Suggestions for exercises

- **EXERCISE 1 (code provided)**

(1) Compute NPCs and the level spacing distribution for a single matrix from a GOE.
(2) Average the values of NPCs for various realizations of matrices and verify that the fluctuations around the expected value dim/3 decreases as we increase the number of realizations.

A matrix from a GOE is obtained as follows:
(i) Write a matrix where all elements are random numbers from a Gaussian distribution with mean 0 and variance 1.
(ii) Add this matrix to its transpose to symmetrize it. The result is a matrix from a GOE

- **The Code for item (1)**

```
(* **************************************************** *)
Clear[dim];
dim = 1000;

(* GOE *)
Clear[rm, goe, Egoe, Vecgoe];

rm = Table[Table[RandomReal[NormalDistribution[0, 1]], {j, 1, dim}], {k, 1, dim}];
goe = rm + Transpose[rm];
Egoe = Eigenvalues[goe];
Vecgoe = Eigenvectors[goe];


(*****   NPC  *****)

Clear[NPCgoe];
Do[
  NPCgoe[j] = 1 / Sum[Abs[Vecgoe[[j, k]] ]^4, {k, 1, dim}];
  , {j, 1, dim}];

Clear[tGOE, lgoe];
tGOE = Table[{Egoe[[j]] / 10., NPCgoe[j]}, {j, 1, dim}];
Print["NPC vs Energy for a single matrix from a GOE"];
lgoe = ListPlot[tGOE, PlotRange → {0, dim / 2}, PlotStyle → Black,
  LabelStyle → Directive[Black, Bold, Medium], AxesLabel → {"E", "NPC"}]



(*****   Level spacing distribution  *****)

(* LEVEL SPACINGS OF THE UNFOLDED SPECTRUM *)
(* Order the eigenvalues from lowest to highest values *)
Clear[Ener];
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```
Ener = Sort[Table[Egoe[[k]], {k, 1, dim}]];

(* Discard ~10% of the eigenvalues located at the borders of the spectrum *)
Clear[percentage, half, spacing];
percentage = 0.1 dim;
half = Floor[percentage / 2.];
Do[
  Clear[average];
  (* Compute the neighboring level spacings
   for the remaining eigenvalues after unfolding them *)
  (* Unfolding here means that the average of each
    group of 10 level spacings = 1 *)
  average = (Ener[[half + 10 j]] - Ener[[half + 10 (j - 1)]]) / 10.;
  Do[spacing[i] = (Ener[[half + i]] - Ener[[(half - 1) + i]]) / average;
   , {i, 1 + 10 (j - 1), 10 j}];
  , {j, 1, Floor[(dim - percentage) / 10]}];

(* HISTOGRAM *)
Clear[spcmin, spcmax, bin, Nofbins];
spcmin = 0.;
spcmax = 8.;
bin = 0.1;
Nofbins = IntegerPart[(spcmax - spcmin) / bin];

Clear[SPChist, Nhist];
SPChist[1] = spcmin;
Do[SPChist[i + 1] = SPChist[i] + bin, {i, 1, Nofbins}];
Do[Nhist[j] = 0., {j, 1, Nofbins}];

(* Nhist[j] gives how many spacings we
 have in the interval SPChist[j+1] and SPChist[j] *)
Do[
  Do[
    If[SPChist[j] < spacing[k] ≤ SPChist[j + 1], Nhist[j] = Nhist[j] + 1];
    , {j, 1, Nofbins}];
  , {k, 1, 10 Floor[(dim - percentage) / 10]}];

(* Normalization *)
Clear[Norma];
Norma = Sum[bin Nhist[j], {j, 1, Nofbins}];
Do[Nhist[j] = Nhist[j] / Norma, {j, 1, Nofbins}];

(* ListPlot with the obtained data *)
Clear[jj, nl];
jj = 0;
nl = {};
Do[jj += 1;
  nl = Append[nl, {SPChist[jj], Nhist[jj]}];
  nl = Append[nl, {SPChist[jj + 1], Nhist[jj]}];
  , {j, 1, Nofbins - 1}];
DataPlot =
```

```
  ListPlot[nl, Joined → True, PlotRange → {{0, 8}, {0, 1}}, PlotStyle → {Black, Thick},
   LabelStyle → Directive[Black, Bold, Medium], AxesLabel → {"s", "P"}];
(* Theoretical curves *)
WignerDyson = Plot[Pi s / 2. Exp[-Pi s^2 / 4.],
   {s, 0, 8}, PlotRange → {0, 1}, PlotStyle → {Red, Thick},
   LabelStyle → Directive[Black, Bold, Medium], AxesLabel → {"s", "P"}];
Poisson = Plot[Exp[-s], {s, 0, 8}, PlotRange → {0, 1}, PlotStyle → {Blue, Thick},
   LabelStyle → Directive[Black, Bold, Medium], AxesLabel → {"s", "P"}];
(* The three curves together *)
Print["Level spacing distribution for a single matrix from a GOE"];
Show[{DataPlot, WignerDyson, Poisson}, PlotRange → {{0, 4}, {0, 1.1}}]
```

