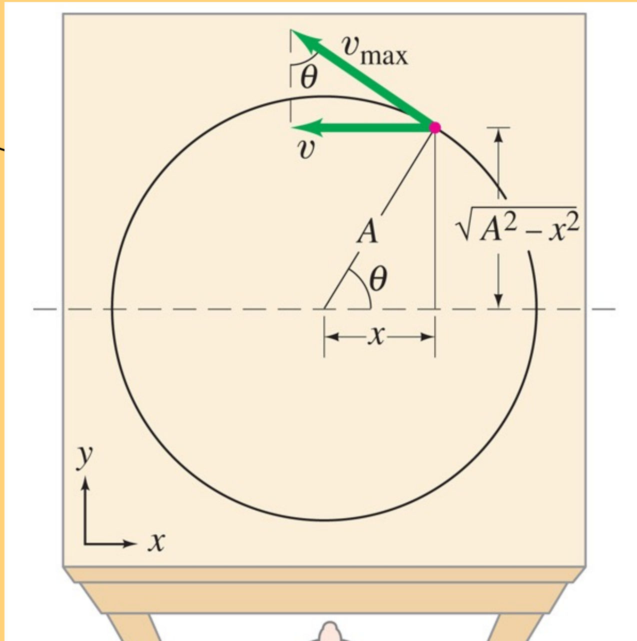
$$T = 2\pi\sqrt{\frac{m}{k}}$$

Notice that the period does not depend on A

Ex. 11-6 A spider of mass 0.30 g waits in its web of negligible mass. A slight movement causes the web to vibrate with a frequency of about 15 Hz. Estimate the value of the spring stiffness constant for the web.

$$k = 2.7\,N/m$$

Position as a Function of Time - SHM

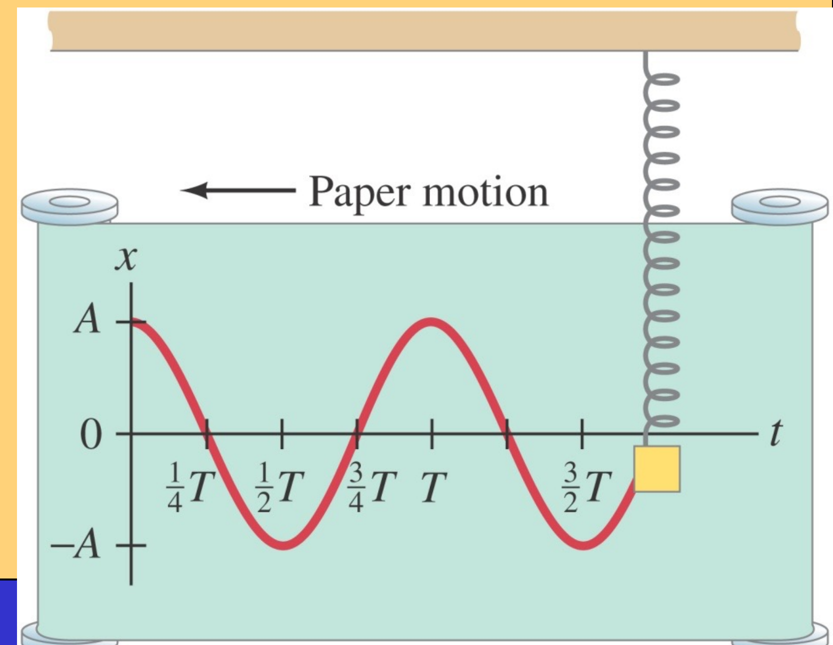


From the figure:

$$\cos\theta = x / A \Rightarrow x = A\cos\theta$$

$$\theta = \omega t \Rightarrow x = A\cos(\omega t)$$

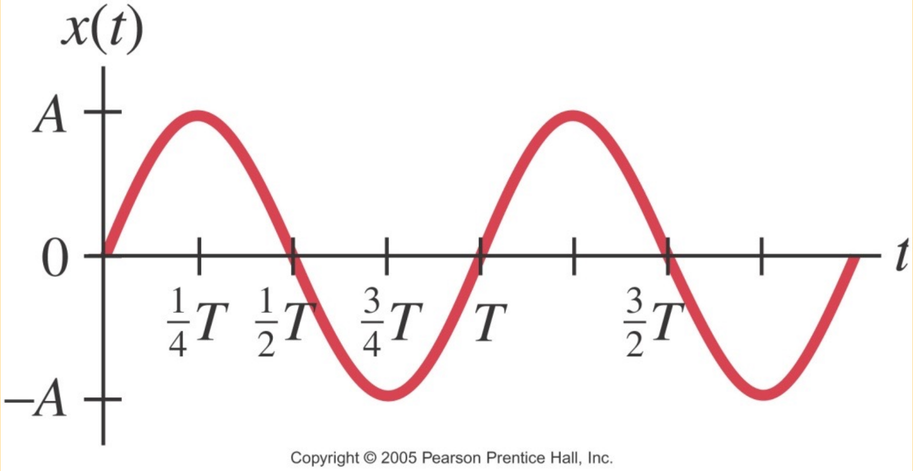
$$\omega = 2\pi f \Rightarrow x = A\cos(2\pi f t) = A\cos(2\pi t / T)$$



Notice that at t=0 we have x=A, and also at t=T. The oscillating object starts from rest (v=0) at its maximum displacement (x=A) at t=0.
The cosine function varies between 1 and –1, so x varies between A and –A.

Sinusoidal Motion - SHM

Other equations for SHM are also possible, depending on the initial conditions. For example, if at t=0 the object is at the equilibrium position and the oscillations are begun by giving the object a push to the right (+x), then

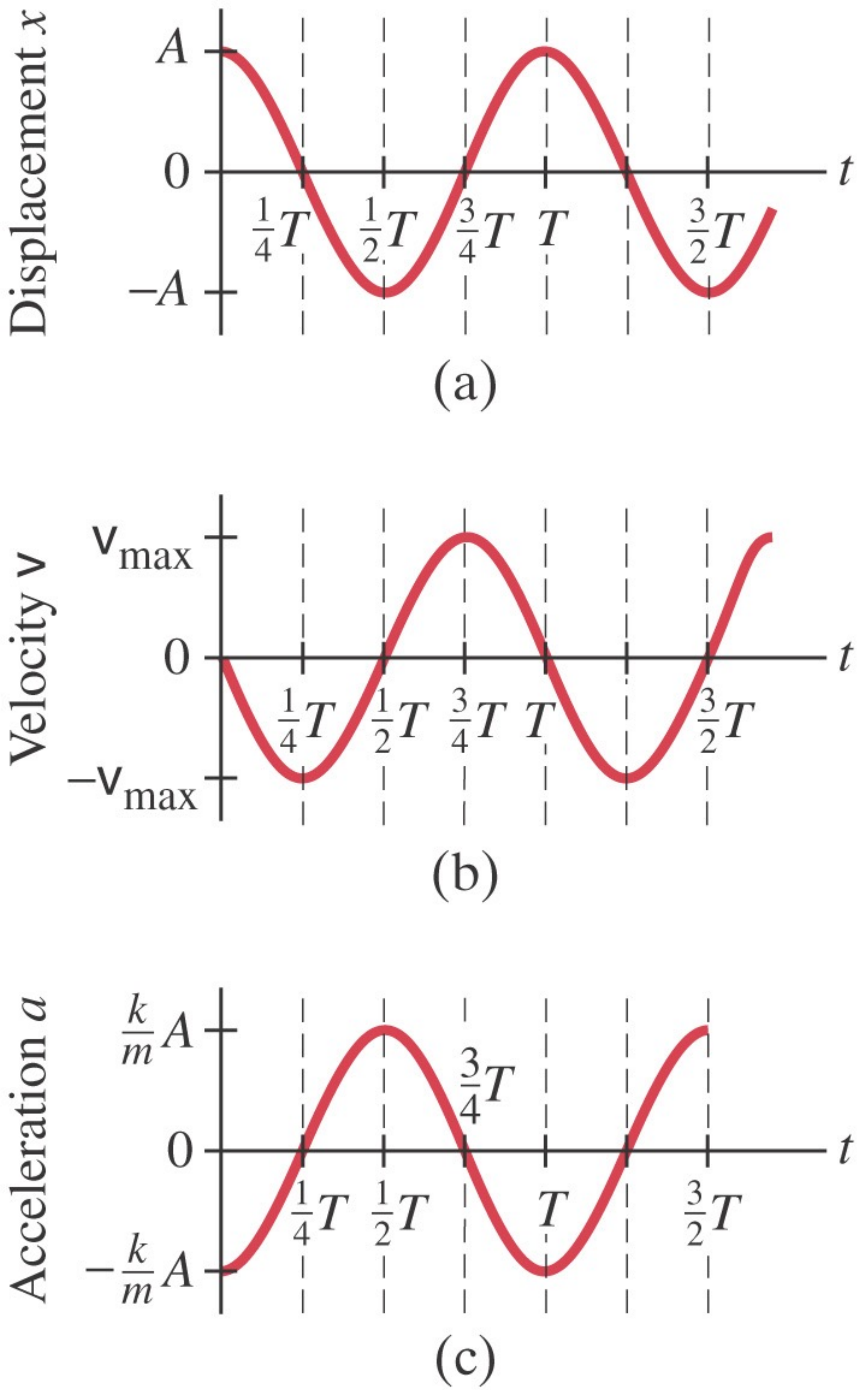


$$x = A\sin(\omega t) = A\sin(2\pi t / T)$$

Both sine and cosine functions are referred to as being **SINUSOIDAL.**

Ex. 11-7 The displacement of an object is described by the following equation, where x is in meters and t in seconds: $x = (0.30m)\cos(8.0t)$
Determine (a) the amplitude, (b) frequency, (c) period, (d) maximum speed, and (e) maximum acceleration.

$(a) A = 0.30m \quad (b) f = 1.27Hz \quad (c) T = 0.79s \quad v_{\max} = 2.4m/s \quad a_{\max} = 19m/s^2$



Equation for v and a

The velocity and acceleration can be calculated as functions of time; the results are below, and are plotted at left.

$$v = -v_{\max} \sin \omega t \qquad \textbf{(11-9)}$$

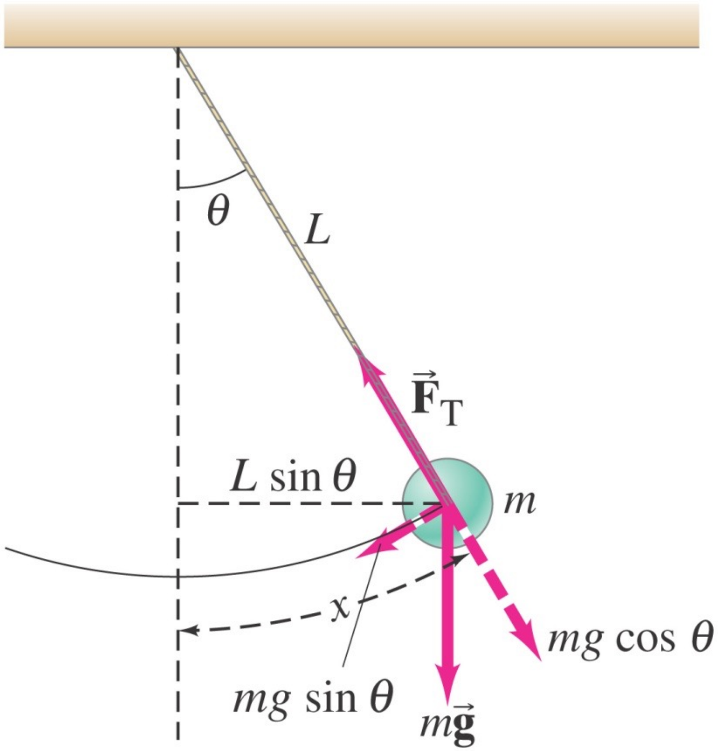
$$v_{\max} = A\sqrt{\frac{k}{m}}$$

$$a = -a_{\max} \cos(2\pi t/T) \qquad \textbf{(11-10)}$$

$$a_{\max} = kA/m$$

EXAMPLE 11-8

The Simple Pendulum



A simple pendulum consists of a mass at the end of a lightweight cord. We assume that the cord does not stretch, and that its mass is negligible.