The Code for item (2)

```
(* ***************************************************** *)
(*****   NPC  averaged over 2 realizations *****)

Clear[dim, tot, Eave, NPCave];
dim = 1000;
tot = 2;
Do[
  Eave[j] = 0;
  NPCave[j] = 0;
  , {j, 1, dim}];

Do[
  Clear[rm, goe, Egoe, keepOrder, Vecgoe];
  rm =
   Table[Table[RandomReal[NormalDistribution[0, 1]], {j, 1, dim}], {k, 1, dim}];
  goe = rm + Transpose[rm];
  Egoe = Sort[Eigenvalues[goe]];
  keepOrder = Ordering[Egoe];
  Vecgoe = Eigenvectors[goe];

  Do[
   NPCgoe = 1 / Sum[Abs[Vecgoe[[keepOrder[[j]], k]] ]^4, {k, 1, dim}];
   NPCave[j] = NPCave[j] + NPCgoe;
   Eave[j] = Eave[j] + Egoe[[j]];
   , {j, 1, dim}];

  , {kk, 1, tot}];

Clear[tGOE, lgoe];
tGOE = Table[{Eave[j] / tot, NPCave[j] / tot}, {j, 1, dim}];
Print["NPC vs Energy for ", tot, " realizations of matrices from a GOE"];
lgoe = ListPlot[tGOE, PlotRange → {0, dim / 2}, PlotStyle → Black,
  LabelStyle → Directive[Black, Bold, Medium], AxesLabel → {"E", "NPC"}]
```

```
(* ****************************************************** *)
(*****   NPC  averaged over 10 realizations *****)

Clear[dim, tot, Eave, NPCave];
dim = 1000;
tot = 10;
Do[
  Eave[j] = 0;
  NPCave[j] = 0;
  , {j, 1, dim}];

Do[
  Clear[rm, goe, Egoe, keepOrder, Vecgoe];
  rm =
   Table[Table[RandomReal[NormalDistribution[0, 1]], {j, 1, dim}], {k, 1, dim}];
  goe = rm + Transpose[rm];
  Egoe = Sort[Eigenvalues[goe]];
  keepOrder = Ordering[Egoe];
  Vecgoe = Eigenvectors[goe];

  Do[
   NPCgoe = 1 / Sum[Abs[Vecgoe[[keepOrder[[j]], k]] ]^4, {k, 1, dim}];
   NPCave[j] = NPCave[j] + NPCgoe;
   Eave[j] = Eave[j] + Egoe[[j]];
   , {j, 1, dim}];

  , {kk, 1, tot}];

Clear[tGOE, lgoe];
tGOE = Table[{Eave[j] / tot, NPCave[j] / tot}, {j, 1, dim}];
Print["NPC vs Energy for ", tot, " realizations of matrices from a GOE"];
lgoe = ListPlot[tGOE, PlotRange → {0, dim / 2}, PlotStyle → Black,
  LabelStyle → Directive[Black, Bold, Medium], AxesLabel → {"E", "NPC"}]
```

EXERCISE 2
(code not provided -- it is a variation of those already given)

Compare the results for NPC for a GOE with the results for a system with two-body interactions.
Consider for the latter :
(i) One of the systems used in the paper;
(ii) The same system chosen in (i) but with random values for the coupling strengths

EXERCISE 3
(code not provided -- it is a variation of those already given)

Consider one of the systems with two-body interactions described in the paper. Divide its spectrum in 3 (or more) equal parts and compute the level spacing distribution for each part. Verify that the distributions involving spacings from the border of the spectrum are more distant from a Wigner-Dyson distribution than those involving spacings from the middle of the spectrum.

EXERCISE 4

(1) Select one of the systems described in the paper and vary the amplitude of the integrability breaking term (IBT) [the value of ϵ_d in the case of the defect or the value of α in the case of next-nearest-neighbor interactions]. Study how the proximity to the Wigner-Dyson distribution depends on the value of IBT.

(2) Quantify how close you are from a Wigner-Dyson distribution by finding the peak of your distribution. The closer it is from ~0.798, the closer you are from a Wigner-Dyson distribution. Make a plot for VALUE_of_PEAK vs VALUE_of_IBT