The restoring force is $F = -mg\sin\theta$

However, if the angle is small, sin $\theta \approx \theta$, and the restoring force becomes proportional to the displacement. We then have **SHM.**

$$F = -mg\sin\theta \approx -mg\theta$$

$$x = L\theta \Rightarrow F \approx -\frac{mg}{L}x$$

which fits Hooke's law, the effective force constant being k=mg/L

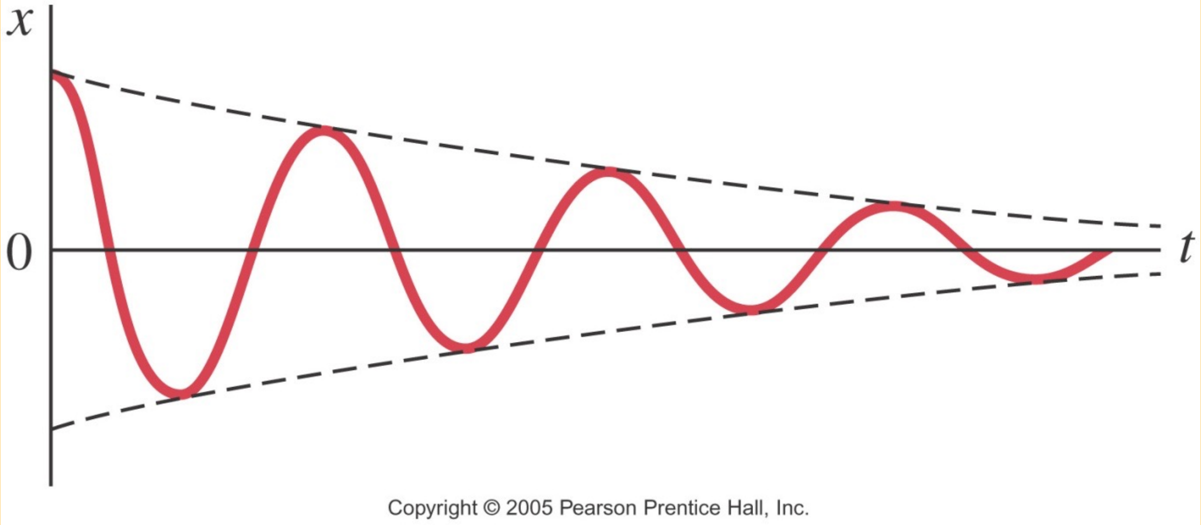
$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}} \Rightarrow T = 2\pi\sqrt{\frac{L}{g}} = \frac{1}{f}$$

The period and frequency do not depend on the mass

Ex.11-9 A geologist uses a simple pendulum that has length of 37.10 cm and a frequency of 0.8190 Hz at a particular location on Earth. What is the acceleration of gravity at that place? $g = 9.824 m/s^2$

Damped Harmonic Motion

Damped harmonic motion is harmonic motion with a frictional or drag force. If the damping is small, we can treat it as an "envelope" that modifies the undamped oscillation.

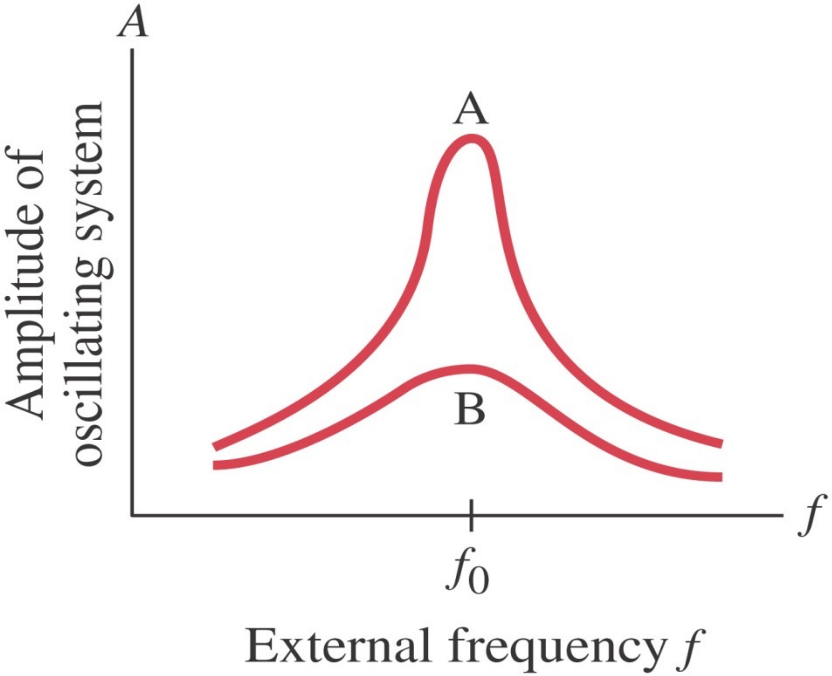


There are systems where damping is unwanted, such as clocks and watches. Then there are systems in which it is wanted, such as automobile shock absorbers and earthquake protection for buildings

Forced Vibrations, Resonance

Forced vibrations occur when there is a **periodic driving force**. This force may or may not have the same period as the natural frequency f_0 of the system.

If the frequency is the same as the natural frequency, the amplitude becomes quite large. This is called **resonance.**

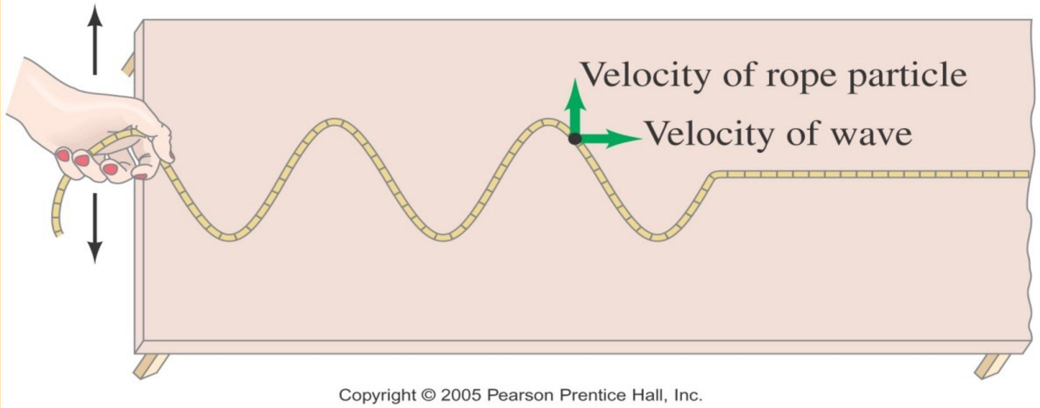


The sharpness of the resonant peak depends on the damping. If the damping is small (A), it can be quite sharp; if the damping is larger (B), it is less sharp.

Examples of resonance: child on a swing, singer shattering a crystal, Tacoma Narrows Bridge.

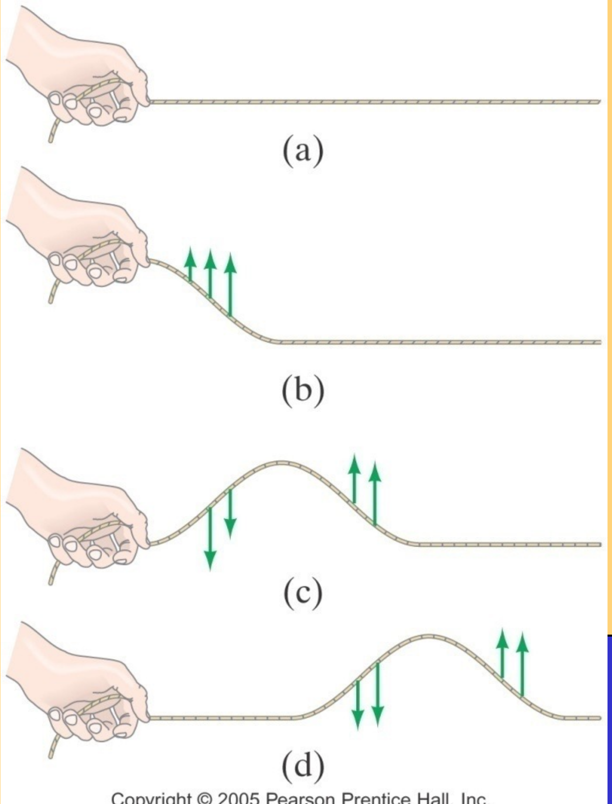
Tacoma Bridge: http://br.youtube.com/watch?v=3mclp9QmCGs

Wave Motion



A wave travels along its medium, but the individual particles just move up and down.

Waves **carry energy** from one place to another.



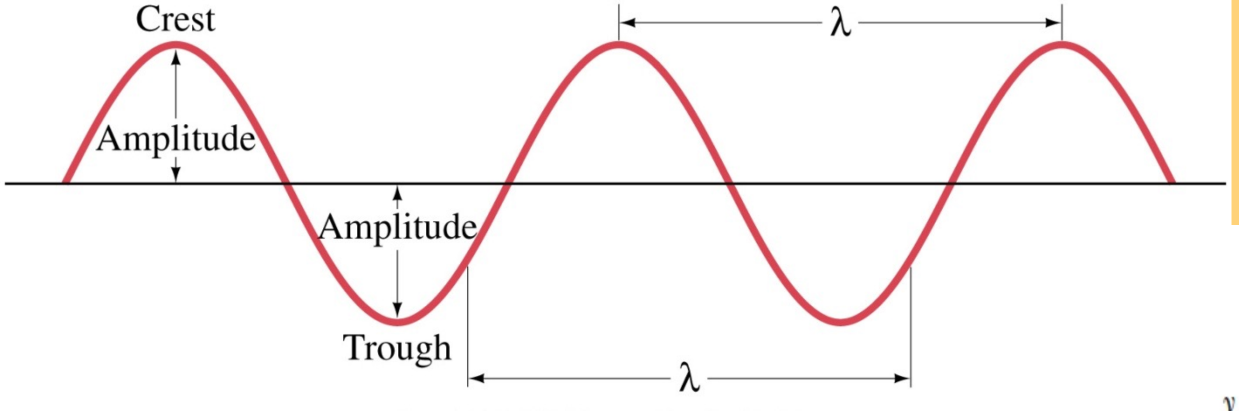
A wave may start with a single **pulse** -- figure. Cohesive forces between adjacent sections of the rope cause the pulse to travel outward. It is similar in other media.

Continuous or periodic waves start with vibrations too, but they are continuous.

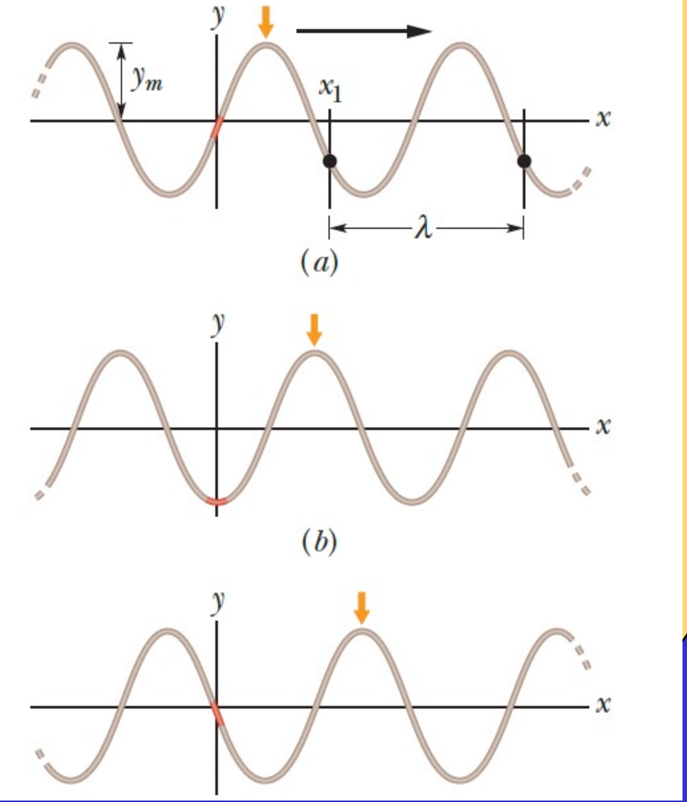
The source of any wave is a vibration.

If the vibration is SHM, then the wave will have a sinusoidal shape.

Wave Motion

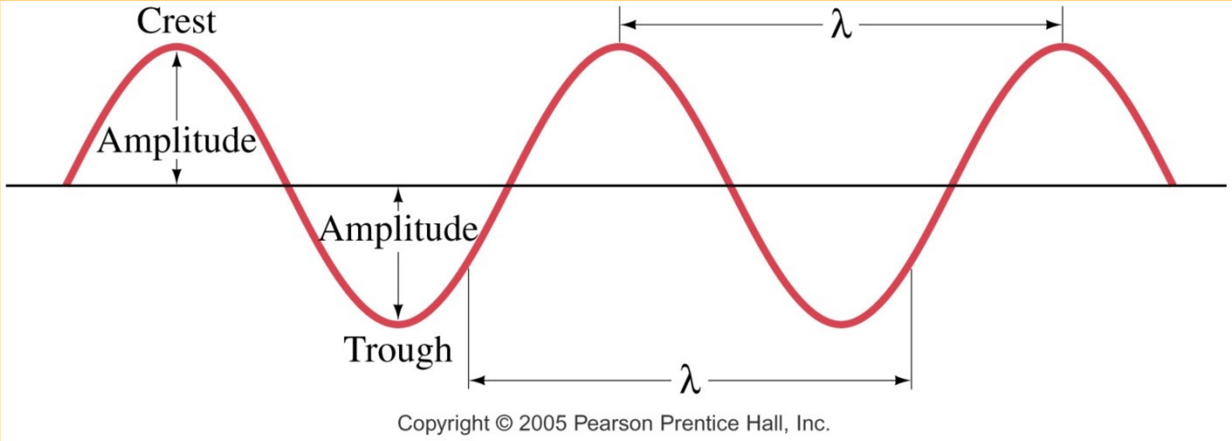


Crest
λ
Amplitude
Amplitude
Trough
λ



Watch this spot in this series of snapshots.
y
y_m
x_1
x
λ
(a)
y
x
(b)
y
x

Wave Motion



Wave characteristics:

•**Amplitude**, *A:* maximum height of a crest or depth of a trough

• **Wavelength**, *λ:* distance between successive crests, or any two successive identical points

• **Frequency** *f* : number of crests (or complete cycles) that pass a given point per unit of time. **Period** *T:* time elapsed between two successive crests

• **Wave velocity**: velocity at which wave crests move. A wave crest travels a distance of one wavelength in a time equal to one period.

$$v = \lambda / T = \lambda f$$

Wave Motion

Wave velocity: velocity at which wave crests move. A wave crest travels a distance of one wavelength in a time equal to one period.

$$v = \lambda / T = \lambda f$$

The speed of a wave on a stretched string or cord depends on the the tension in the cord as

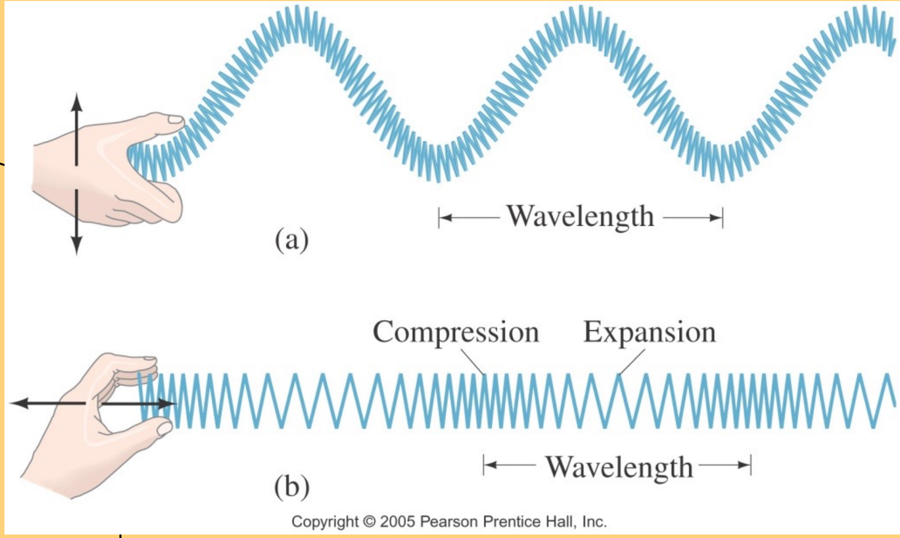
$$v = \sqrt{\frac{F_T}{m / L}}$$

***CAREFUL**: wave velocity is different from the velocity of a particle in the medium!*

Ex. 11-11 A wave whose wavelength is 0.30 m is traveling down a 300-m-long wire whose total mass is 15 kg. If the wire is under tension of 1000 N, what are the speed and frequency of this wave?

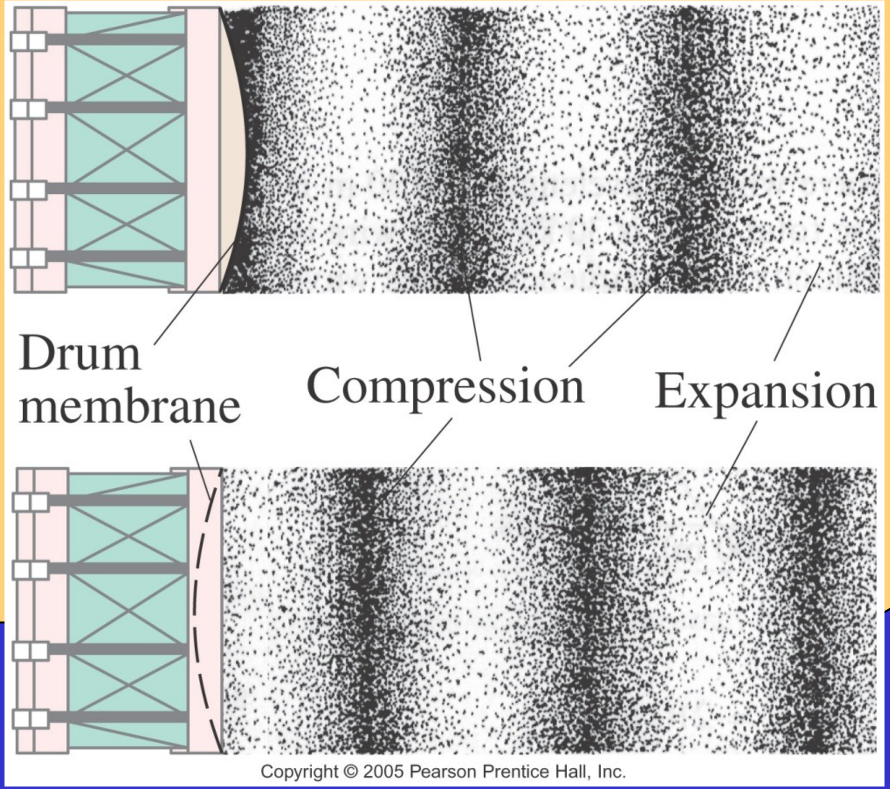
v = 140 m/s and f = 470 Hz

Transverse and Longitudinal Waves

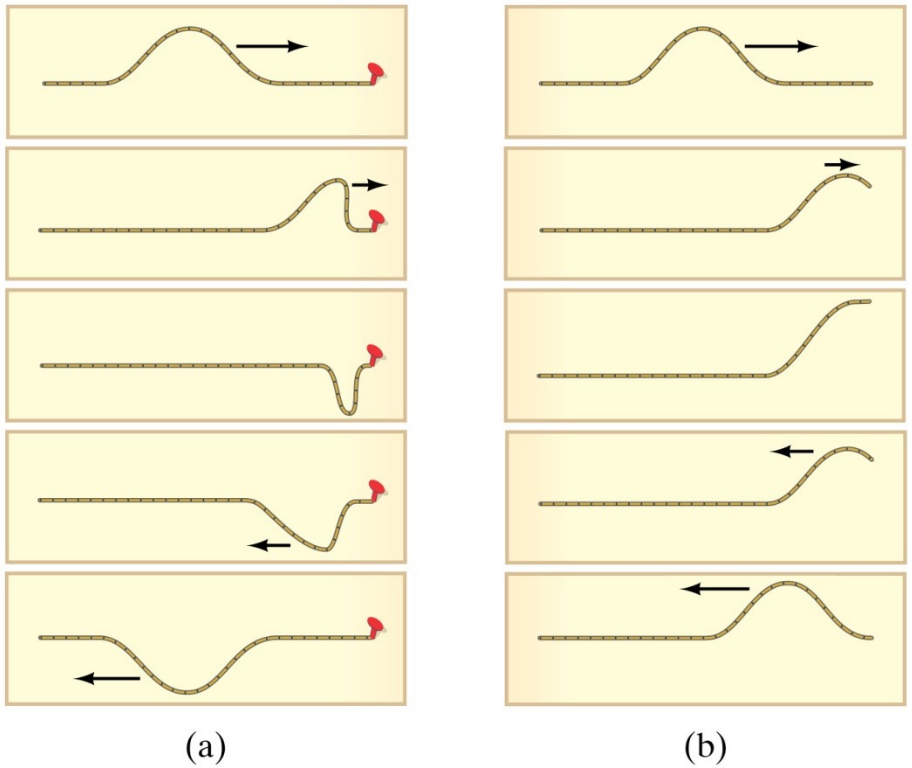


The motion of particles in a wave can either be perpendicular to the wave direction (**transverse**) or parallel to it (**longitudinal**).

Sound waves are longitudinal waves:



Reflection and Transmission of Waves



When a wave strikes an obstacle or come to the end of the medium it is traveling in, at least part of it is reflected. Echo is an example of reflection

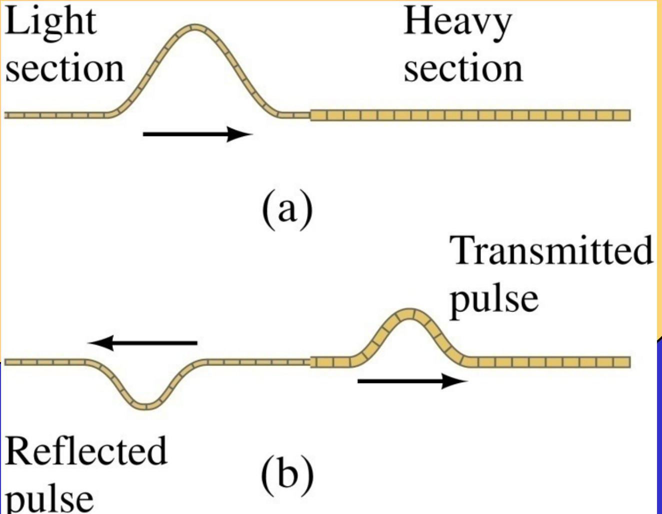
http://www.kettering.edu/~drussell/Demos/reflect/reflect.html

A wave hitting an **obstacle** will be reflected (a), and its reflection will be **inverted**. (action and reaction)

A wave reaching the end of its medium, but where the medium is still **free** to move, will be reflected (b), and its reflection will be **upright.**

A wave encountering a denser medium will be partly reflected and partly transmitted; if the wave speed is less in the denser medium, the wavelength will be shorter, because f does not change

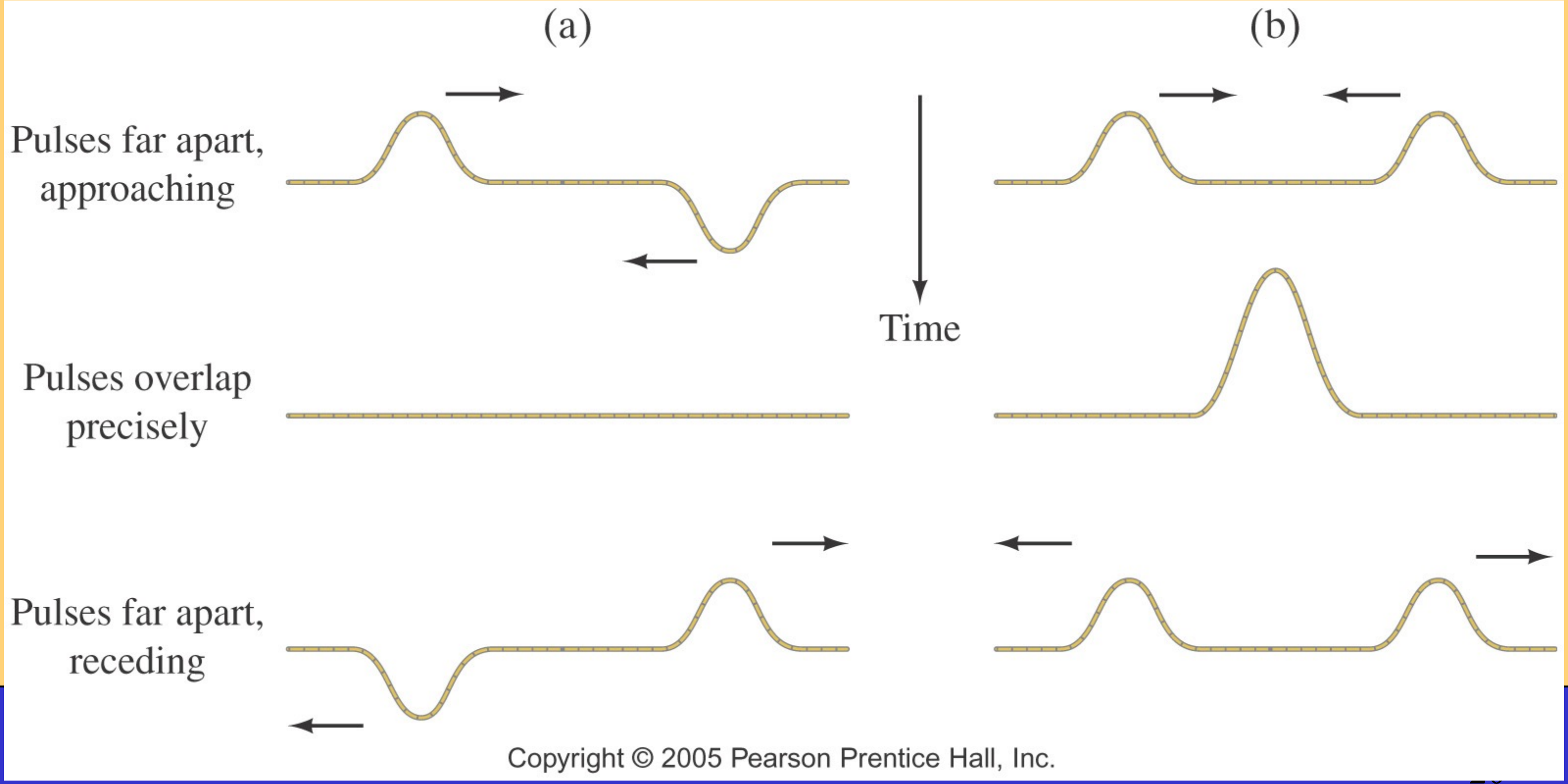
$$\lambda = v / f$$



Interference;Principle of Superposition

The **superposition principle** says that when two waves pass through the same point, the displacement is the arithmetic sum of the individual displacements.

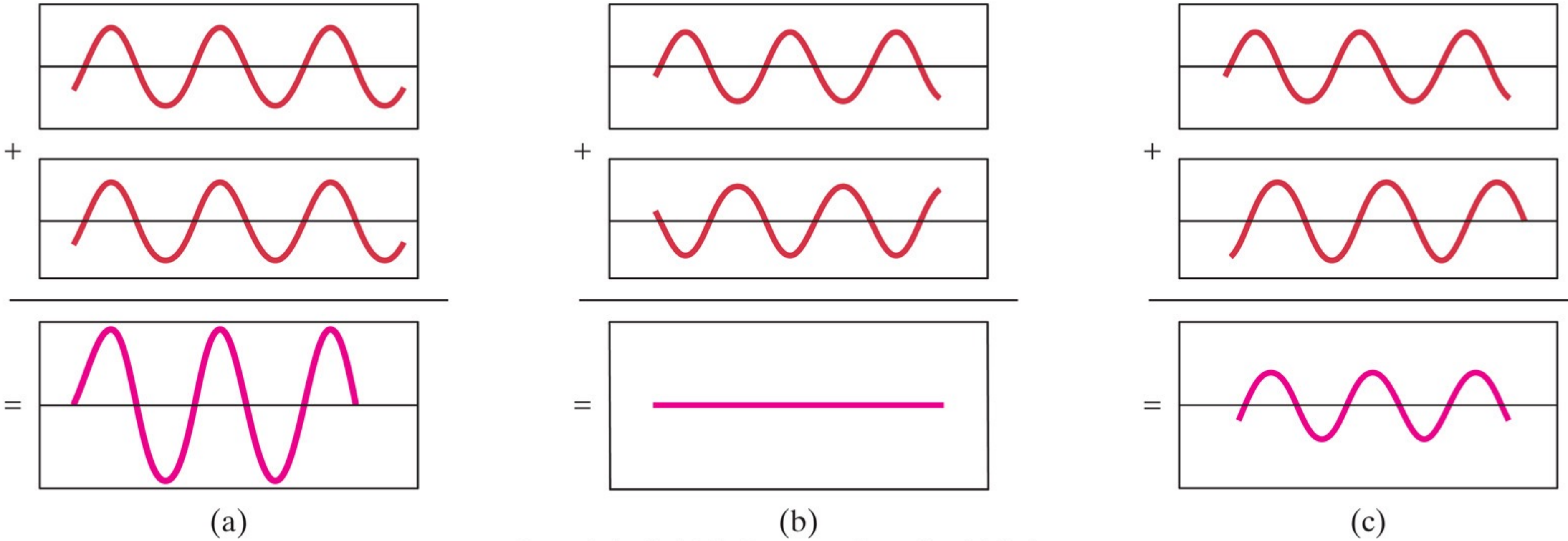
In the figure below, (a) exhibits **destructive** interference and (b) exhibits **constructive** interference.



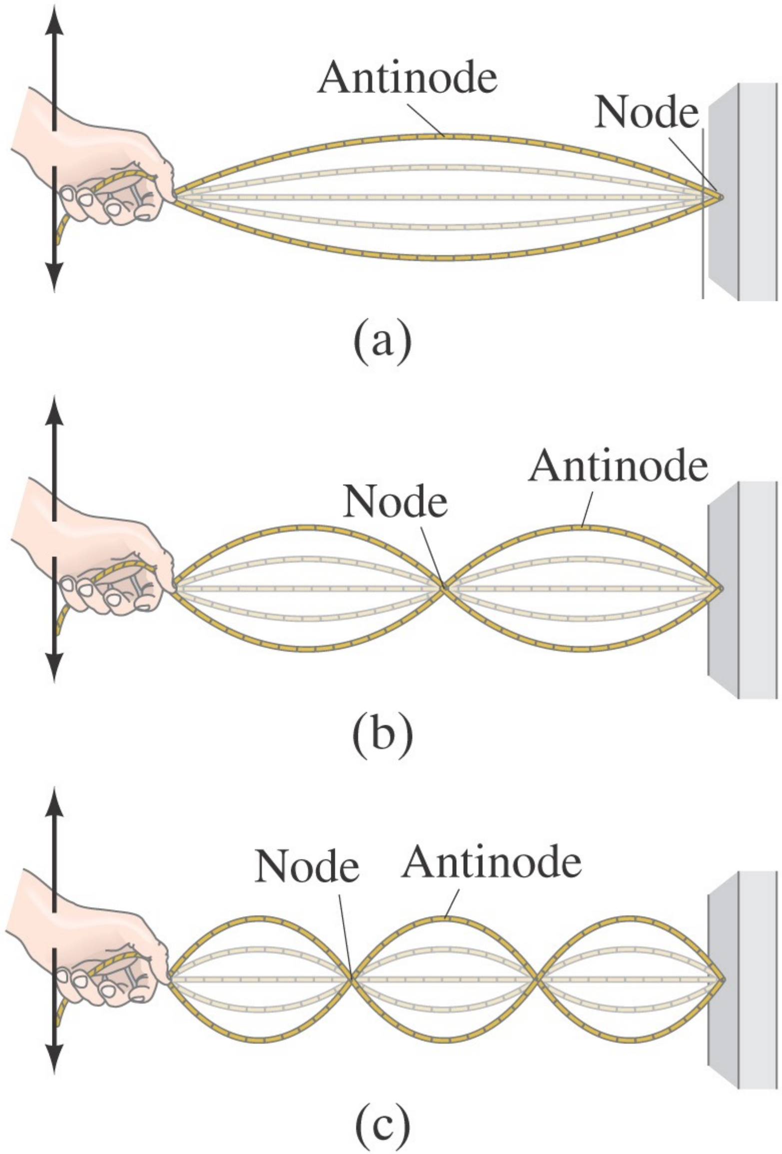
Interference

These figures show the sum of two waves.
(a) they add **constructively**, the two waves are said to be **in phase**;
(b) they add **destructively,** the two waves are said to be **out phase**;
(c) they add partially destructively.

If the amplitudes of two interfering waves are not equal, fully destructive interference does not occur.



Standing Waves, Resonance



If you shake one end of a cord and the other is kept fixed, waves will travel in both directions.

If you vibrate at the right frequency, the two traveling waves will interfere in such a way that a large-amplitude **standing wave** is produced.

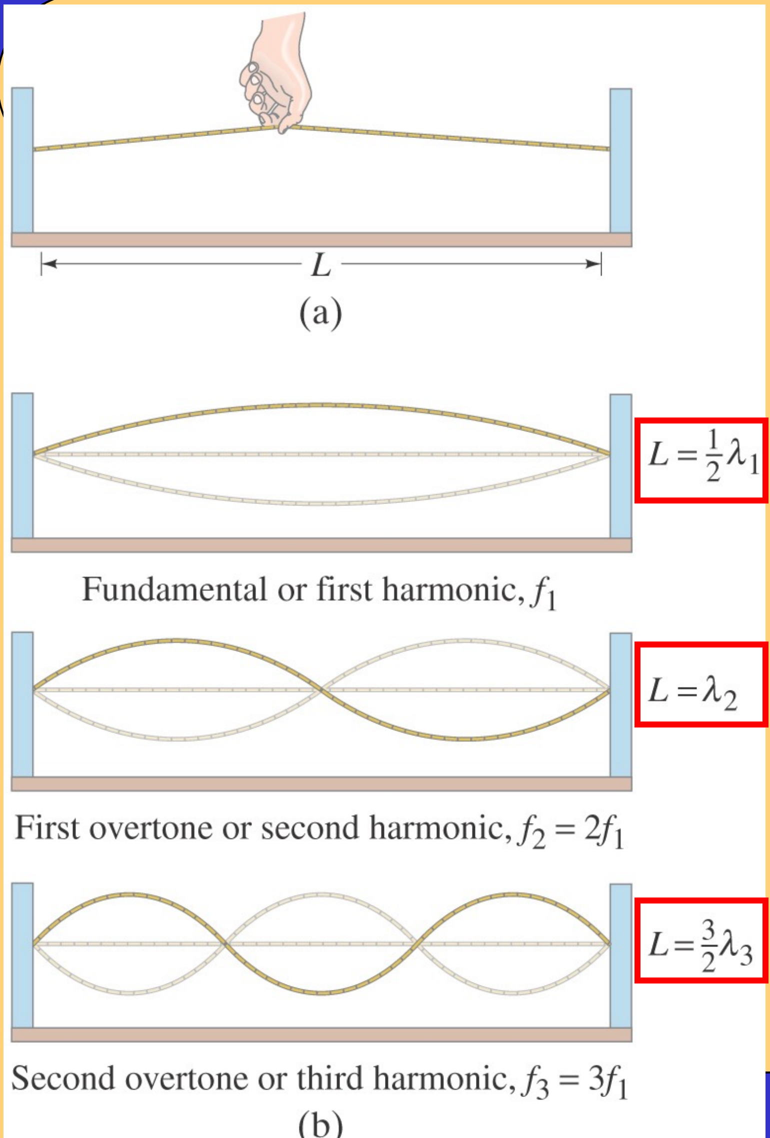
Standing waves do not appear to travel.

Nodes are points of destructive interference, where the cord remains still all the time

Antinodes are points of constructive interference, where the cord oscillates with maximum amplitude.

(a) Lowest frequency
(b) Twice the lowest frequency
(c) Three times the lowest frequency

Standing Waves, Resonance



The frequencies at which standing waves are produced are called **natural frequencies** or **resonant frequencies**.

The lowest frequency is called fundamental frequency. It corresponds to one antinode (or loop). The next mode of vibration has two loops. See wavelength in figure.

Natural frequencies are also called **harmonics**.
First harmonic = **fundamental**
Second harmonic or first **overtone** = twice the fundamental; etc

For a vibrating string **overtones** are whole-number (integral) multiples of the fundamental.

Standing Waves, Resonance

In general we can write: $L = \frac{n\lambda_n}{2} \quad n = 1,2,3,\ldots$

Therefore, the wavelengths and frequencies of standing waves are:

$$\lambda_n = \frac{2L}{n}, \qquad n = 1, 2, 3, \cdots$$

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = nf_1, \qquad n = 1, 2, 3, \cdots$$

Because a standing wave is equivalent to two traveling waves moving in opposite directions, the concept of wave velocity still makes sense

$$v = \sqrt{\frac{F_T}{m/L}}$$

Ex 11-14 A piano string is 1.10 m long and has a mass of 9.00 g. (a) How much tension must the string be under if it is to vibrate at a fundamental frequency of 131 Hz? (b) What are the frequencies of the first four harmonics? (a) 679 N (b) 131, 262, 393, and 524 Hz